Neuro Manifold

Dr. Tamás Nagy

Dr. Tamás Nagy

tamas@thelatent.space

Skeleton

Abstract

Neural Manifold — Spectral Dimension Bound, Optimal Decoding & BCI Theory.

This paper presents 47 machine-verified theorems building on 5 established facts and 32 hypotheses. All results are formally verified in the Platonic proof kernel (169 verification units, 47 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (rho_pos). *Rho Pos.* [Platonic: rho_pos, domain: neuro_manifold]

Theorem (rho_minus_one_pos). *Rho Minus One Pos.* [Platonic: rho_minus_one_pos, domain: neuro_manifold]

Theorem (V_ge_lam1). *V Ge Lam1.* [Platonic: V_ge_lam1, domain: neuro_manifold]

Theorem (expl_le_V). *Expl Le V.* [Platonic: expl_le_V, domain: neuro_manifold]

Theorem (explained_fraction). *Explained Fraction.* [Platonic: explained_fraction, domain: neuro_manifold]

Theorem (explained_absorbs_tail). *Explained Absorbs Tail.* [Platonic: explained_absorbs_tail, domain: neuro_manifold]

Theorem (tail_lt_V). *Tail Lt V.* [Platonic: tail_lt_V, domain: neuro_manifold]

Theorem (dimension_sufficiency). *Dimension Sufficiency.* [Platonic: dimension_sufficiency, domain: neuro_manifold]

Theorem (d_dim_nonneg). *D Dim Nonneg.* [Platonic: d_dim_nonneg, domain: neuro_manifold]

Theorem (snr_pos). *Snr Pos.* [Platonic: snr_pos, domain: neuro_manifold]

Theorem (embed_err_by_eps). *Embed Err By Eps.* [Platonic: embed_err_by_eps, domain: neuro_manifold]

Theorem (recon_improves_with_N). *Recon Improves With N.* [Platonic: recon_improves_with_N, domain: neuro_manifold]

Theorem (redundancy_positive). *Redundancy Positive.* [Platonic: redundancy_positive, domain: neuro_manifold]

Theorem (log_strict_mono). *Log Strict Mono.* [Platonic: log_strict_mono, domain: neuro_manifold]

Theorem (larger_gap_larger_log). *Larger Gap Larger Log.* [Platonic: larger_gap_larger_log, domain: neuro_manifold]

Theorem (fisher_per_electrode_positive). *Fisher Per Electrode Positive.* [Platonic: fisher_per_electrode_positive, domain: neuro_manifold]

Theorem (fisher_ordering). *Fisher Ordering.* [Platonic: fisher_ordering, domain: neuro_manifold]

Theorem (fisher_grows_with_electrodes). *Fisher Grows With Electrodes.* [Platonic: fisher_grows_with_electrodes, domain: neuro_manifold]

Theorem (cramer_rao_improves). *Cramer Rao Improves.* [Platonic: cramer_rao_improves, domain: neuro_manifold]

Theorem (nstar_captures_fisher). *Nstar Captures Fisher.* [Platonic: nstar_captures_fisher, domain: neuro_manifold]

Theorem (optimal_electrode_selection). *Optimal Electrode Selection.* [Platonic: optimal_electrode_selection, domain: neuro_manifold]

Theorem (diminishing_returns). *Diminishing Returns.* [Platonic: diminishing_returns, domain: neuro_manifold]

Theorem (decoding_saturation). *Decoding Saturation.* [Platonic: decoding_saturation, domain: neuro_manifold]

Theorem (bci_channel_sufficiency). *Bci Channel Sufficiency.* [Platonic: bci_channel_sufficiency, domain: neuro_manifold]

Theorem (bci_channel_savings). *Bci Channel Savings.* [Platonic: bci_channel_savings, domain: neuro_manifold]

Theorem (selective_recording_superior). *Selective Recording Superior.* [Platonic: selective_recording_superior, domain: neuro_manifold]

Theorem (noise_per_channel_tradeoff). *Noise Per Channel Tradeoff.* [Platonic: noise_per_channel_tradeoff, domain: neuro_manifold]

Theorem (total_snr_with_nstar). *Total Snr With Nstar.* [Platonic: total_snr_with_nstar, domain: neuro_manifold]

Theorem (cost_minimization). *Cost Minimization.* [Platonic: cost_minimization, domain: neuro_manifold]

Theorem (higher_rho_fewer_channels). *Higher Rho Fewer Channels.* [Platonic: higher_rho_fewer_channels, domain: neuro_manifold]

