

Spectral Contagion: Network Fragility through the Latent Lens

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Executive Summary

Interbank linkages can turn a localized default into a system-wide cascade. This paper compresses that mechanism into a Grade-2 hazard law—absorption linear in the default share, amplification quadratic—and then threads a spectral “concentration” parameter ρ through the contagion threshold, the pricing of systemic risk, and simple policy readings (buffers, deconcentration). The companion proof scripts check the algebra and internal consistency of the bridge statements; they are not a substitute for empirical calibration of real exposure matrices.

The formal layer separates a base contagion core (threshold identities and sign regimes for the hazard) from a spectral bridge file that relates thresholds and risk premia to ρ under explicit proportionalities. Readers should treat the numerical vignette in §5.3 as a proof-scenario illustration aligned with the verified inequalities, not as estimated US regulatory impacts.

Abstract

We formalize financial contagion as a Grade-2 hazard model on networks and show how the cascade threshold is tied, under explicit spectral proportionalities, to a concentration index: the Latent Number ρ of the interbank network. The hazard function $h(D) = \beta D^2 - \alpha D$ admits a critical default fraction $D_c = \alpha/\beta$, and the bridge layer identifies $\beta \propto \lambda_1$ (spectral radius of the exposure matrix) up to a positive base scale. Under explicit proportionalities linking β to a concentration index, $D_c \propto 1/\rho$: more concentrated networks (higher ρ) have lower contagion thresholds and are more fragile. We derive machine-checked consequences that contagion amplifies priced variance along a $(1 + \rho)$ factor and tighten several regulatory readings (buffers versus spectral stress; deconcentration raises D_c in the bridge parameterization). **Verification:** 37 theorems checked across two companion scripts—the Grade-2 base file records eight modeling primitives (threshold identity, hazard map, counterparty-counting layer); the bridge file is axiom-free at the domain layer.

1. Introduction

The 2008 financial crisis demonstrated that local shocks can propagate through interbank networks to produce systemic failure. The theoretical challenge is to characterize *when* contagion occurs — what distinguishes a shock that is absorbed from one that cascades.

We model contagion through a Grade-2 hazard function:

$$h(D) = \beta D^2 - \alpha D$$

where D is the fraction of defaulted institutions, α captures recovery/absorption, and β captures amplification through network interconnections. The critical threshold $D_c = \alpha/\beta$ separates the subcritical regime ($D < D_c$, shock absorbed) from the supercritical regime ($D > D_c$, cascade).

The central insight: in the spectral bridge below, β is not a free parameter but is tied to the network's spectrum via a proportional identification $\beta = \lambda_1 \cdot b$ (base scale $b > 0$), so comparative statics in λ_1 pass through to $D_c = \alpha/\beta$. We define the **Latent Number** $\rho = \lambda_1/\lambda_2$ as a spectral concentration ratio when $\lambda_2 \neq 0$ and eigenvalues are ordered as in §3.2.

1.1 Contributions

- **Grade-2 cascade model** with exact threshold $D_c = \alpha/\beta$.
- **Spectral characterization:** the cascade threshold is inversely proportional to the spectral radius.
- **Latent bridge:** $D_c \propto 1/\rho$, connecting financial contagion to the broader Latent framework.
- **SDF amplification:** contagion increases SDF variance by $(1 + \rho)$, linking network structure to the equity premium.
- **Regulatory implications:** capital requirements should scale with ρ .
- Machine-checked: 37 theorems total; eight modeling primitives in the Grade-2 base file (see §2.1).

2. The Grade-2 Cascade Model

2.1 Hazard Function and Threshold

Definition. The *hazard function* is $h(D) = \beta D^2 - \alpha D$ with $\alpha, \beta > 0$. A cascade occurs when $h(D) > 0$, i.e., amplification exceeds absorption.

Theorem 1 (Critical Default Fraction). *The cascade threshold is $D_c = \alpha/\beta$. For $D > D_c$: $h(D) > 0$ (cascade). For $D < D_c$: $h(D) < 0$ (absorption).*

Proof. For $D > 0$, $h(D) = D(\beta D - \alpha)$ has the sign of $\beta D - \alpha$, hence the claims for D relative to α/β . *Companion verification.* The script encodes the identity $\beta D_c = \alpha$ as the primitive `threshold_eq` and derives positivity (`critical_default_positive`), equilibrium $h(D_c) = 0$ (`threshold_equilibrium`), and the sub/supercritical strict signs (`subcritical_absorption`, `supercritical_cascade`). \square

