

# American Basket Gym

*Dr. Tamás Nagy*

Dr. Tamás Nagy

tamas@thel latent.space

Skeleton

## Abstract

American Basket Gym — ProofEnv proofs (eigen-COS / FW / GH error bounds).

This paper presents 67 machine-verified theorems. All results are formally verified in the Platonic proof kernel (79 verification units, 67 proved statements) and exportable to Lean 4.

---

## 1. Introduction

## 2. Further Results

**Theorem** (per\_step\_nonneg). *Per Step Nonneg.* [Platonic: per\_step\_nonneg, domain: american\_basket\_gym]

**Theorem** (backward\_nonneg). *Backward Nonneg.* [Platonic: backward\_nonneg, domain: american\_basket\_gym]

**Theorem** (backward\_le\_total). *Backward Le Total.* [Platonic: backward\_le\_total, domain: american\_basket\_gym]

**Theorem** (total\_nonneg). *Total Nonneg.* [Platonic: total\_nonneg, domain: american\_basket\_gym]

**Theorem** (cos\_reduction\_helps). *Cos Reduction Helps.* [Platonic: cos\_reduction\_helps, domain: american\_basket\_gym]

**Theorem** (fw\_reduction\_helps). *Fw Reduction Helps.* [Platonic: fw\_reduction\_helps, domain: american\_basket\_gym]

**Theorem** (gh\_reduction\_helps). *Gh Reduction Helps.* [Platonic: gh\_reduction\_helps, domain: american\_basket\_gym]

**Theorem** (per\_step\_add\_comm). *Per Step Add Comm.* [Platonic: per\_step\_add\_comm, domain: american\_basket\_gym]

**Theorem** (mul\_nonneg\_left). *Mul Nonneg Left.* [Platonic: mul\_nonneg\_left, domain: american\_basket\_gym]

**Theorem** (total\_error\_split). *Total Error Split.* [Platonic: total\_error\_split, domain: american\_basket\_gym]

**Theorem** (total\_error\_monotone\_in\_gh). *Total Error Monotone In Gh.* [Platonic: total\_error\_monotone\_in\_gh, domain: american\_basket\_gym]

**Theorem** (wave2\_mul\_add\_distrib\_M). *Wave2 Mul Add Distrib M*. [Platonic: wave2\_mul\_add\_distrib\_M, domain: american\_basket\_gym]

**Theorem** (wave2\_backward\_ge\_M\_dc). *Wave2 Backward Ge M Dc*. [Platonic: wave2\_backward\_ge\_M\_dc, domain: american\_basket\_gym]

**Theorem** (wave2\_backward\_ge\_M\_df). *Wave2 Backward Ge M Df*. [Platonic: wave2\_backward\_ge\_M\_df, domain: american\_basket\_gym]

**Theorem** (wave2\_te\_monotone\_in\_M). *Wave2 Te Monotone In M*. [Platonic: wave2\_te\_monotone\_in\_M, domain: american\_basket\_gym]

**Theorem** (wave2\_dc\_nonneg\_implies\_Mdc\_nonneg). *Wave2 Dc Nonneg Implies Mdc Nonneg*. [Platonic: wave2\_dc\_nonneg\_implies\_Mdc\_nonneg, domain: american\_basket\_gym]

**Theorem** (wave2\_df\_nonneg\_implies\_Mdf\_nonneg). *Wave2 Df Nonneg Implies Mdf Nonneg*. [Platonic: wave2\_df\_nonneg\_implies\_Mdf\_nonneg, domain: american\_basket\_gym]

**Theorem** (wave2\_sub\_self\_dc). *Wave2 Sub Self Dc*. [Platonic: wave2\_sub\_self\_dc, domain: american\_basket\_gym]

**Theorem** (wave2\_sub\_self\_df). *Wave2 Sub Self Df*. [Platonic: wave2\_sub\_self\_df, domain: american\_basket\_gym]

**Theorem** (wave2\_sub\_self\_dg). *Wave2 Sub Self Dg*. [Platonic: wave2\_sub\_self\_dg, domain: american\_basket\_gym]

**Theorem** (wave2\_sub\_self\_M). *Wave2 Sub Self M*. [Platonic: wave2\_sub\_self\_M, domain: american\_basket\_gym]

**Theorem** (wave2\_neg\_doubles\_backstep). *Wave2 Neg Doubles Backstep*. [Platonic: wave2\_neg\_doubles\_backstep, domain: american\_basket\_gym]

**Theorem** (wave2\_total\_plus\_neg\_te). *Wave2 Total Plus Neg Te*. [Platonic: wave2\_total\_plus\_neg\_te, domain: american\_basket\_gym]

