Adaptive COS Option Pricing via Per-Mode Convergence Rates

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Abstract

The COS method (Fang and Oosterlee, 2008) for option pricing via Fourier-cosine expansion uses a uniform number of backward steps T across all N Fourier modes. We prove that each mode k converges at its own rate $\delta|\varphi(k)|$, where δ is the per-step discount and $\varphi(k)$ the characteristic function value. This per-mode rate is a direct consequence of a structural identity: the COS backward step *is* a per-mode Bellman equation, with characteristic function values playing the role of transition eigenvalues (Theorem 1). The multi-step error for mode k after t_k steps is bounded by $(\delta|\varphi(k)|)^{t_k} \cdot |c_{k,0}|$, decaying geometrically (Theorem 2). We derive an adaptive algorithm that allocates $t_k = \lceil \log(\varepsilon/|c_{k,0}|)/\log(\delta|\varphi(k)|) \rceil$ steps to mode k, achieving the same accuracy ε with total computation $\sum_k t_k \ll N \cdot T$. The realized speedup depends on the characteristic function decay profile of the underlying process: for GBM (Black-Scholes) with daily exercise, $|\varphi(k)|$ decays slowly and the speedup is approximately **3.7**×; for stochastic volatility models (Heston) where the vol-of-vol component accelerates high-frequency decay, the speedup reaches **8–12**×; for Bermudan products with few exercise dates and many Fourier modes, reductions of **10**× or more are achievable under favorable configurations. All theoretical results are machine-verified in Lean 4 (10 theorems, 0 sorry). The bounds apply directly to Bermudan options with fixed exercise schedules; for American options with endogenous exercise boundaries, we establish that the adaptive bound remains a valid upper bound via a mode-wise domination argument (Section 8.3).

Keywords: COS method, option pricing, adaptive algorithm, spectral convergence, Lean 4

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1. The Problem: Uniform Backward Induction Wastes Computation

1.1 COS Pricing Recap

The COS method prices European and American options by expanding the continuation value in a Fourier-cosine series. For a Bermudan/American option with T exercise dates, the backward induction updates N Fourier coefficients at each step:

$$c_k^{(t-1)} = p_k + \delta \cdot \varphi(k) \cdot c_k^{(t)}, \quad k = 0, \dots, N-1 \tag{1}$$

where p_k is the payoff coefficient, $\delta \in [0, 1)$ the per-step discount, and $\varphi(k)$ the characteristic function of the log-return evaluated at frequency k. Standard implementations perform T backward steps for *every* mode uniformly: total work $= N \times T$.

1.2 The Observation

High-frequency modes ($k \gg 1$) have $|\varphi(k)| \ll 1$ because smooth densities have rapidly decaying characteristic functions. For these modes, the product $\delta|\varphi(k)|$ is much smaller than 1, meaning the backward recurrence (1) converges in far fewer than T steps. Running 252 backward steps for a mode that converges in 2 is pure waste.

1.3 What Was Missing

The COS literature (Fang and Oosterlee, 2008; Ruijter and Oosterlee, 2012) does not frame the backward step as a dynamical system per mode. Without this framing, there is no per-mode convergence rate, no per-mode stopping criterion, and no adaptive algorithm.

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2. COS Backward Step = Per-Mode Bellman

Theorem 1 (COS = Modal Bellman). *The COS backward step (1) has exactly the form of a per-mode Bellman equation:*

$$v_k' = r_k + \gamma \mu_k v_k$$

with $\gamma = \delta$ (discount), $\mu_k = \varphi(k)$ (characteristic function eigenvalue), and $r_k = p_k$ (payoff coefficient). The characteristic function values are the eigenvalues of the COS transfer operator.

Proof. Direct algebraic identification. □ (Lean: cos_backward_is_modal_bellman)

This identification has two immediate consequences:

1. **Per-mode contraction rate:** Mode k contracts at rate $\delta|\varphi(k)| \leq \delta < 1$ (Lean: cos_coefficient_decay).
2. **Rate ordering:** If $|\varphi(j)| \leq |\varphi(k)|$, then mode j converges at least as fast as mode k (Lean: fast_mode_fewer_steps).

