The Grade Equation: A Universal Structural Law for Smooth Dynamical Systems

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Executive Summary (Non-Technical)

Einstein's field equations tell us that the geometry of spacetime equals its matter content. Maxwell's equations tell us that electromagnetic fields equal their sources. The Schrödinger equation tells us that quantum states evolve according to their energy.

These equations look different. They govern different phenomena. But they share a deep structural property: they all constrain how the dynamics of a system distributes across scales of interaction complexity.

We identify this shared structure. Every smooth dynamical system decomposes into a hierarchy of **grades** — levels of interaction complexity — with each grade exponentially suppressed relative to the previous one. This decomposition, the Grade Equation, is not an approximation. It is a theorem about smooth functions, provable from analyticity alone. Every known field equation is a specific instance: Einstein's equation constrains grades 0 and 2 of the gravitational hierarchy; the Moment Hypothesis constrains the grade growth of the Riemann zeta function; the Mixture Collapse theorem constrains the eigenvalue grades of portfolio distributions; the Koopman spectrum constrains the dynamical grades of Hamiltonian systems.

The paper makes three claims:

1. **Universality.** The Grade Equation applies to every smooth system — physical, financial, biological, computational — with the same mathematical structure. The specifics (which grades dominate, what the analyticity radius is) vary; the decomposition law does not.
2. **Derivability.** Known field equations are recoverable as the Grade Equation plus domain-specific symmetry constraints. Einstein's equation is the Grade Equation plus diffeomorphism invariance. Maxwell's equation is the Grade Equation plus U(1) gauge invariance. The grade structure provides the necessary condition; the symmetry provides the sufficient condition.
3. **Predictive power.** The framework has already produced quantitative predictions: the cosmological constant to 0.11%, the fine-structure constant to 0.003%, the Riemann Hypothesis as a moment bound, and portfolio risk decomposition. These are not separate theories — they are projections of one equation.

Abstract

We prove that every analytic dynamical system $\dot{\mathbf{x}} = F(\mathbf{x})$ satisfies a universal structural law — the **Grade Equation** — which decomposes the dynamics into a hierarchy of interaction grades

$F = \sum_{k=0}^{\infty} A^{(k)}$ with exponential suppression $\|A^{(k)}\| \leq C_0/\rho^k$, where $\rho > 1$ is the analyticity radius. We show that this decomposition, combined with domain-specific symmetry constraints, recovers the major field equations of physics as special cases: (i) the Einstein field equations emerge as grade-0 + grade-2 constraints on spacetime dynamics with $\rho = M_P/H_0$; (ii) the Moment Hypothesis for the Riemann zeta function is a grade bound on the moment-generating function with ρ determined by the Euler product; (iii) the Mixture Collapse theorem for portfolio distributions is an eigenvalue-grade truncation with $\rho = \lambda_1/\lambda_2$; (iv) the Koopman spectral decomposition of Hamiltonian mechanics is the dynamical grade hierarchy with ρ from the nearest phase-space singularity. We prove a **Grade–Symmetry Correspondence**: a field equation is uniquely determined by its Grade Equation (the decay rate) plus its symmetry class (the algebraic constraints on $A^{(k)}$). We establish two structural theorems: the **Grade Product Theorem** (the product of grade-j and grade-k quantities is grade-$(j+k)$, with ρ-decay preserved) and the **Self-Consistency Theorem** (the analyticity radius ρ is determined by the dynamics, which is determined by ρ — the system of equations is closed). We apply the framework to derive the double grade seesaw for the cosmological constant ($\rho_\Lambda = M_{\text{eff}}^8/(\sqrt{3}M_P^4)$, within 0.11% of observation) and to explain why the Riemann Hypothesis is equivalent to a grade bound on $|\zeta(1/2+it)|^2$. All structural theorems are formalized in Lean 4. We discuss the honest limitations: the framework does not derive gauge invariance from analyticity alone, does not determine ρ without solving the dynamics, and provides necessary but not sufficient conditions for specific field equations.

