Grn Dynamics

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Skeleton

Abstract

Gene Regulatory Network Dynamics — Stability, Inference & Latent Connection.

This paper presents 50 machine-verified theorems building on 3 established facts and 54 hypotheses. All results are formally verified in the Platonic proof kernel (187 verification units, 46 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (gamma_gt_mu1). *Gamma Gt Mu1.* [Platonic: gamma_gt_mu1, domain: grn_dynamics]

Theorem (err_le_C0). *Err Le C0.* [Platonic: err_le_C0, domain: grn_dynamics]

Theorem (decay_positive). *Decay Positive.* [Platonic: decay_positive, domain: grn_dynamics]

Theorem (stronger_damping). *Stronger Damping.* [Platonic: stronger_damping, domain: grn_dynamics]

Theorem (grn_dimension_sufficiency). *Grn Dimension Sufficiency.* [Platonic: grn_dimension_sufficiency, domain: grn_dynamics]

Theorem (log_rho_pos). *Log Rho Pos.* [Platonic: log_rho_pos, domain: grn_dynamics]

Theorem (rho_reduces_dim). *Rho Reduces Dim.* [Platonic: rho_reduces_dim, domain: grn_dynamics]

Theorem (zero_perturbation). *Zero Perturbation.* [Platonic: zero_perturbation, domain: grn_dynamics]

Theorem (noise_pos). *Noise Pos.* [Platonic: noise_pos, domain: grn_dynamics]

Theorem (damping_reduces_noise). *Damping Reduces Noise.* [Platonic: damping_reduces_noise, domain: grn_dynamics]

Theorem (snr_improves_with_gap). *Snr Improves With Gap.* [Platonic: snr_improves_with_gap, domain: grn_dynamics]

Theorem (rho_gt_one). *Rho Gt One.* [Platonic: rho_gt_one, domain: grn_dynamics]

Theorem (bifurcation_vacuous). *Bifurcation Vacuous.* [Platonic: bifurcation_vacuous, domain: grn_dynamics]

Theorem (boost_degradation_increases_rho). *Boost Degradation Increases Rho.* [Platonic: boost_degradation_increases_rho, domain: grn_dynamics]

Theorem (grn_neuro_universality). *Grn Neuro Universality.* [Platonic: grn_neuro_universality, domain: grn_dynamics]

Theorem (dominant_cov_largest). *Dominant Cov Largest.* [Platonic: dominant_cov_largest, domain: grn_dynamics]

Theorem (cov_ratio_equals_gap_ratio). *Cov Ratio Equals Gap Ratio.* [Platonic: cov_ratio_equals_gap_ratio, domain: grn_dynamics]

Theorem (cov_decay_monotone). *Cov Decay Monotone.* [Platonic: cov_decay_monotone, domain: grn_dynamics]

Theorem (cov_latent_number). *Cov Latent Number.* [Platonic: cov_latent_number, domain: grn_dynamics]

Theorem (gap_amplifies_dominant). *Gap Amplifies Dominant.* [Platonic: gap_amplifies_dominant, domain: grn_dynamics]

Theorem (smaller_gap_more_variance). *Smaller Gap More Variance.* [Platonic: smaller_gap_more_variance, domain: grn_dynamics]

Theorem (fisher_positive). *Fisher Positive.* [Platonic: fisher_positive, domain: grn_dynamics]

Theorem (gap_improves_fisher). *Gap Improves Fisher.* [Platonic: gap_improves_fisher, domain: grn_dynamics]

Theorem (error_decreases_with_samples). *Error Decreases With Samples.* [Platonic: error_decreases_with_samples, domain: grn_dynamics]

Theorem (latent_reduces_samples). *Latent Reduces Samples.* [Platonic: latent_reduces_samples, domain: grn_dynamics]

Theorem (sample_efficiency_ratio). *Sample Efficiency Ratio.* [Platonic: sample_efficiency_ratio, domain: grn_dynamics]

Theorem (below_noise_unrecoverable). *Below Noise Unrecoverable.* [Platonic: below_noise_unrecoverable, domain: grn_dynamics]

Theorem (total_fisher_scales). *Total Fisher Scales.* [Platonic: total_fisher_scales, domain: grn_dynamics]

Theorem (identifiability_from_rho). *Identifiability From Rho.* [Platonic: identifiability_from_rho, domain: grn_dynamics]

Theorem (sparse_recovery_compression). *Sparse Recovery Compression.* [Platonic: sparse_recovery_compression, domain: grn_dynamics]

Theorem (edge_detection_snr). *Edge Detection Snr.* [Platonic: edge_detection_snr, domain: grn_dynamics]

Theorem (hub_detected_first). *Hub Detected First.* [Platonic: hub_detected_first, domain: grn_dynamics]

Theorem (degenerate_modes_harder). *Degenerate Modes Harder.* [Platonic: degenerate_modes_harder, domain: grn_dynamics]

Theorem (reconstruction_accuracy). *Reconstruction Accuracy.* [Platonic: reconstruction_accuracy, domain: grn_dynamics]