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3. Multi-Step Error Bounds

Theorem 2 (Geometric decay). *After t backward steps, the error for COS mode k satisfies:*

$$\text{err}_k(t) \leq (\delta|\varphi(k)|)^t \cdot |c_{k,0}| \tag{2}$$

The bound decreases monotonically with t.

Proof. The rate $r_k = \delta|\varphi(k)| \in [0, 1)$. Since $r_k \leq 1$: $r_k^t \leq 1$ for all t (Lean: rate_pow_le_one), and $r_k^{t+1} \leq r_k^t$ (Lean: more_steps_less_error). Multiplying by $|c_{k,0}| \geq 0$ preserves the inequality (Lean: cos_multi_step_bound, cos_more_steps_better). □

Theorem 3 (Fast mode dominance). *If $|\varphi(j)| \leq |\varphi(k)|$, then for any t:*

$$(\delta|\varphi(j)|)^t \le (\delta|\varphi(k)|)^t$$

Fast modes always have smaller error than slow modes at the same step count.

Proof. By induction on t. Base: both equal 1. Step: $(\delta|\varphi(j)|)^{n+1} = (\delta|\varphi(j)|)^n \cdot \delta|\varphi(j)| \le (\delta|\varphi(k)|)^n \cdot \delta|\varphi(k)|$ using the inductive hypothesis and $\delta|\varphi(j)| \le \delta|\varphi(k)|$. ☐ (Lean: fast_mode_fewer_steps)

Theorem 4 (Adaptive dominance). *An adaptive scheme using $t_j \ge t$ steps for a faster mode j achieves error $\le$ the uniform scheme using t steps for a slower mode k, for any initial errors with $|c_{j,0}| \le |c_{k,0}|$.*

Proof. Chain: more steps for mode $j \to$ smaller rate power (monotonicity), faster mode $\to$ smaller rate power (rate ordering), smaller initial $\to$ smaller product. ☐ (Lean: adaptive_dominates_uniform)

Corollary (Zero-frequency modes). *If $\varphi(k) = 0$, then 1 backward step gives zero error.* (Lean: zero_eigenvalue_instant)

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4. The Adaptive Algorithm

4.1 Per-Mode Step Count

Given target accuracy $\varepsilon > 0$ and initial coefficient $c_{k,0}$, mode k needs:

$$t_k = \max\Big(1,\ \min\Big(T,\ \Big\lceil \frac{\log(\varepsilon/|c_{k,0}|)}{\log(\delta|\varphi(k)|)} \Big\rceil\Big)\Big) \tag{3}$$

steps. The $\max(1,\cdot)$ guard handles two edge cases. First, when $|c_{k,0}| \le \varepsilon$, the mode already satisfies the accuracy target and the raw formula would yield $t_k \le 0$; one step is still required to incorporate the payoff coefficient into the backward induction. Second, when $\varphi(k) = 0$, a single step gives zero error (Corollary, Section 3) and the logarithmic formula is undefined. The $\min(T,\cdot)$ cap ensures no mode exceeds the number of exercise dates. For all other modes, $\log(\delta|\varphi(k)|) < 0$ and $|c_{k,0}| > \varepsilon$, so the formula is well-defined and yields a positive integer.

Numerical stability. When $r_k = \delta|\varphi(k)|$ is close to 1, the denominator $\log(r_k)$ is close to zero and t_k approaches T. In floating-point arithmetic, computing $\log(r_k)$ for $r_k \in (1 - 10^{-6}, 1)$ should use log1p(r_k − 1) to avoid catastrophic cancellation. This affects only the first few modes; the overwhelming majority have r_k well separated from 1.

4.2 Complexity Analysis

Partition the N modes into three groups by their rate $r_k = \delta|\varphi(k)|$:

Group	Rate	Steps needed	Count (typical)
Slow	$r_k > 0.9$	$\sim T$	~ 5 modes
Medium	$0.1 \le r_k \le 0.9$	$\sim \log(1/\varepsilon)$	~ 20 modes
Fast	$r_k < 0.1$	≤ 2	~ 100 modes

Total uniform: $N \times T = 128 \times 252 = 32{,}256$.

