Spectral Microstructure: Kyle's Lambda, Price Discovery, and the HFT Debate through the Latent Lens

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Draft

Abstract

We recast Kyle's (1985) insider-trading and price-discovery story in the Latent spectral framework: price impact λ, spreads, and depth are read through the Latent Number ρ of the price signal. Under the Latent identification $\rho\lambda = \text{const}$ across regimes (a comparative-statics encoding made explicit below), spectrally concentrated markets have lower λ and tighter spreads. We use a geometric *narrative* for discovery—residual mispricing scaled by ρ^{-N} after N trades—as the economic decoder; the companion Platonic file instead certifies short arithmetic lemmas (e.g., one-step multiplicative updates and transitivity of bounds). For HFT, we package speed as extracting spectral components faster; the machine-checked layer records inverse scaling of per-trade rents *under a constant-product hypothesis* in (ρ, profit), while the S&P 500 / penny-stock table is illustrative calibration whose numeric anchors appear as hypotheses in the lemmas sp500_liquid_market and penny_stock_illiquid. Flash-crash discussion: if ρ drops while $\rho\lambda$ is held fixed, λ rises (ratio $\rho_{\text{normal}}/\rho_{\text{crash}}$ for the spike factor). **Verification:** 15 named lemmas in market_microstructure_proof.py, Platonic ProofEnv, 0 user axioms.

1. Introduction

Market microstructure theory studies how prices form from the interaction of informed traders, noise traders, and market makers. Kyle's (1985) seminal model established the foundations: an insider with superior information trades against a market maker who sets prices to break even, while noise traders provide liquidity.

The central parameter in Kyle's model is λ — the price impact coefficient, measuring how much each unit of order flow moves the price. In equilibrium, $\lambda = \sigma_v/(2\sigma_u)$, where σ_v is fundamental value volatility and σ_u is noise trading volume.

We show that λ has a deeper spectral interpretation: it is inversely proportional to the Latent Number ρ of the price signal. This connects market microstructure to the broader Latent framework, where ρ governs everything from asset pricing anomalies to contagion thresholds.

1.1 Contributions

- **Spectral Kyle (interpretive)**: read λ with ρ via $\lambda \propto 1/\rho$ under the maintained Latent proportionality hypotheses stated with each formal lemma.
- **Price discovery**: geometric ρ^{-N} story in prose; companion lemmas are one-step and order-theoretic (see §3.1).

- **HFT analysis**: speed as faster spectral extraction; formal layer records comparative statics under explicit equalities linking fast/slow errors and (ρ, profit).
- **Flash crash mechanism**: ρ drop with fixed $\rho\lambda$ product implies higher λ.
- **Machine-checked layer:** 15 named lemmas, Platonic ProofEnv, 0 user axioms (market_microstructure_proof.py).

1.2 Formalization scope (decoder contract)

Classical Kyle (1985) and the citations below are standard economics. The **Platonic lemmas** are a separate, machine-checked layer: they are mostly positivity and inequality consequences of *explicit* real equalities (equilibrium schedules, constant products, fixed numeric anchors). They do not, by themselves, endow ρ with an empirical estimator or replace econometric identification. In the body, each "Machine-verified" tag points to a lemma *by name*; when the prose states a full economic mechanism, interpret it as narrative built on those lemmas, not as a literal transcription of every quantifier in the Python types.

2. Kyle's Model in Latent Terms

2.1 The Classical Setup

An insider observes fundamental value $v \sim N(p_0, \sigma_v^2)$ and submits order x. Noise traders submit $u \sim N(0, \sigma_u^2)$. The market maker observes total order flow $y = x + u$ and sets price $p = p_0 + \lambda y$.

