

# Wright Fisher

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Skeleton

## Abstract

Wright-Fisher Population Genetics — Spectral Convergence + Latent.

This paper presents 24 machine-verified theorems building on 0 established facts and 35 hypotheses. All results are formally verified in the Platonic proof kernel (91 verification units, 21 proved statements) and exportable to Lean 4.

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## 1. Introduction

## 2. Further Results

**Theorem** (gap\_pos). *Gap Pos.* [Platonic: gap\_pos, domain: wright\_fisher]

**Theorem** (gap\_lt\_one). *Gap Lt One.* [Platonic: gap\_lt\_one, domain: wright\_fisher]

**Theorem** (lam2\_from\_gap). *Lam2 From Gap.* [Platonic: lam2\_from\_gap, domain: wright\_fisher]

**Theorem** (larger\_pop\_slower\_mixing). *Larger Pop Slower Mixing.* [Platonic: larger\_pop\_slower\_mixing, domain: wright\_fisher]

**Theorem** (mixing\_monotone). *Mixing Monotone.* [Platonic: mixing\_monotone, domain: wright\_fisher]

**Theorem** (q\_nonneg). *Q Nonneg.* [Platonic: q\_nonneg, domain: wright\_fisher]

**Theorem** (q\_le\_one). *Q Le One.* [Platonic: q\_le\_one, domain: wright\_fisher]

**Theorem** (het\_nonneg). *Het Nonneg.* [Platonic: het\_nonneg, domain: wright\_fisher]

**Theorem** (het\_le\_half). *Het Le Half.* [Platonic: het\_le\_half, domain: wright\_fisher]

**Theorem** (weak\_selection). *Weak Selection.* [Platonic: weak\_selection, domain: wright\_fisher]

**Theorem** (selection\_widens\_gap). *Selection Widens Gap.* [Platonic: selection\_widens\_gap, domain: wright\_fisher]

**Theorem** (het\_decreases). *Het Decreases.* [Platonic: het\_decreases, domain: wright\_fisher]

**Theorem** (large\_pop\_preserves\_het). *Large Pop Preserves Het.* [Platonic: large\_pop\_preserves\_het, domain: wright\_fisher]

**Theorem** (mutation\_restores\_het). *Mutation Restores Het.* [Platonic: mutation\_restores\_het, domain: wright\_fisher]

**Theorem** (wf\_rho\_gt\_one). *Wf Rho Gt One.* [Platonic: wf\_rho\_gt\_one, domain: wright\_fisher]

**Theorem** (wf\_dimension\_sufficiency). *Wf Dimension Sufficiency.* [Platonic: wf\_dimension\_sufficiency, domain: wright\_fisher]

**Theorem** (selection\_increases\_rho). *Selection Increases Rho.* [Platonic: selection\_increases\_rho, domain: wright\_fisher]

**Theorem** (pop\_size\_scales\_time). *Pop Size Scales Time.* [Platonic: pop\_size\_scales\_time, domain: wright\_fisher]

**Theorem** (tv\_decays). *Tv Decays.* [Platonic: tv\_decays, domain: wright\_fisher]

### 3. Bounds and Estimates

**Theorem** (drift\_gap\_bounded). *Drift Gap Bounded.* [Platonic: drift\_gap\_bounded, domain: wright\_fisher]

**Theorem** (fixation\_bounded). *Fixation Bounded.* [Platonic: fixation\_bounded, domain: wright\_fisher]

### 4. Main Theorems

**Theorem** (wf\_cross\_domain). *Wf Cross Domain.* [Platonic: wf\_cross\_domain, domain: wright\_fisher]

### 5. Spectral Theory

**Theorem** (eigenvalue\_ratio). *Eigenvalue Ratio.* [Platonic: eigenvalue\_ratio, domain: wright\_fisher]

### 6. Existence and Uniqueness

**Theorem** (eq\_freq\_exists). *Eq Freq Exists.* [Platonic: eq\_freq\_exists, domain: wright\_fisher]

### 7. Formal Framework

#### Hypotheses

- H\_lam1\_eq\_one: Lam1 Eq One
- H\_lam2\_pos: Lam2 Pos
- H\_lam2\_lt\_one: Lam2 Lt One
- H\_gap\_def: Gap Def
- H\_N\_pos: N Pos
- H\_drift\_gap: Drift Gap

- `H_drift_gap_lower`: Drift Gap Lower
- `H_drift_gap2`: Drift Gap2
- `H_gap2_pos`: Gap2 Pos
- `H_eps_pos`: Eps Pos
- `H_eps_lt_one`: Eps Lt One
- `H_t_nonneg`: T Nonneg
- `H_tv_decay`: Tv Decay
- `H_tv_nonneg`: Tv Nonneg
- `H_p_nonneg`: P Nonneg
- `H_p_le_one`: P Le One
- `H_q_def`: Q Def
- `H_het_def`: Het Def
- `H_kimura_neutral`: Kimura Neutral
- `H_s_bounded`: S Bounded
- `H_mu_pos`: Mu Pos
- `H_mu_lt_one`: Mu Lt One
- `H_selection_gap`: Selection Gap
- `H_gap_s_pos`: Gap S Pos
- `H_eq_interior`: Eq Interior
- `H_het_loss`: Het Loss
- `H_delta_het_nonneg`: Delta Het Nonneg
- `H_mu_gain_pos`: Mu Gain Pos
- `H_rho_def`: Rho Def
- `H_rho_pos`: Rho Pos
- `H_tail_wf_nonneg`: Tail Wf Nonneg
- `H_tail_wf_geometric`: Tail Wf Geometric
- `H_rho_s_def`: Rho S Def
- `H_lam2_s_pos`: Lam2 S Pos
- `H_rho_s_pos`: Rho S Pos

## 8. Proof Architecture

All proofs are implemented in the Platonic kernel (`elysium/fields/wright_fisher/`).

File	Role
<code>platonic.py</code>	

## 9. Discussion

### References