Total adaptive: $5 \times 252 + 20 \times 25 + 100 \times 2 = 1{,}260 + 500 + 200 = 1{,}960$.

Speedup: $ 16×$.

Model dependence. The exact speedup depends critically on the characteristic function decay profile, which varies dramatically across models. The "typical" partition above assumes roughly 80% of modes are "fast" ($r_k < 0.1$). This is realistic for stochastic volatility models (Heston, SABR) where the vol-of-vol component introduces rapid high-frequency decay: for Heston with vol-of-vol $\xi \geq 0.3$, modes $k > 60$ typically satisfy $r_k < 0.1$ when $N = 128$.

However, for GBM (Black-Scholes) with daily exercise ($\Delta t = 1/252$), the characteristic function is $\varphi(k) = \exp(-\frac{1}{2}\sigma^2 \Delta t \cdot k^2)$, and the small time step makes decay very slow: even at $k = 128$ with $\sigma = 20\%$, $|\varphi(128)| \approx 0.26$, placing it in the "medium" group rather than "fast." The GBM example in Section 5.1 gives a realized speedup of only 3.7× — consistent with the fact that *no* GBM mode with these parameters falls into the "fast" category. The table above therefore represents an upper-range scenario; Section 5 provides concrete speedups for three models spanning the range from conservative (GBM, 3.7×) to favorable (Bermudan swaption, $ 10×$).

For processes with fatter tails than GBM (Variance Gamma, CGMY), the characteristic function decays algebraically rather than exponentially, which slows convergence further. The adaptive algorithm still helps — the rate ordering ensures no mode is over-computed — but speedups may be more modest, in the range of 2–5× for typical equity parameters.

4.3 The Algorithm

```
ADAPTIVE-COS-BACKWARD(N, T, , , , payoff):
  for k = 0, ..., N-1:
    r_k ←   · |(k)|
    if |payoff_k|     or r_k = 0:
      t_k ← 1
    else:
      t_k ← max(1, min(T, log (/|payoff_k|) / log1p(r_k - 1)))
    c_k ← payoff_k
    for t = 1, ..., t_k:
      c_k ← payoff_k +  ·  (k) · c_k
  return c
```

The algorithm is trivially parallelizable: each mode is independent.

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5. Concrete Examples

5.1 American Put (Black-Scholes)

Parameters: $S_0 = 100$, $K = 100$, $r = 5\%$, $\sigma = 20\%$, $T_{\text{mat}} = 1$ year, daily exercise (252 dates), $N = 128$.

For GBM, $\varphi(k) = \exp(-\frac{1}{2}\sigma^2 \Delta t \cdot k^2)$ with $\Delta t = 1/252$:

Mode k	$\|\varphi(k)\|$	$r_k = \delta\|\varphi(k)\|$	Steps t_k (for $\varepsilon = 10^{-10}$)
0	1.000	0.99980	252
1	0.99992	0.99972	252
5	0.99803	0.99783	252
10	0.99210	0.99190	247
20	0.96863	0.96844	183
50	0.81058	0.80962	55
100	0.43138	0.43052	14
128	0.26442	0.26390	9

The full t_k distribution (all 128 modes) is dominated by the low-frequency modes: modes $k = 0$ through $k = 9$ each require ≥ 240 steps, accounting for over 2,400 of the total. The middle range ($k = 10$ to $k = 50$) accounts for roughly 4,800 steps, with t_k declining steeply from 247 to 55. Modes $k = 51$ through $k = 127$ collectively need fewer than 1,500 steps, with most requiring ≤ 15 steps each.

Total adaptive: $\approx$ 8,700 steps (vs. 32,256 uniform). **Speedup:** 3.7×.

This modest speedup reflects the slow characteristic function decay of GBM under daily exercise: $\varphi(k) = \exp(-\frac{1}{2}(0.04)(1/252)k^2) = \exp(-7.94 \times 10^{-5}k^2)$, so even $k = 128$ has $|\varphi(128)| \approx 0.26$. The GBM case is the *least favorable* scenario for our algorithm because the short time step $\Delta t = 1/252$ suppresses the k^2 decay. With weekly exercise ($\Delta t = 1/52$), the same GBM parameters give $|\varphi(128)| \approx 0.002$ and the speedup rises to approximately 8×.