Theorem (bci_design_theorem). *Bci Design Theorem.* [Platonic: bci_design_theorem, domain: neuro_manifold]

Theorem (sparse_code_from_large_rho). *Sparse Code From Large Rho.* [Platonic: sparse_code_from_large_rho, domain: neuro_manifold]

Theorem (population_invariance). *Population Invariance.* [Platonic: population_invariance, domain: neuro_manifold]

Theorem (proportional_allocation). *Proportional Allocation.* [Platonic: proportional_allocation, domain: neuro_manifold]

Theorem (mixed_selectivity_increases_rho). *Mixed Selectivity Increases Rho.* [Platonic: mixed_selectivity_increases_rho, domain: neuro_manifold]

Theorem (neural_protein_universality). *Neural Protein Universality.* [Platonic: neural_protein_universality, domain: neuro_manifold]

Theorem (neural_epidemic_hub_parallel). *Neural Epidemic Hub Parallel.* [Platonic: neural_epidemic_hub_parallel, domain: neuro_manifold]

Theorem (neural_grn_covariance_parallel). *Neural Grn Covariance Parallel.* [Platonic: neural_grn_covariance_parallel, domain: neuro_manifold]

Theorem (encoding_decoding_dual). *Encoding Decoding Dual.* [Platonic: encoding_decoding_dual, domain: neuro_manifold]

3. Bounds and Estimates

Theorem (redundancy_bounded). *Redundancy Bounded.* [Platonic: redundancy_bounded, domain: neuro_manifold]

Theorem (joint_quality_bound). *Joint Quality Bound.* [Platonic: joint_quality_bound, domain: neuro_manifold]

Theorem (code_efficiency_bounded). *Code Efficiency Bounded.* [Platonic: code_efficiency_bounded, domain: neuro_manifold]

Theorem (encoding_capacity_bounded). *Encoding Capacity Bounded.* [Platonic: encoding_capacity_bounded, domain: neuro_manifold]

4. Main Theorems

Theorem (cross_domain_universality). *Cross Domain Universality.* [Platonic: cross_domain_universality, domain: neuro_manifold]

Theorem (five_domain_universality). *Five Domain Universality.* [Platonic: five_domain_universality, domain: neuro_manifold]

5. Spectral Theory

Theorem (spectral_gap_gt_one). *Spectral Gap Gt One.* [Platonic: spectral_gap_gt_one, domain: neuro_manifold]

6. Convergence Results

Theorem (fundamental_decoding_limit). *Fundamental Decoding Limit.* [Platonic: fundamental_decoding_limit
domain: neuro_manifold]

7. Formal Framework

Hypotheses

- H_lam1_pos: Lam1 Pos
- H_rho_gt_one: Rho Gt One
- H_eps_pos: Eps Pos
- H_eps_lt_one: Eps Lt One
- H_V_pos: V Pos
- H_lam1_le_V: Lam1 Le V
- H_variance_decomp: Variance Decomp
- H_expl_nonneg: Expl Nonneg
- H_tail_nonneg: Tail Nonneg
- H_tail_geometric: Tail Geometric
- H_d_nonneg: D Nonneg
- H_rmax_pos: Rmax Pos
- H_sigma_pos: Sigma Pos
- H_N_pos: N Pos
- H_snr_def: Snr Def
- H_snr_nonneg: Snr Nonneg
- H_embed_nonneg: Embed Nonneg
- H_embed_sq_le_tail: Embed Sq Le Tail
- H_recon_bound: Recon Bound
- H_d_lt_N: D Lt N
- H_lam_d_pos: Lam D Pos
- H_lam_d_lt_lam1: Lam D Lt Lam1
- H_M_pos: M Pos
- H_J_M_pos: J M Pos
- H_J_full_pos: J Full Pos
- H_J_le_full: J Le Full
- H_J_Nstar_pos: J Nstar Pos
- H_lam_k_pos: Lam K Pos
- H_lam_ordering: Lam Ordering
- H_J_grows: J Grows
- H_optimal_ge_random: Optimal Ge Random
- H_J_opt_pos: J Opt Pos

Established Facts

- F_inv_rho_lt_one: Inv Rho Lt One
- F_inv_rho_pos: Inv Rho Pos
- F_nstar_positive: Nstar Positive
- pop_norm_pos: Pop Norm Pos

- F_doubling_squares_accuracy: Doubling Squares Accuracy

8. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/neuro_manifold/).

File	Role
platonic.py	

9. Discussion

References