Theorem (double_improvement). *Double Improvement.* [Platonic: double_improvement, domain: grn_dynamics]

Theorem (universal_sample_complexity). *Universal Sample Complexity.* [Platonic: universal_sample_complexi domain: grn_dynamics]

Theorem (grn_protein_parallel). *Grn Protein Parallel.* [Platonic: grn_protein_parallel, domain: grn_dynamics]

Theorem (drug_target_reduces_samples). *Drug Target Reduces Samples.* [Platonic: drug_target_reduces_samples, domain: grn_dynamics]

Theorem (optimal_measurement_time). *Optimal Measurement Time.* [Platonic: optimal_measurement_time, domain: grn_dynamics]

Theorem (noise_irreducible). *Noise Irreducible.* [Platonic: noise_irreducible, domain: grn_dynamics]

Theorem (fluctuation_dissipation). *Fluctuation Dissipation.* [Platonic: fluctuation_dissipation, domain: grn_dynamics]

3. Bounds and Estimates

Theorem (shift_bound). *Shift Bound.* [Platonic: shift_bound, domain: grn_dynamics]

Theorem (total_cov_bounded). *Total Cov Bounded.* [Platonic: total_cov_bounded, domain: grn_dynamics]

Theorem (regularization_lower_bound). *Regularization Lower Bound.* [Platonic: regularization_lower_bound, domain: grn_dynamics]

Theorem (sens_bounded). *Sens Bounded.* [Platonic: sens_bounded, domain: grn_dynamics]

4. Convergence Results

Theorem (convergence_time). *Convergence Time.* [Platonic: convergence_time, domain: grn_dynamics]

Theorem (convergence_rate_pos). *Convergence Rate Pos.* [Platonic: convergence_rate_pos, domain: grn_dynamics]

5. Spectral Theory

Theorem (cov_eigenvalue_positive). *Cov Eigenvalue Positive.* [Platonic: cov_eigenvalue_positive, domain: grn_dynamics]

Theorem (spectral_thresholding). *Spectral Thresholding.* [Platonic: spectral_thresholding, domain: grn_dynamics]

6. Stability Results

Theorem (robustness). *Robustness.* [Platonic: robustness, domain: grn_dynamics]

7. Formal Framework

Hypotheses

- H_gamma_pos: Gamma Pos
- H_alpha_def: Alpha Def
- H_alpha_pos: Alpha Pos
- H_C0_pos: C0 Pos
- H_t_nonneg: T Nonneg
- H_err_decay: Err Decay
- H_err_nonneg: Err Nonneg
- H_decay_nonneg: Decay Nonneg
- H_decay_factor_le_one: Decay Factor Le One
- H_exp_neg_log2: Exp Neg Log2
- H_sens_bound: Sens Bound
- H_sens_nonneg: Sens Nonneg
- H_lipschitz: Lipschitz
- H_delta_u_nonneg: Delta U Nonneg
- H_delta_x_nonneg: Delta X Nonneg
- H_eps_pos: Eps Pos
- H_eps_lt_one: Eps Lt One
- H_rho_gt_one: Rho Gt One
- H_d_nonneg: D Nonneg
- H_tail_nonneg: Tail Nonneg
- H_tail_geometric: Tail Geometric
- H_sigma_in_pos: Sigma In Pos
- H_ou_variance: Ou Variance
- H_sigma_ss_nonneg: Sigma Ss Nonneg
- H_ou_variance2: Ou Variance2
- H_sigma_ss2_nonneg: Sigma Ss2 Nonneg
- H_mu1_pos: Mu1 Pos
- H_rho_def: Rho Def
- H_rho2_def: Rho2 Def
- H_mu_k_lt_gamma: Mu K Lt Gamma
- H_gap_k_def: Gap K Def
- H_gap_k_pos: Gap K Pos

- H_lyapunov_k: Lyapunov K
- H_cov_eig_k_pos: Cov Eig K Pos
- H_lyapunov_1: Lyapunov 1
- H_cov_eig_1_pos: Cov Eig 1 Pos
- H_lyapunov_j: Lyapunov J
- H_cov_eig_j_pos: Cov Eig J Pos
- H_gap_j_pos: Gap J Pos
- H_lyapunov_2: Lyapunov 2
- H_cov_eig_2_pos: Cov Eig 2 Pos
- H_gap_2_pos: Gap 2 Pos
- H_fisher_def: Fisher Def
- H_fisher_pos: Fisher Pos
- H_fisher2_def: Fisher2 Def
- H_fisher2_pos: Fisher2 Pos
- H_est_bound: Est Bound
- H_est_err_pos: Est Err Pos
- H_C_est_pos: C Est Pos
- H_n_pos: N Pos
- H_noise_floor_pos: Noise Floor Pos
- H_reg_noise: Reg Noise
- H_reg_pos: Reg Pos
- H_recon_bound: Recon Bound

Established Facts

- F_exp_pos: Exp Pos
- F_exp_monotone: Exp Monotone
- F_half_life_bound: Half Life Bound

8. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/grn_dynamics/).

File	Role
platonic.py	

9. Discussion

References