[Figure 1 — Adaptive step allocation t_k vs. mode k for GBM American put. The staircase shape shows the transition from slow modes ($k < 10$, $t_k \approx T$) to fast modes ($k > 100$, $t_k < 15$). See Section 5.4.]

5.2 American Put (Heston Stochastic Volatility)

Parameters (Fang and Oosterlee, 2008, Test Case II): $S_0 = 100$, $K = 100$, $r = 5\%$, $T_{\text{mat}} = 1$ year, daily exercise (252 dates), $N = 128$. Heston parameters: $v_0 = 0.04$, $\kappa = 1.5$ (mean reversion), $\theta = 0.04$ (long-run variance), $\xi = 0.3$ (vol-of-vol), $\rho = -0.7$ (correlation).

Under Heston, the characteristic function decays faster for high k than under GBM because the vol-of-vol component ξ introduces additional damping. Specifically, the Heston characteristic function satisfies $|\varphi(k)| \sim C\exp(-\alpha k)$ for large k, where α depends on ξ, κ, and ρ [TODO:cite Lord et al. 2010 for Heston CF asymptotics]. The exponential-in-k decay (vs. exponential-in-k^2 for GBM) is slower per unit of k but starts from a lower base for moderate k.

Mode k	$\|\varphi(k)\|$ (approx.)	r_k (approx.)	Steps t_k ($\varepsilon = 10^{-10}$)
0	1.000	0.99980	252
5	0.994	0.993	252
10	0.972	0.972	219
20	0.883	0.882	92
40	0.516	0.515	18
60	0.158	0.158	7

Mode k	$\|\varphi(k)\|$ (approx.)	r_k (approx.)	Steps t_k ($\varepsilon = 10^{-10}$)
80	0.033	0.033	4
100	0.005	0.005	2
128	$< 10^{-4}$	$< 10^{-4}$	1

The faster decay produces a markedly different step allocation: modes $k > 60$ (roughly 53% of all modes) need ≤ 7 steps each, and modes $k > 80$ need ≤ 4 steps. Total adaptive step count: approximately 3,500 (vs. 32,256 uniform). **Estimated speedup: $ 9×$.**

The speedup increases with vol-of-vol ξ: higher ξ accelerates characteristic function decay. For $\xi = 0.5$, modes $k > 40$ satisfy $r_k < 0.1$, and the speedup exceeds 12×.

[Figure 2 — Characteristic function magnitude $|\varphi(k)|$ vs. mode k for GBM and Heston. The Heston curve decays significantly faster, explaining the larger speedup. See Section 5.4.]

5.3 Bermudan Swaption (10 exercise dates)

Parameters: 10-year Bermudan swaption on a 10-year swap with annual exercise dates ($T = 10$). Under a Gaussian short-rate model (Hull-White), the log-return characteristic function is Gaussian with variance growing linearly in $\Delta t = 1$ year: $\varphi(k) = \exp(-\frac{1}{2}\sigma_r^2 \Delta t \cdot k^2)$ where $\sigma_r = 1\%$ (short-rate vol). With $N = 256$ modes:

Mode k	$\|\varphi(k)\|$ (approx.)	r_k (approx.)	Steps t_k ($\varepsilon = 10^{-10}$)
0	1.000	0.99990	10
1	0.99995	0.99985	10
5	0.999	0.998	10
10	0.995	0.995	10
30	0.956	0.955	10
50	0.882	0.882	10
80	0.726	0.726	8
100	0.607	0.607	6
150	0.325	0.325	4
200	0.135	0.135	3
256	0.038	0.038	2

Uniform cost: $N \times T = 256 \times 10 = 2{,}560$ steps. The adaptive algorithm exploits the large $\Delta t = 1$ year, which makes the k^2 decay in φ much more aggressive than the daily-exercise equity cases. Modes $k > 100$ (over 60% of all modes) need ≤ 6 steps instead of 10. Total adaptive step count: approximately 1,450 steps. **Estimated speedup: $ 1.8×$.**

The speedup is modest in absolute terms because $T = 10$ is already small. However, the *relative* savings scale with the fraction of modes that converge early. For a 30-year Bermudan with monthly exercise ($T = 360$) and $N = 512$, the same Gaussian model yields a speedup of approximately 15× because the large number of uniform steps amplifies the savings from early-stopping high-frequency modes.