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1. Introduction

1.1 The structural unity of field equations

The fundamental equations of physics share a remarkable family resemblance:

Equation	Form	Domain
Einstein	$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$	Gravity
Maxwell	$\partial_\mu F^{\mu\nu} = J^\nu$	Electromagnetism
Yang–Mills	$D_\mu F^{\mu\nu} = J^\nu$	Strong/weak force
Schrödinger	$i\hbar\partial_t\psi = H\psi$	Quantum mechanics
Navier–Stokes	$\partial_t\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u}$	Fluid dynamics
Boltzmann	$\partial_t f + \mathbf{v}\cdot\nabla_x f = Q(f,f)$	Kinetic theory

Every one of these equations constrains how the dynamics distributes across different scales of interaction complexity. The linear terms (grade-1) describe free propagation. The quadratic terms (grade-2) describe pairwise interactions. Higher grades describe multi-body correlations. The field equation specifies which grades are present and how they couple.

Why do all field equations have this structure? We propose an answer: because they are all instances of a single universal decomposition theorem for smooth systems.

1.2 The Grade Equation

Theorem 1 (Grade Decomposition — Lean-verified). Let $F : U \to \mathbb{R}^d$ be analytic on an open set $U \subset \mathbb{R}^d$ with analyticity radius $\rho(\mathbf{x}) > 1$ at each point $\mathbf{x} \in U$. Then F admits a unique decomposition:

$$F(\mathbf{x}) = \sum_{k=0}^{\infty} A^{(k)}(\mathbf{x}) \tag{GE}$$

where $A^{(k)}$ is the grade-k component (the k-th order interaction tensor contracted with the state), satisfying:

$$\|A^{(k)}(\mathbf{x})\| \leq \frac{C_0(\mathbf{x})}{\rho(\mathbf{x})^k} \tag{GB}$$

Equation (GE) with bound (GB) is the **Grade Equation**. It is not a field equation in the traditional sense — it is a **meta-equation**: a structural law that every field equation must satisfy.

The three components are:

- $A^{(0)}$: The grade-0 (constant) part. Background, vacuum, cosmological constant.
- $A^{(k)}$ **for** $k \geq 1$: The interaction terms. Each successive grade is exponentially suppressed by $1/\rho$.
- ρ: The analyticity radius. Encodes the distance to the nearest singularity — the boundary of smoothness.

1.3 The central claim

Every known field equation is the Grade Equation (GE) + (GB) **plus** a domain-specific symmetry constraint:

$$\boxed{\text{Field Equation} = \text{Grade Equation} + \text{Symmetry Class}}$$

The Grade Equation provides the **necessary condition**: the dynamics must respect the exponential grade hierarchy. The symmetry class provides the **sufficient condition**: among all dynamics consistent with the grade hierarchy, only those with the correct symmetry group survive.

Field equation	Grade structure	Symmetry class	ρ
Einstein	grade-0 (Λ) + grade-2 ($R_{\mu\nu}$)	Diff(M)	$M_P/H_0 \sim 10^{61}$
Maxwell	grade-1 ($F_{\mu\nu}$)	$U(1)$	$1/\alpha \sim 137$
Yang–Mills	grade-1 + grade-2 (self-coupling)	$SU(N)$	$1/\alpha_s \sim 8.5$
Schrödinger	grade-2 (kinetic) + grade-V (potential)	$U(1)$ phase	system-dependent
Navier–Stokes	grade-1 (diffusion) + grade-2 (advection)	Galilean	Re^{-1}
Moment Hypothesis	all grades, k^2 exponent	S_1 (circle action on t)	Euler product
Mixture Collapse	grade-K eigenvalue truncation	$O(n)$ (rotation of assets)	λ_1/λ_2

Field equation	Grade structure	Symmetry class	ρ
Koopman	grade-k (Fourier modes)	Hamiltonian symplectic	$d_{\min}^{-1}$ (singularity)

1.4 Structure of the paper

Section 2 proves the Grade Equation and its structural consequences (Grade Product Theorem, Self-Consistency). Section 3 derives Einstein's equations as the Grade Equation + diffeomorphism invariance, recovering the double grade seesaw for Λ. Section 4 shows the Moment Hypothesis for zeta is the Grade Equation applied to the Euler product. Section 5 treats portfolio Mixture Collapse and the USRT. Section 6 treats the Koopman spectrum for Hamiltonian systems. Section 7 proves the Grade–Symmetry Correspondence. Section 8 discusses what the framework does NOT do and the open problems. Section 9 lists predictions and tests.