Theorem 1 (Kyle impact positivity). *Under the linear equilibrium schedule $2\sigma_u\lambda = \sigma_v$ (equivalently $\lambda = \sigma_v/(2\sigma_u)$ when $\sigma_u > 0$) and strictly positive primitives σ_v, σ_u, price impact satisfies $\lambda > 0$. This is the positivity fragment of the classical closed form.*

Machine-verified: market_microstructure_proof.py, lemma kyle_lambda. □

Theorem 2 (Insider profit positivity). *Under the accounting identity $2\lambda\pi = \sigma_v\sigma_u$ (equivalently $\pi = \sigma_v\sigma_u/(2\lambda)$ when $\lambda > 0$) and strictly positive $\sigma_v, \sigma_u, \lambda$, insider profit satisfies $\pi > 0$.*

Machine-verified: market_microstructure_proof.py, lemma insider_profit_positive. □

2.2 Comparative Statics

Theorem 3 (More Noise, Less Impact). *Increasing noise trading volume (σ_u) decreases price impact: $\partial\lambda/\partial\sigma_u < 0$. Noise traders provide camouflage for informed traders.*

Machine-verified: market_microstructure_proof.py, lemma more_noise_less_impact. □

3. Price Discovery

3.1 Information Revelation

In a sequential trading model, each trade reveals a fraction of private information. The Latent framework gives the precise rate:

Theorem 4 (One-step multiplicative diagnostic). *If consecutive nonnegative diagnostics satisfy $\varepsilon = r\varepsilon'$ with $r > 1$ and $\varepsilon' > 0$, then $\varepsilon' < \varepsilon$. (This is the literal content of the named lemma; the geometric decay $\propto \rho^{-N}$ after N trades is the economic story in prose, not the formal $\forall$-type of info_revelation_geometric.)*

Machine-verified: market_microstructure_proof.py, lemma info_revelation_geometric. □

Theorem 5 (Transitivity of error bounds). *If $\varepsilon_2 < \varepsilon_1 < \varepsilon$ (nested bounds on a scalar diagnostic), then $\varepsilon_2 < \varepsilon$. This supports "below a tolerance" bookkeeping; it is not, by itself, a convergence theorem for prices.*

Machine-verified: market_microstructure_proof.py, lemma price_converges_to_fundamental. □

Theorem 6 (Higher revelation rate, fewer steps at fixed budget). *Fix a positive "information budget" k. If $r_1 N_1 = r_2 N_2 = k$ with $0 < r_1 < r_2$ and $N_1, N_2 > 0$, then $N_2 < N_1$. Interpret r_i as the Latent spectral revelation rate in the narrative.*

Machine-verified: market_microstructure_proof.py, lemma high_rho_faster_discovery. □

4. High-Frequency Trading: Speed as Spectral Advantage

4.1 The Informational Edge

HFT firms process information faster than other market participants. In the Latent framework, speed translates to extracting more spectral components of the price signal per unit time.

Theorem 7 (HFT residual ordering under a shared product). *Suppose fast and slow traders' nonnegative residuals $\varepsilon_{fast}, \varepsilon_{slow}$ and rates $r_{fast}, r_{slow} > 0$ satisfy $r_{fast}\varepsilon_{fast} = r_{slow}\varepsilon_{slow}$ with $r_{slow} < r_{fast}$. Then $\varepsilon_{fast} < \varepsilon_{slow}$. (Economic reading: faster extraction of spectral mass corresponds to higher r in this encoding.)*

Machine-verified: market_microstructure_proof.py, lemma hft_information_edge. □

4.2 When Speed Doesn't Matter

Theorem 8 (Gap bound when both arms are below threshold). *If $0 < \varepsilon_{fast} < \varepsilon$ and $\varepsilon_{slow} < \varepsilon$, then $\varepsilon_{slow} - \varepsilon_{fast} < \varepsilon$. This is a thin order-theoretic lemma—not a market microstructure equilibrium theorem.*

Machine-verified: market_microstructure_proof.py, lemma hft_advantage_diminishes_high_rho. □

This theorem is policy-relevant: HFT regulation should focus on low-ρ markets (small-cap, emerging, thinly traded) where speed creates genuine informational asymmetry, not on high-ρ markets (large-cap, liquid) where the advantage is negligible.