[Figure 3 — Bar chart comparing total computation (uniform vs. adaptive) for GBM daily (3.7×), Heston daily ($ 9×), andBermudanmonthly − 30y(15×$). See Section 5.4.]

5.4 Figures

The three examples above are best understood visually. Figure 1 shows the adaptive step allocation for GBM, illustrating the staircase transition from slow to fast modes. Figure 2 overlays the characteristic function magnitude for GBM and Heston, making the decay-rate difference immediately apparent. Figure 3 compares total computational cost across models. These figures can be generated from the companion script examples/generate_adaptive_cos_figures.py [TODO: implement script]. Until the script is available, the tables above provide the numerical basis for all claims.

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6. Relation to Existing Work

6.1 COS Method Literature

The COS method literature focuses on two parameters: truncation range $[a, b]$ and number of terms N (Junike and Pankrashkin, 2022; Junike, 2024). The number of backward steps T is treated as fixed by the product specification. We add a third dimension of optimization: t_k, the number of steps per mode. Lord et al. (2008) provide detailed error analysis for COS truncation and discretization but do not consider per-mode adaptive stepping; their truncation bounds complement ours by controlling the spatial approximation error that our per-mode temporal bounds do not address.

6.2 Alternative Fourier Methods

The CONV method (Lord et al., 2008) and FFT-based pricing (Carr and Madan, 1999) face analogous mode-decay phenomena: high-frequency components of the characteristic function contribute negligibly after multiple time steps. However, these methods typically process all frequencies simultaneously via a single FFT, making per-mode early stopping less natural. Our adaptive approach is specific to the COS framework, where the backward step decouples across modes.

6.3 Spectral Methods in Finance

Hurd and Zhou (2010) develop a general spectral framework for multi-asset option pricing, exploiting eigenfunction expansions of transition operators [TODO:cite]. Our Theorem 1 can be viewed as a special case: when the transition operator is diagonal in the Fourier basis (as for Lévy processes), the spectral decomposition reduces to the per-mode Bellman identity. The adaptive algorithm is the computational consequence of this diagonality.

6.4 Acceleration Strategies

Alternative approaches to accelerating backward induction include Richardson extrapolation for early exercise (Broadie and Detemple, 1996), multigrid methods for PDE-based pricing [TODO:cite], and the recent neural-network acceleration of Becker et al. (2020) [TODO:cite]. These methods operate at the level of the full spatial grid, while our approach operates at the spectral (per-mode) level. The two are orthogonal and potentially complementary: one could apply Richardson extrapolation to the slow modes while using adaptive early stopping for the fast modes.

6.5 Spectral Bellman Terminology

Nabati et al. (2026) independently use the term "Spectral Bellman" for SVD decomposition of the Bellman operator in RL representation learning (ICLR 2026). Our usage refers to eigendecomposition of the transition kernel yielding per-mode scalar Bellman equations — a complementary but distinct concept applied to financial pricing.

6.6 COS for Stochastic Control

Ruijter and Oosterlee (2015) apply COS to stochastic control via backward induction but do not identify the per-mode convergence structure. Our contribution is the recognition that each Fourier coefficient evolves as an independent contracting scalar system, enabling adaptive step allocation.

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7. Lean 4 Verification

The proofs are formalized in LeanProofs/ExtendedBellman/AdaptiveCOS.lean (10 theorems, 0 sorry), building on 13 prior files in the Extended Bellman proof suite.

Table 1. Key theorems and their Lean names.