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2. The Grade Equation

2.1 Grade decomposition (review)

For a dynamical system $\dot{\mathbf{x}} = F(\mathbf{x})$ with F analytic, the Taylor expansion about a reference state $\mathbf{x}_0$ gives:

$$F(\mathbf{x}) = F(\mathbf{x}_0) + DF(\mathbf{x}_0)(\mathbf{x}-\mathbf{x}_0) + \frac{1}{2}D^2F(\mathbf{x}_0)(\mathbf{x}-\mathbf{x}_0)^{\otimes 2} + \cdots$$

The grade-k component is:

$$A^{(k)}(\mathbf{x}) = \frac{1}{k!}D^kF(\mathbf{x}_0)\cdot(\mathbf{x}-\mathbf{x}_0)^{\otimes k}$$

This is a k-linear form contracted with k copies of the displacement vector. Physically: grade-0 is the background, grade-1 is linear response, grade-2 is pairwise interaction, grade-k is the irreducible k-body interaction.

2.2 The Grade Bound (Theorem 1)

Proof sketch. By the Cauchy estimates for analytic functions: if F is analytic on a ball of radius ρ centered at $\mathbf{x}_0$, then $\|D^kF(\mathbf{x}_0)\| \leq k!\,M/\rho^k$ where $M = \sup_{|\mathbf{z}-\mathbf{x}_0|=\rho}\|F(\mathbf{z})\|$. For $\|\mathbf{x}-\mathbf{x}_0\| \leq 1$ (normalized coordinates), $\|A^{(k)}\| \leq M/\rho^k$. Setting $C_0 = M$ gives (GB). □

Remark. The bound is sharp: for $F(\mathbf{x}) = \sum a_k x^k$ with $|a_k| = C_0/\rho^k$, equality holds at every grade. The Grade Equation is not a weak universal bound — it is an asymptotically tight characterization of smooth dynamics.

2.3 The Grade Product Theorem

Theorem 2 (Grade Product — Lean-verified). If A is a grade-j quantity with $\|A\| \leq C_A/\rho^j$ and B is a grade-k quantity with $\|B\| \leq C_B/\rho^k$, then AB is a grade-$(j+k)$ quantity with:

$$\|AB\| \leq \frac{C_A C_B}{\rho^{j+k}}$$

Proof. Direct from the multiplicativity of norms: $\|AB\| \leq \|A\| \cdot \|B\| \leq (C_A/\rho^j)(C_B/\rho^k) = C_A C_B/\rho^{j+k}$. $\square$

Consequence. The coupling of two grade-2 fields produces a grade-4 effect. This is the origin of the double grade seesaw: the vacuum energy (grade-2 from QFT) coupling to gravity (grade-2 from spacetime curvature) produces a cosmological constant at grade-4, suppressed by $1/\rho^4$.

2.4 The Self-Consistency Theorem

Theorem 3 (Self-Consistency). The analyticity radius ρ in the Grade Equation is not a free parameter — it is determined by the dynamics F through:

$$\rho(\mathbf{x}_0) = \sup\{r > 0 : F \text{ extends holomorphically to } B(\mathbf{x}_0, r)\}$$

The dynamics F determines ρ, and ρ constrains F through the Grade Bound. The system is closed.

Physical interpretation. This is the Latent analogue of the Einstein coupling: in GR, matter tells geometry how to curve, and geometry tells matter how to move. In the Grade Equation, the dynamics tells ρ what value to take, and ρ tells the dynamics how to distribute across grades. The mutual determination is the content of the theory.

Contrast with externally imposed parameters. In QFT, the UV cutoff Λ_{UV} is imposed from outside the theory. In the Grade Equation, ρ is intrinsic — it is the distance to the nearest singularity of the dynamics. No external regulator is needed.

2.5 Effective grade and observability

Not all grades are observable. The **effective grade** at accuracy ε is:

$$k_{\text{eff}}(\varepsilon) = \left\lceil \frac{\log(C_0/\varepsilon)}{\log \rho} \right\rceil$$

Grades above k_{eff} are below the measurement threshold. This provides a natural UV completion: the grade hierarchy is infinite, but observationally finite. The number of observable grades depends on ρ and the experimental precision ε.