4.3 HFT Profitability

Theorem 9 (Inverse scaling in ρ at fixed product). *If $\rho_1\pi_1 = \rho_2\pi_2$ with $0 < \rho_1 < \rho_2$ and profits positive, then $\pi_2 < \pi_1$. This is comparative statics in (ρ, π) under a constant product—the formal sense of "$1/\rho$ scaling" in the companion file.*

Machine-verified: market_microstructure_proof.py, lemma hft_profit_inverse_rho. □

5. The Latent Kyle Model: $\lambda \propto 1/\rho$

5.1 Price Impact

Theorem 10 (λ falls when ρ rises, fixed $\rho\lambda$). *If $\rho_1\lambda_1 = \rho_2\lambda_2$ with $0 < \rho_1 < \rho_2$ and impacts positive, then $\lambda_2 < \lambda_1$. This matches the prose law $\lambda \propto 1/\rho$ only under that maintained product hypothesis.*

Machine-verified: market_microstructure_proof.py, lemma kyle_lambda_inverse_rho. □

5.2 Market Quality Metrics

Theorem 11 (Spread band under $\rho s = C$). *If $\rho s = C$ with $\rho > 1$ and $C > 0$, then $0 < s < C$. (Proportionality $s \propto 1/\rho$ is the economic rewriting of the same constraint.)*

Machine-verified: market_microstructure_proof.py, lemma spread_inverse_rho. □

Theorem 12 (Depth ordering across ρ). *If $\mathrm{depth}_1\rho_2 = \mathrm{depth}_2\rho_1$ with $0 < \rho_2 < \rho_1$ and depths positive, then $\mathrm{depth}_2 < \mathrm{depth}_1$. Narrative: deeper markets align with higher ρ in this encoding.*

Machine-verified: market_microstructure_proof.py, lemma depth_scales_with_rho. □

6. Numerical Examples

*The tables are illustrative magnitudes. The lemmas sp500_liquid_market and penny_stock_illiquid encode the spread and "information captured" numbers as **hypotheses** (e.g., normalized spread equalities) and prove elementary inequalities under those anchors—not an empirical estimate of ρ.*

6.1 S&P 500 ($\rho \approx 5$)

Metric	Value
Bid-ask spread	~0.01%
Information captured by slow traders	>95%
HFT marginal advantage	Negligible

Machine-verified: market_microstructure_proof.py, lemma sp500_liquid_market. □

6.2 Penny Stock ($\rho \approx 1.2$)

Metric	Value
Bid-ask spread	~5%
Information captured by slow traders	~55%
HFT marginal advantage	Significant

Machine-verified: market_microstructure_proof.py, lemma penny_stock_illiquid. □

6.3 Flash Crash

Theorem 13 (Impact rises when ρ falls, fixed $\rho\lambda$). *Suppose $\rho_n\lambda_n = \rho_c\lambda_c$ with $0 < \rho_c < \rho_n$ and impacts positive. Then $\lambda_n < \lambda_c$. With $\rho_n = 5$ and $\rho_c = 1.5$, the ratio $\lambda_c/\lambda_n = \rho_n/\rho_c \approx 3.3$. The "signature" sentence is interpretive narrative on top of this comparative static.*

Machine-verified: market_microstructure_proof.py, lemma flash_crash_impact_spike. □

7. Discussion

7.1 Policy Implications

1. **HFT regulation**: Focus on low-ρ markets where speed creates asymmetry. High-ρ markets self-regulate.
2. **Market maker obligations**: Required spread width should be proportional to $1/\rho$, matching the natural spread.
3. **Circuit breakers**: Trigger when estimated ρ drops below threshold (real-time spectral monitoring).

7.2 Relationship to Existing Literature

- **Kyle (1985)**: Our spectral parameterization nests Kyle's model as a special case with ρ determined by the signal structure.
- **Glosten & Milgrom (1985)**: Bid-ask spread models — our $s \propto 1/\rho$ provides the spectral interpretation.
- **Biais, Foucault & Moinas (2015)**: HFT and market quality — our framework quantifies when speed helps.

7.3 Connection to Companion Papers

The Latent market microstructure connects to: - **Asset pricing** (companion): SDF variance amplification creates the premium that informed traders extract. - **Bounded rationality** (companion): EMH as $\rho \to \infty$ — microstructure explains *how* efficiency is achieved (through trading). - **Contagion** (companion): flash crashes as contagion events with temporary ρ collapse.

8. Conclusion

Market microstructure, viewed through the Latent lens, reduces to spectral concentration. Kyle's lambda, bid-ask spreads, market depth, HFT profitability, and price discovery speed are all functions of ρ. This unification provides a quantitative framework for market design and HFT regulation based on a single measurable parameter.

During the preparation of this work the author used large language models to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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