**Theorem** (wave2\_per\_step\_triangle). *Wave2 Per Step Triangle*. [Platonic: wave2\_per\_step\_triangle, domain: american\_basket\_gym]

**Theorem** (wave2\_ratio\_one). *Wave2 Ratio One*. [Platonic: wave2\_ratio\_one, domain: american\_basket\_gym]

**Theorem** (wave2\_le\_of\_eq\_backward). *Wave2 Le Of Eq Backward*. [Platonic: wave2\_le\_of\_eq\_backward, domain: american\_basket\_gym]

**Theorem** (wave2\_symm\_total\_error). *Wave2 Symm Total Error*. [Platonic: wave2\_symm\_total\_error, domain: american\_basket\_gym]

**Theorem** (wave2\_expand\_te\_twice). *Wave2 Expand Te Twice*. [Platonic: wave2\_expand\_te\_twice, domain: american\_basket\_gym]

**Theorem** (wave2\_dc\_le\_sum). *Wave2 Dc Le Sum*. [Platonic: wave2\_dc\_le\_sum, domain: american\_basket\_gym]

**Theorem** (wave2\_df\_le\_sum). *Wave2 Df Le Sum*. [Platonic: wave2\_df\_le\_sum, domain: american\_basket\_gym]

**Theorem** (wave3\_diff\_implies\_measurable). *Wave3 Diff Implies Measurable.* [Platonic: wave3\_diff\_implies\_measurable, domain: american\_basket\_gym]

**Theorem** (wave3\_vec\_dot\_comm\_inst). *Wave3 Vec Dot Comm Inst.* [Platonic: wave3\_vec\_dot\_comm\_inst, domain: american\_basket\_gym]

**Theorem** (wave3\_vec\_norm\_sq\_nonneg\_inst). *Wave3 Vec Norm Sq Nonneg Inst.* [Platonic: wave3\_vec\_norm\_sq\_nonneg\_inst, domain: american\_basket\_gym]

**Theorem** (wave3\_vec\_dot\_smul\_left\_inst). *Wave3 Vec Dot Smul Left Inst.* [Platonic: wave3\_vec\_dot\_smul\_left\_inst, domain: american\_basket\_gym]

**Theorem** (wave3\_finset\_sum\_nonneg\_inst). *Wave3 Finset Sum Nonneg Inst.* [Platonic: wave3\_finset\_sum\_nonneg\_inst, domain: american\_basket\_gym]

**Theorem** (wave3\_finset\_prod\_nonneg\_inst). *Wave3 Finset Prod Nonneg Inst.* [Platonic: wave3\_finset\_prod\_nonneg\_inst, domain: american\_basket\_gym]

**Theorem** (wave15\_two\_source\_per\_step). *Wave15 Two Source Per Step.* [Platonic: wave15\_two\_source\_per\_step, domain: american\_basket\_gym]

**Theorem** (wave15\_per\_step\_le\_twice\_max). *Wave15 Per Step Le Twice Max.* [Platonic: wave15\_per\_step\_le\_twice\_max, domain: american\_basket\_gym]

**Theorem** (wave15\_fw\_approx\_eq\_c\_cv\_cubed). *Wave15 Fw Approx Eq C Cv Cubed.* [Platonic: wave15\_fw\_approx\_eq\_c\_cv\_cubed, domain: american\_basket\_gym]

**Theorem** (wave15\_fw\_error\_nonneg). *Wave15 Fw Error Nonneg.* [Platonic: wave15\_fw\_error\_nonneg, domain: american\_basket\_gym]

**Theorem** (wave15\_fw\_error\_monotone\_cv). *Wave15 Fw Error Monotone Cv.* [Platonic: wave15\_fw\_error\_monotone\_cv, domain: american\_basket\_gym]

**Theorem** (wave15\_fw\_error\_at\_zero\_cv). *Wave15 Fw Error At Zero Cv.* [Platonic: wave15\_fw\_error\_at\_zero\_cv, domain: american\_basket\_gym]

**Theorem** (wave15\_conditioning\_reduces\_fw\_error). *Wave15 Conditioning Reduces Fw Error.* [Platonic: wave15\_conditioning\_reduces\_fw\_error, domain: american\_basket\_gym]

**Theorem** (wave15\_residual\_variance\_nonneg). *Wave15 Residual Variance Nonneg.* [Platonic: wave15\_residual\_variance\_nonneg, domain: american\_basket\_gym]

**Theorem** (wave15\_conditional\_variance\_reduction). *Wave15 Conditional Variance Reduction.* [Platonic: wave15\_conditional\_variance\_reduction, domain: american\_basket\_gym]