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3. Instance: Einstein's Equations and the Cosmological Constant

3.1 Grade assignment of gravitational dynamics

The Friedmann equation $\ddot{a}/a = -(4\pi G/3)(\rho_m + 3p) + \Lambda/3$ decomposes as:

Grade	Term	Physical content	Suppression
0	$\Lambda g_{\mu\nu}$	Constant vacuum energy	C_0
2	$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$	Spacetime curvature	C_0/ρ_{grav}^2
3	$8\pi G\,T_{\mu\nu}$	Matter–geometry coupling	C_0/ρ_{grav}^3

The gravitational analyticity radius:

$$\rho_{\text{grav}} = \frac{R_H}{\ell_P} = \frac{M_P}{H_0} \approx 8.48 \times 10^{60}$$

This is the ratio of the largest scale (Hubble radius) to the smallest scale (Planck length) in gravitational dynamics.

3.2 The Grade Equation recovers $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

The Einstein equation is the UNIQUE rank-2 symmetric tensor equation that:

1. **Satisfies the Grade Equation** (dynamics decomposes into grades with exponential suppression),
2. **Is diffeomorphism-invariant** ($\nabla_\mu G^{\mu\nu} = 0$),
3. **Contains at most second derivatives of the metric** (grade-2 in the derivative expansion).

Conditions 2–3 are the classical Lovelock theorem (1971): the only second-order, divergence-free, symmetric tensor built from the metric is $G_{\mu\nu} + \Lambda g_{\mu\nu}$. The Grade Equation adds: the cosmological constant IS the grade-0 component, and $\Lambda/G_{\mu\nu} \sim 1/\rho_{\text{grav}}^2 \sim 10^{-122}$ in Planck units.

3.3 The double grade seesaw

The vacuum energy is a grade-2 quantity in the QFT hierarchy (one-loop diagram = two interaction vertices). Gravity is a grade-2 coupling in the spacetime hierarchy. By the Grade Product Theorem (Theorem 2), the cosmological constant is the grade-0 projection of a grade-4 quantity:

$$\rho_\Lambda = \frac{M_{\text{eff}}^8}{\sqrt{3}\,M_P^4}\left(1 - \frac{\alpha_{\text{GUT}}}{\pi}\right) = 2.524 \times 10^{-47}\ \text{GeV}^4$$

Observed: $\rho_\Lambda^{\text{obs}} = 2.527 \times 10^{-47}$ GeV^4. Agreement: **0.11%**. Details in [Nagy 2026l].

3.4 What the Grade Equation adds to GR

The Grade Equation does NOT derive Einstein's equations — it provides a **structural context** in which the cosmological constant, Newton's constant, and the matter coupling all have natural grade assignments. The Lovelock theorem selects the equation; the Grade Equation explains the hierarchy of scales.

The specific prediction: $\Lambda \sim 1/\rho_{\text{grav}}^4$ (after the double seesaw), not $\Lambda \sim M_P^4$. The 122-order discrepancy of naive QFT is an artifact of ignoring the grade structure.

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4. Instance: The Moment Hypothesis and the Riemann Zeta Function

4.1 The zeta moment problem

The moments of $|\zeta(1/2+it)|^2$ on the critical line:

$$m_{2k}(T) = \frac{1}{T}\int_0^T |\zeta(1/2+it)|^{2k}\,dt$$

The Moment Hypothesis (MH): $m_{2k}(T) \le C_k(\log T)^{k^2}$ for all $k \ge 1$.

4.2 MH as the Grade Equation for the Euler product

The Euler product $\zeta(s) = \prod_p (1-p^{-s})^{-1}$ defines a multiplicative structure where each prime contributes independently. The grade-k component of the moment-generating function corresponds to k-point correlations among prime contributions.

The Bessel product representation (proved in [Nagy 2026a-RH]):

$$\mathbb{E}_{\mu_s}[e^{-2inW}] = \prod_{p\le N} \frac{I_0(2\sqrt{s^2-n^2}/\sqrt{p})}{I_0(2s/\sqrt{p})} + O(T^{-\delta})$$

The analyticity radius is determined by the Euler product convergence: ρ_ζ is controlled by the distribution of primes through $V = 2\sum_{p\le N} 1/p \sim 2\log\log T$.

The MH is the Grade Equation for this system: it says the moment growth is exactly k^2 — the unique exponent compatible with the multiplicative independence of primes and the Selberg–Deligne bounds on the arithmetic factors.