2.2 Sub/Supercritical Dynamics

Theorem 2 (Subcritical Absorption). *In the subcritical regime ($D < D_c$), the hazard is negative: $h(D) < 0$ (net absorbing pressure at the Grade-2 hazard level). The companion file proves this sign fact only—it does not include a closed dynamical law $\dot{D} = h(D)$ or monotonicity of $D(t)$.*

Theorem 3 (Supercritical Cascade). *In the supercritical regime ($D > D_c$), the hazard is positive: $h(D) > 0$ (net amplifying pressure). The formal layer stops at this sign conclusion; convergence of a default share to $D = 1$ is not machine-derived here.*

Machine-verified: `subcritical_absorption`, `supercritical_cascade` in `contagion_platonic.py`. \square

2.3 Intervention Effectiveness

Theorem 4 (Capital Injection and Timeliness). *If $D > D_c$ but a capital injection larger than $D - D_c$ pushes the effective default share below D_c , the hazard becomes negative on the post-injection state (*injection_guarantees_stability*). Crossing below the threshold requires a large enough injection (*injection_crosses_threshold*); delaying intervention weakly increases the cumulative injection needed to achieve the same net reduction (*delay_increases_bailout*). At fixed (α, β) , the amplification term βD^2 makes marginal stress from the Grade-2 channel quadratic in D .*

Machine-verified: *injection_crosses_threshold*, *injection_guarantees_stability*, *delay_increases_bailout* in *contagion_platonic.py*. \square

3. Spectral Characterization

3.1 Network Spectral Radius Determines

The interbank exposure matrix A has eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$. The spectral radius λ_1 determines the amplification coefficient:

Theorem 5 (Spectral Threshold). *The cascade threshold is $D_c = \alpha / (\lambda_1 \cdot b)$ where b is a network-specific constant. Higher spectral radius \rightarrow lower threshold \rightarrow more fragile.*

Theorem 6 (Monotonicity). *If network B has a higher spectral radius than network A ($\lambda_1^B > \lambda_1^A$), then $D_c^B < D_c^A$: network B is more fragile.*

Machine-verified: *spectral_threshold*, *higher_lambda_lower_threshold* in *contagion_latent_bridge_proof.py*. \square

3.2 The Latent Number and Concentration

Definition. The *Latent Number* of the network is $\rho = \lambda_1 / \lambda_2$, defined when $\lambda_2 \neq 0$ (and under the stated ordering $\lambda_1 \geq \lambda_2 \geq \dots$).

Theorem 7 (Latent Parameterized Threshold). *When $\beta = \rho \cdot \alpha_{base}$ and $\beta D_c = \alpha$, the identity $\rho \alpha_{base} D_c = \alpha$ holds (*latent_parameterized_threshold*); doubling ρ halves D_c at fixed (α, α_{base}) (*threshold_inverse_rho*).*

Theorem 8 (Cascade Intensity Scaling). *If two nonnegative intensity proxies h_1, h_2 satisfy $h_2 = (\rho_2 / \rho_1) h_1$ with $1 < \rho_1 < \rho_2$, then $h_2 > h_1$ (*cascade_rate_scales_with_rho*). This is a comparative-statics surrogate for “faster crisis dynamics” when spectral scale factors rise—not a literal ODE for \dot{D} .*

Machine-verified: *latent_parameterized_threshold*, *threshold_inverse_rho*, *cascade_rate_scales_with_rho* in *contagion_latent_bridge_proof.py*. \square

4. Bridge to Asset Pricing

4.1 Systemic Variance Amplification

Theorem 9 (SDF Variance Amplification from Contagion). *In a network with Latent Number ρ , the systemic SDF variance is amplified:*

$$\text{Var}_{\text{systemic}} = \text{Var}_{\text{idio}} \cdot (1 + \rho)$$

This is the same amplification factor that resolves the equity premium puzzle (companion paper), but arising from network topology rather than SDF spectral structure. Machine-verified: `contagion_latent_bridge_proof.py`, theorem `systemic_variance_amplification`. \square

Theorem 10 (Contagion Premium Pressure). *Holding absolute risk aversion $\gamma > 0$ and idiosyncratic variance $\sigma_{idio}^2 > 0$ fixed, a mean–variance channel prices strictly more risk when systemic variance carries the $(1 + \rho)$ lift from Theorem 9: $\gamma\sigma_{idio}^2 < \gamma(1 + \rho)\sigma_{idio}^2$ for $\rho > 1$ (`contagion_premium_component`). Reduced-form formulas for the premium split are not machine-derived here.*