**Theorem** (wave15\_variance\_monotone\_in\_captured). *Wave15 Variance Monotone In Captured.* [Platonic: wave15\_variance\_monotone\_in\_captured, domain: american\_basket\_gym]

**Theorem** (wave15\_residual\_zero\_at\_full\_rank). *Wave15 Residual Zero At Full Rank.* [Platonic: wave15\_residual\_zero\_at\_full\_rank, domain: american\_basket\_gym]

**Theorem** (wave15\_residual\_scales\_with\_total). *Wave15 Residual Scales With Total.* [Platonic: wave15\_residual\_scales\_with\_total, domain: american\_basket\_gym]

**Theorem** (wave15\_two\_nonneg\_sum\_and\_comm). *Wave15 Two Nonneg Sum And Comm.* [Platonic: wave15\_two\_nonneg\_sum\_and\_comm, domain: american\_basket\_gym]

**Theorem** (total\_error\_halved\_when\_components\_halved). *Total Error Halved When Components Halved.* [Platonic: total\_error\_halved\_when\_components\_halved, domain: american\_basket\_gym]

**Theorem** (error\_reduction\_factor\_positive). *Error Reduction Factor Positive.* [Platonic: error\_reduction\_factor\_positive, domain: american\_basket\_gym]

**Theorem** (vega\_error\_from\_vol\_bump). *Vega Error From Vol Bump.* [Platonic: vega\_error\_from\_vol\_bump, domain: american\_basket\_gym]

**Theorem** (cos\_terms\_improve\_accuracy). *Cos Terms Improve Accuracy.* [Platonic: cos\_terms\_improve\_accuracy, domain: american\_basket\_gym]

**Theorem** (cos\_error\_positive). *Cos Error Positive.* [Platonic: cos\_error\_positive, domain: american\_basket\_gym]

**Theorem** (gh\_nodes\_improve\_accuracy). *Gh Nodes Improve Accuracy.* [Platonic: gh\_nodes\_improve\_accuracy, domain: american\_basket\_gym]

**Theorem** (gh\_weight\_nonneg\_implies\_sum\_nonneg). *Gh Weight Nonneg Implies Sum Nonneg.* [Platonic: gh\_weight\_nonneg\_implies\_sum\_nonneg, domain: american\_basket\_gym]

**Theorem** (backward\_error\_accumulates). *Backward Error Accumulates.* [Platonic: backward\_error\_accumulates, domain: american\_basket\_gym]

**Theorem** (optimal\_stopping\_gap). *Optimal Stopping Gap.* [Platonic: optimal\_stopping\_gap, domain: american\_basket\_gym]

### 3. Bounds and Estimates

**Theorem** (delta\_error\_bounded\_by\_price\_error). *Delta Error Bounded By Price Error.* [Platonic: delta\_error\_bounded\_by\_price\_error, domain: american\_basket\_gym]

**Theorem** (gamma\_error\_bounded). *Gamma Error Bounded.* [Platonic: gamma\_error\_bounded, domain: american\_basket\_gym]

**Theorem** (truncation\_ratio\_bounded). *Truncation Ratio Bounded.* [Platonic: truncation\_ratio\_bounded, domain: american\_basket\_gym]

**Theorem** (exercise\_decision\_error\_bounded). *Exercise Decision Error Bounded.* [Platonic: exercise\_decision\_error\_bounded, domain: american\_basket\_gym]

### 4. Regularity

**Theorem** (wave3\_diff\_implies\_continuous). *Wave3 Diff Implies Continuous.* [Platonic: wave3\_diff\_implies\_continuous, domain: american\_basket\_gym]

**Theorem** (wave3\_continuous\_id\_inst). *Wave3 Continuous Id Inst.* [Platonic: wave3\_continuous\_id\_inst, domain: american\_basket\_gym]

**Theorem** (wave3\_continuous\_const\_any). *Wave3 Continuous Const Any.* [Platonic: wave3\_continuous\_const\_any, domain: american\_basket\_gym]

## 5. Spectral Theory

**Theorem** (eigenvalue\_tail\_nonneg). *Eigenvalue Tail Nonneg.* [Platonic: eigenvalue\_tail\_nonneg, domain: american\_basket\_gym]

**Theorem** (more\_eigenvalues\_less\_truncation\_error). *More Eigenvalues Less Truncation Error.* [Platonic: more\_eigenvalues\_less\_truncation\_error, domain: american\_basket\_gym]

## 6. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/american\_basket\_gym/).

File	Role
american_basket_gym_proof.py	

## 7. Discussion

## References