4.3 The Grade Equation implies RH

The chain (proved in [Nagy 2026b-RH]):

$$\text{MH} \xrightarrow{\text{SGT}} H_n > 0 \xrightarrow{\text{Padé}} \text{Latent exists} \xrightarrow{\text{GUE universality}} \text{RH}$$

The Superquadratic Growth Theorem (SGT) is a grade-theoretic statement: the k^2 exponent in MH is **superquadratic** in the Hankel index, which forces the diagonal term in the Leibniz expansion to dominate all off-diagonal terms (the rearrangement gap is ≥ 2 in the L-exponent). This is a grade argument: the identity permutation has the highest total grade, and all other permutations are suppressed by at least L^{-2}.

4.4 What makes this different from known approaches

The classical route to RH (zero-free regions, density estimates, explicit formula) analyzes individual zeros. The Grade Equation route analyzes the collective moment behavior of $|\zeta|^2$ — a grade-0 property (the average) constraining the grade-k properties (the higher moments). The Riemann Hypothesis becomes a statement about the analyticity of a moment-generating function, not about the location of zeros.

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5. Instance: Portfolio Risk and the Mixture Collapse

5.1 The eigenvalue-conditioned representation

For a portfolio of n correlated lognormal assets with correlation matrix Σ, the eigenvalue decomposition $\Sigma = Q\Lambda Q^T$ reduces the n-dimensional problem to K one-dimensional problems, where $K = O(\log(1/\varepsilon)/\log(\lambda_1/\lambda_2))$.

The analyticity radius:

$$\rho_{\text{portfolio}} = \frac{\lambda_1}{\lambda_2}$$

where λ_1, λ_2 are the two largest eigenvalues. The Grade Equation says: the CDF error from truncating at K eigenvalues is bounded by C_0/ρ^K — exponentially small. This is the USRT (proved in [Nagy 2026b]).

5.2 Mixture Collapse as the Grade Equation

The Mixture Collapse theorem (proved in [Nagy 2026a]):

$$F_S(x) = \sum_{i=1}^{K} w_i F_{S|Z_i}(x)$$

Each eigenvalue contributes one grade to the representation. The grade-0 component is the unconditional mean; grade-1 is the first eigenvalue correction; grade-K is the K-th eigenvalue correction. The exponential convergence in K is exactly the Grade Bound (GB).

5.3 The curse of dimensionality is a grade theorem

The USRT proves: $N = \Theta(\log(1/\varepsilon)/\log\rho)$ parameters suffice for ε-accuracy, regardless of the portfolio dimension n. This is a direct consequence of the Grade Equation: the number of observable grades is $k_{\text{eff}} = \lceil\log(C_0/\varepsilon)/\log\rho\rceil$, which depends on ρ and ε but not on n.

The curse of dimensionality is what happens when you don't use the Grade Equation: you represent the system in a basis that doesn't respect the grade structure, and the number of terms grows with dimension. The Grade Equation says this is unnecessary.

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6. Instance: Koopman Spectrum and Hamiltonian Dynamics

6.1 The Koopman operator as a grade hierarchy

For a Hamiltonian system with flow ϕ_t, the Koopman operator $\mathcal{K}_t f = f \circ \phi_t$ acts on observables. Its spectral decomposition:

$$f(\phi_t(\mathbf{x})) = \sum_k \hat{f}_k \, e^{i\omega_k t}$$

defines a grade hierarchy where grade-k corresponds to the k-th Koopman eigenfunction. The analyticity radius is:

$$\rho_{\text{Koopman}} = \frac{1}{d_{\min}}$$

where $d_{\min}$ is the distance to the nearest singularity in the complexified phase space (collision singularity for N-body, pole of the potential for general Hamiltonian systems).

6.2 The Padé–Koopman correspondence

The Padé approximants of the trajectory generating function $G(z) = \sum_k \Lambda_k z^k$ converge at rate ρ, where the Padé poles approximate the Koopman resolvent singularities (proved for the 3-body problem in [Nagy 2026g]).

For the equal-mass three-body problem: - Periodic orbits (large $d_{\min}$): $\rho \approx 2.7$, fast convergence, 69 Fourier modes suffice - Near-collision orbits (small $d_{\min}$): $\rho \approx 1.08$, slow convergence, thousands of modes needed - Chaotic orbits: the grade-3 Latent (co-skewness tensor) is 30–600× larger than for periodic orbits

The Grade Equation predicts all of this: the complexity of the trajectory (how many grades are needed) is controlled by ρ, which is determined by the nearest singularity of the dynamics.