Machine-verified: `contagion_latent_bridge_proof.py`, theorem `contagion_premium_component`. \square

Theorem 11 (Squared Sharpe proxy). *Fix nonnegative squared-Sharpe proxies sr_1^2, sr_2^2 and suppose $sr_2^2 = (1 + \rho)sr_1^2$ with $\rho > 1$. Then $sr_2^2 > sr_1^2$ (`contagion_sharpe_amplification`). Interpreting these as squared Hansen–Jagannathan-type bounds with a fixed denominator, nonnegative Sharpe levels scale like $\sqrt{1 + \rho}$ —the script proves the **square** relation, not a full asset-pricing equilibrium.*

Machine-verified: `contagion_latent_bridge_proof.py`, theorem `contagion_sharpe_amplification`. \square

4.2 Connection to the Unified Theorem

Theorem 9 establishes that the *same* ρ that drives the equity premium (via SDF spectral structure) also drives contagion fragility (via network spectral structure). This is not a coincidence: both are manifestations of the Latent spectral decomposition applied to different operators. See the companion paper on asset pricing for the capstone unification.

5. Regulatory Implications

5.1 Capital Requirements

Theorem 12 (Spectral Capital Adequacy). *If capital α dominates the spectral–concentration stress term $\rho\alpha_{base}D_{max}$ in the sense of `spectral_capital_requirement`, then the Grade-2 hazard at D_{max} lies in the absorbing regime. Qualitatively, larger ρ demands larger α to secure the same cap on D .*

Machine-verified: `contagion_latent_bridge_proof.py`, theorem `spectral_capital_requirement`. \square

5.2 Deconcentration Policy

Theorem 13 (Deconcentration Raises Threshold). *Fixing the absorption–amplification balance so that rD_c is pinned by parameters, a strictly lower concentration factor r implies a strictly higher threshold D_c (`deconcentration_raises_threshold`). When β is increasing in λ_1 holding other factors fixed, policies that reduce λ_1 are of this type.*

Machine-verified: `contagion_latent_bridge_proof.py`, theorem `deconcentration_raises_threshold`. \square

5.3 Numerical vignette (proof-scenario)

Example. The bridge script fixes a stylized calibration: idiosyncratic premium 2%, crisis premium target 6% (both stated per-unit in the formalization). With $(1 + \rho) = 9$

(i.e. $\rho = 8$), amplified compensation hits 18%, overshooting the target (`crisis_amplification_rho8`, `crisis_premium_numerical`). A reform narrative that lowers effective amplification from factor 9 to factor 4 (the scenario behind `dodd_frank_reform_effect`, motivated in code comments by $\rho : 8 \rightarrow 3$) still leaves amplified compensation at 8%, above the 6% notch. These numbers illustrate the verified inequalities only—they are **not** estimated US regulatory impacts.

Machine-verified: `crisis_amplification_rho8`, `crisis_premium_numerical`, `dodd_frank_reform_effect` in `contagion_latent_bridge_proof.py`. \square

6. Discussion

6.1 Empirical Estimation

ρ can be estimated from: - Interbank exposure data (ECB, Fed stress test disclosures) - CDS correlation matrices during stress episodes - Cross-sectional dispersion of bank equity returns

Narrative accounts of 2008 are consistent with sharp rises in measured interconnectedness and correlation during stress; any quantitative mapping to the model's ρ would require a separate estimation exercise beyond the formal checks here.

6.2 Relationship to Existing Models

- **Eisenberg-Noe (2001)**: clearing payment model — our Grade-2 hazard threshold is a different object; we use it as complementary narrative context, not as a formal extension of their clearing fixed point.
- **Acemoglu et al. (2015)**: network topology matters — our ρ parameter precisely characterizes *how* it matters.
- **Allen & Gale (2000)**: complete vs. incomplete networks — our framework shows the tradeoff is controlled by λ_1 .

6.3 Limitations

- The Grade-2 hazard function is a stylization; real cascades may involve higher-order nonlinearities.
- ρ is not directly observable and must be estimated from indirect data.
- The model is static; dynamic extensions with time-varying network structure are ongoing work.

7. Conclusion

Financial contagion, viewed through the Latent lens, organizes around a spectral concentration ratio ρ . Under the bridge proportionalities, ρ tightens the cascade threshold, enters the systemic variance lift $(1 + \rho)$, and sharpens capital-adequacy inequalities. The companion scripts machine-check 37 theorem statements given eight explicit modeling primitives in the Grade-2 base file—useful scaffolding for policy language, not a substitute for data work on real networks.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author

reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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