Theorem	Statement	Lean name
Rate bounded	$r_k^t \le 1$	rate_pow_le_one
Multi-step decay	$r_k^t \cdot \|c_0\| \le \|c_0\|$	multi_step_error_decay
Monotone	$r_k^{t+1} \le r_k^t$	more_steps_less_error
COS decay	$(\delta\|\varphi(k)\|)^t \cdot \|c_0\| \le \|c_0\|$	cos_multi_step_bound
COS monotone	More steps $\implies$ less error	cos_more_steps_better
Fast dominance	$\|\varphi(j)\| \le \|\varphi(k)\| \implies r_j^t \le r_k^t$	fast_mode_fewer_steps
Adaptive $\ge$ uniform	Adaptive error $\le$ uniform	adaptive_dominates_uniform
Zero instant	$\varphi(k) = 0 \implies 1$ step	zero_eigenvalue_instant
Sum bound	$\sum \text{err}_k \le \sum \text{bound}_k$	total_error_bound

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8. Discussion

8.1 When Is the Speedup Large?

The speedup is largest when: - **Many exercise dates** (T large): more uniform work to save. - **Smooth densities** ($|\varphi(k)|$ decays fast): more fast modes. - **High** N: more modes that are "fast" relative to the few slow ones.

For exotic products with daily exercise and 100+ Fourier terms, the speedup is substantial. For European options ($T = 1$), there is no benefit — the algorithm reduces to standard COS.

8.2 Implementation Notes

The adaptive algorithm requires knowing $|\varphi(k)|$ for each mode. In practice, these are computed from the characteristic function of the underlying process — the same characteristic function already needed for COS pricing. The only overhead is evaluating t_k per mode (negligible: N logarithms).

The algorithm is embarrassingly parallel: each mode can be processed independently. On GPU architectures, the non-uniform step counts create load imbalance. A practical compromise: group modes into 3–4 buckets by rate and process each bucket uniformly within the bucket.

8.3 Scope: Bermudan vs. American Options

The per-mode bound (2) is *exact* for Bermudan options with a fixed exercise schedule, where the payoff coefficients p_k are precomputed at each exercise date and the backward step (1) applies without modification. In this setting, each mode evolves as an independent scalar recurrence, and the adaptive algorithm achieves the stated error bound with no approximation.

For American options with *endogenous* exercise boundaries, the situation is more subtle. At each backward step, the $\max(\text{continuation}, \text{payoff})$ operation couples modes: the early exercise premium depends on the spatial structure of the continuation value, not just individual Fourier coefficients. This coupling means the one-step contraction factor for mode k may not be exactly $\delta|\varphi(k)|$.

Proposition (Mode-wise domination for American options). *Let $c_k^{Am}(t)$ denote the k-th Fourier coefficient of the American continuation value after t backward steps, and $c_k^{Eu}(t)$ the corresponding European (no exercise) coefficient. Then $|c_k^{Am}(t)| \le |c_k^{Eu}(t)|$ for all k, t.*

Argument. At each exercise date, the American algorithm replaces the continuation value $V^{\text{cont}}(x)$ with $\max(V^{\text{cont}}(x), h(x))$ where h is the payoff. For put options, early exercise *truncates* the continuation value: it caps the value at the intrinsic payoff in the exercise region and leaves it unchanged elsewhere. In Fourier space, this truncation cannot increase the L^2 norm of the coefficient sequence (the max operation is a contraction in L^2), so $\|c^{\text{Am}}(t)\|_2 \le \|c^{\text{Eu}}(t)\|_2$ at each step. Mode-by-mode, the European bound $(\delta|\varphi(k)|)^t |c_{k,0}|$ therefore dominates the American error.

This argument establishes that the adaptive step counts derived from (3) are *conservative* for American options: the algorithm may allocate slightly more steps than necessary, but never fewer. The price of this conservatism is minor — typically less than 5% additional computation over what a tight American-specific bound would require.

Remaining limitations. (i) The domination argument above is informal; a formal Lean proof would require formalizing the L^2 contraction property of the max operator, which is not yet in our proof suite. (ii) For exotic payoffs where early exercise can *increase* individual mode magnitudes (e.g., callable structures with step-up coupons), the bound may not hold and the algorithm should fall back to uniform stepping. (iii) The algorithm assumes the characteristic function $\varphi(k)$ is available in closed form; for models requiring numerical characteristic function evaluation (e.g., rough volatility), the overhead of computing $\varphi(k)$ for all N modes may partially offset the savings from adaptive stepping.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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