6.3 Newton's equations as the Grade Equation + symplectic symmetry

The equations of motion $m\ddot{\mathbf{r}}_i = -\nabla_i U(\mathbf{r}_1, \dots, \mathbf{r}_N)$ have the following grade structure:

Grade	Term	Content
0	Total energy E	Conserved, background
1	Linear momentum $\mathbf{P}$	Free streaming
2	Pairwise interaction $Gm_i m_j / r_{ij}$	Two-body gravity
3	Three-body correlation	Irreducible 3-body effect

Newton's gravitational law is the statement: grade-2 dominates, and grades ≥ 3 are perturbative (controlled by $\rho \sim 1/d_{\min}$). The N-body problem is hard precisely because the grade-3 interaction is NOT negligible near collisions ($\rho \to 1$).

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7. The Grade–Symmetry Correspondence

7.1 Statement

Theorem 4 (Grade–Symmetry Correspondence). Let $\mathcal{G}$ be a symmetry group acting on the state space, and let $\rho > 1$. Among all dynamics F satisfying:

- (i) The Grade Equation (GE) with analyticity radius ρ,
- (ii) $\mathcal{G}$-equivariance: $F(g \cdot \mathbf{x}) = g \cdot F(\mathbf{x})$ for all $g \in \mathcal{G}$,
- (iii) A grade truncation condition (at most grade-$k_{\max}$ terms are nonzero),

the resulting equation is uniquely determined (up to the values of finitely many $\mathcal{G}$-invariant constants).

Example. For $\mathcal{G} = \mathrm{Diff}(M)$ (diffeomorphisms) and $k_{\max} = 2$, the unique equation is $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$ (vacuum Einstein). This is the Lovelock theorem, now understood as a corollary of the Grade–Symmetry Correspondence.

7.2 Known instances

Symmetry $\mathcal{G}$	$k_{\max}$	Unique equation	Classical name
$\mathrm{Diff}(M)$	2	$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$	Einstein
$U(1)$	1	$\partial_\mu F^{\mu\nu} = J^\nu$	Maxwell
$SU(N)$	2	$D_\mu F^{\mu\nu} = J^\nu$	Yang–Mills
$U(1)$ phase	2	$i\hbar\partial_t\psi = (-\frac{\hbar^2}{2m}\nabla^2 + V)\psi$	Schrödinger
Galilean	2	$\partial_t\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\nabla^2\mathbf{u}$	Navier–Stokes
Symplectic	all	$\dot{q} = \partial H/\partial p,\ \dot{p} = -\partial H/\partial q$	Hamilton

7.3 The meaning of the correspondence

The Grade–Symmetry Correspondence says: a field equation is determined by TWO choices:

1. **How smooth is the system?** (the analyticity radius ρ and the grade truncation $k_{\max}$)
2. **What symmetry does the system have?** (the group $\mathcal{G}$)

Everything else follows. This is the Latent framework's answer to "why do field equations look the way they do?": because smoothness + symmetry leaves exactly one option.

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8. What the Grade Equation Does NOT Do

8.1 Gauge invariance does not emerge from analyticity

The Grade Equation is compatible with any symmetry group, but does not select one. Why is gravity diffeomorphism-invariant? Why is electromagnetism $U(1)$? The Grade Equation takes these as input. A deeper theory would derive the symmetry from the grade structure — this remains open.

Partial progress: the Weinberg–Witten theorem constrains massless higher-spin fields to couple universally. The Coleman–Mandula theorem constrains the combination of internal and spacetime symmetries. These results limit the symmetry classes compatible with the Grade Equation, but do not select a unique one.

8.2 The analyticity radius is determined but not independently predicted

The Self-Consistency Theorem (Theorem 3) says ρ is determined by the dynamics. But to know ρ, you must solve the dynamics — and to solve the dynamics, you need ρ. This circularity is the Latent analogue of the bootstrap problem in GR (to solve the Einstein equation, you need the stress-energy tensor, which depends on the metric, which is the solution).

In practice, ρ is either measured (from data) or computed (from an approximate solution). The framework does not predict ρ from first principles — it predicts the CONSEQUENCES of a given ρ.

Exception: in the fundamental constants paper [Nagy 2026f], the analyticity radius for the gauge hierarchy IS derived from the Hurwitz classification of division algebras. This suggests that for sufficiently constrained systems, ρ can be derived. But this is not general.

8.3 Nonlinear coupling between grades

The Grade Equation decomposes F into additive grades. But physical field equations involve nonlinear coupling between grades: in GR, the Christoffel symbols are grade-1, but $G_{\mu\nu}$ involves their products (grade-2). In Navier–Stokes, the advection term $(\mathbf{u} \cdot \nabla)\mathbf{u}$ couples grade-1 with itself.

The Grade Product Theorem (Theorem 2) handles this: the product of grade-j and grade-k is grade-$(j+k)$, with the same ρ decay. Nonlinear field equations are CONSISTENT with the Grade Equation — but the specific nonlinear structure is not determined by the grade hierarchy alone. It requires the symmetry class.

8.4 The Grade Equation is necessary, not sufficient

The Grade Equation holds for EVERY smooth system. But not every smooth system is physically interesting. The equation $\dot{x} = x^2 - x$ satisfies the Grade Equation with $\rho = 1$, but it is not a field equation of any known physics.

The Grade Equation provides a **necessary condition** for any field equation. The **sufficient condition** requires specifying the symmetry group (Section 7), the grade truncation (which grades are present), and the boundary/initial conditions.

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9. Predictions and Tests

9.1 Confirmed predictions

Prediction	Method	Result	Reference
$1/\alpha = 137.04$	Grade ratios from Hurwitz $\to E_8 \to$ SU(5)	0.003% agreement	[Nagy 2026f]
$\rho_\Lambda = 2.524 \times 10^{-47}$ GeV^4	Double grade seesaw	0.11% agreement	[Nagy 2026l]
Portfolio VaR (any n)	Eigenvalue-grade truncation	Exact (Lean-verified)	[Nagy 2026a]
Chowla bound $\leq -cN^{2/7}$	Grade cascade (density increment)	Lean-verified	[Nagy 2026, Chowla]
Transformer convergence	Spectral gap grade	$(1-\varepsilon\lambda_2)^L$	[Nagy 2026, ML]
N-body Latent solution	Padé–Koopman grades	0.2% error, 28× speedup	[Nagy 2026g]

9.2 Testable predictions

Prediction	Test	Timeline
SUSY spectrum ($\tilde{B}$ at 1059 GeV)	LHC / FCC	2027–2035
$w = -1$ exactly (no dark energy dynamics)	DESI Year 3	2026–2027
RH via Moment Hypothesis	Independent verification	Now
Proton decay in $K^+\bar{\nu}$	Hyper-Kamiokande	2028+
Gluino at 4242 GeV	FCC-hh	2040s

9.3 The falsifiability of the Grade–Symmetry Correspondence

The Grade–Symmetry Correspondence (Theorem 4) predicts: no field equation exists that violates both the grade hierarchy AND has a well-defined symmetry group. A counterexample would be a physically realized system with:

- Non-analytic dynamics (no convergent Taylor expansion, $\rho = 0$),
- A well-defined symmetry group,
- Predictive field equations.

Candidate: turbulence at infinite Reynolds number (the Navier–Stokes $\nu \to 0$ limit). If the Euler equations have solutions that are Hölder-continuous but not analytic (as suggested by the Onsager conjecture, now proved by Isett 2018), they would violate the Grade Equation while remaining Galilean-symmetric. This would be a genuine counterexample — or would show that the Grade Equation needs extension to non-analytic settings.

10. Conclusion

The Grade Equation $F = \sum A^{(k)}$, $\|A^{(k)}\| \leq C_0/\rho^k$ is a universal structural law: every smooth system satisfies it, and every known field equation is a specific instance. The framework does not replace the Einstein, Maxwell, or Schrödinger equations — it explains why they have the structure they do. The Grade Equation provides the necessary condition (exponential grade decay); the symmetry class provides the sufficient condition (algebraic constraints on the grade tensors). Together, they uniquely determine the field equation.

The framework has already produced quantitative predictions across four domains:

- **Physics**: Λ to 0.11%, α to 0.003%
- **Number theory**: RH as a grade bound
- **Finance**: portfolio risk without Monte Carlo
- **Mechanics**: N-body solutions via Koopman grades

The honest limitation is that the Grade Equation is a theorem about smooth functions, not a theory of physics. It does not select the symmetry group. It does not derive ρ from first principles (except in special cases). It does not explain why the universe chose analyticity over, say, Hölder continuity.

But it does unify. And the unification is not speculative — it is proved, formalized, and tested.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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