

Pricing Basket Options via Eigenvalue-Conditional Black-Scholes Mixing

Dr. Tamás Nagy

tnagyphd@gmail.com

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Abstract

Pricing European basket options requires the distribution of a weighted sum of correlated lognormals — the Fenton Distribution (Nagy, 2026a) — which has no closed form. We propose **eigenvalue-conditional Black-Scholes mixing**: decompose the correlation matrix, condition on dominant factor realizations via Gauss-Hermite quadrature, and price each conditional basket using Black-Scholes. We prove that conditioning reduces the conditional coefficient of variation by $\sqrt{1 - \tau_K}$ (Proposition 1), with total pricing error bounded by $|V - \hat{V}| \leq \varepsilon_{\text{FW}}(K) + \varepsilon_{\text{GH}}$ (Theorem 1). The method achieves 10/10 across a graduated test suite spanning 2–500 assets, with 1.76% mean error. A 500-asset basket prices in 61 ms; a 1,558-point correlation-volatility stress surface computes in 8.7 seconds (vs. \$ \$26 minutes for Monte Carlo). The eigenvalue-conditional architecture unifies risk measurement and derivative pricing within a single decomposition framework, validated at 60/60 across six independent test suites.

1. Introduction

1.1 The Basket Option Problem

A European basket call option pays $\max(S_T - K, 0)$ at maturity T , where $S_T = \sum_i w_i S_i(T)$ is the weighted sum of n asset prices. Under geometric Brownian motion, each $\ln S_i(T)$ is normally distributed, making S_T a sum of correlated lognormals — a distribution with no closed form. Pricing requires:

$$V = e^{-rT} \mathbb{E}[\max(S_T - K, 0)] = e^{-rT} \int_K^\infty (x - K) f_{S_T}(x) dx$$

where f_{S_T} is the density of the basket value.

1.2 Related Work

Moment matching. Levy (1992) and Gentle (1993) approximate S_T as a single lognormal via Fenton-Wilkinson moment matching. Deelstra, Liinev, and Vanmaele (2004) price arithmetic basket options by conditioning on a weighted sum of asset prices, deriving sharp lower and upper bounds — a conditioning technique that is the closest precedent to our eigenvalue-conditioning approach. These methods are fast but lose accuracy for OTM options and when the basket distribution departs from lognormality.

Monte Carlo. Glasserman (2004) provides the definitive treatment. Accurate but slow: 10^5 – 10^6 paths are needed, with standard errors decreasing only as $O(1/\sqrt{N})$.

COS method. Fang and Oosterlee (2008) introduced the Fourier-cosine expansion for single-asset option pricing, achieving exponential convergence when the characteristic function is analytically known. Fang and Oosterlee (2009) extended this to early-exercise and barrier options. Ruijter and Oosterlee (2012) developed the two-dimensional COS method via tensor-product grids for multi-asset payoffs. Lord (2010) applied the COS framework to lognormal sums using a complex logarithm transformation. The present work differs from these extensions: rather than extending the COS grid to multiple dimensions (which scales as $O(N^d)$), we use the eigenvalue conditioning of Nagy (2026a) to reduce the problem to one dimension, then bypass the COS pricing formula entirely in favor of per-scenario Black-Scholes evaluation.

Lower bounds. Curran (1994) proposed geometric conditioning for Asian options; Rogers and Shi (1995) derived tight lower bounds. These give useful benchmarks but not exact prices.

1.3 Why COS Payoff Projection Fails for Baskets

The COS method expresses the option price as a dot product of density coefficients A_k and payoff coefficients V_k :

$$V = e^{-rT} \sum_{k=0}^{N-1} A_k \cdot V_k^{\text{payoff}}$$

For single-asset options this works because the characteristic function is known analytically and the domain $[a, b]$ can be chosen tightly around the log-price. For baskets, COS payoff projection produces systematic pricing errors because the wide domain required for tail probability capture — the primary use case of the Spectral Fenton Distribution (Nagy, 2026a) — dilutes spectral resolution near the strike, overestimating the variance in the density coefficients A_k and generating 8–25% pricing bias. We quantify this mechanism in Section 5.4.

The COS payoff coefficients — $\cos(k\pi) = (-1)^k$, vanishing of $\sin(k\pi)$, and the pricing dot-product $\sum_k A_k V_k$ — are Lean-verified (`PayoffCoefficients.lean`; see Nagy, 2026a, Appendix D). The error lies in the density coefficients A_k , not the pricing formula.

Remark. The COS method excels at CDF computation on the same wide domain: the Spectral Fenton Distribution achieves 30/30 graduated test levels for VaR (Nagy, 2026b). The CDF $F(x) = P(S \leq x)$ integrates the density globally, where mean bias is negligible, while the option payoff $\max(S - K, 0)$ amplifies variance errors near the strike.

1.4 Our Approach: Eigenvalue-Conditional Mixing

Instead of Fourier-inverting the basket density on a wide domain and projecting the payoff, we decompose the pricing problem via eigenvalue conditioning — the same infrastructure that computes VaR and ES in Nagy (2026a):

1. **Eigendecompose** the correlation matrix to identify K dominant factors.
2. **Condition** on each factor realization z_q via Gauss-Hermite quadrature.

3. **Price** each conditional basket using the **Black-Scholes formula**, since conditioning reduces the per-scenario volatility sufficiently that the Fenton-Wilkinson lognormal approximation becomes accurate (Proposition 1).
4. **Mix** the conditional prices with quadrature weights.

The key insight is that eigenvalue conditioning — developed for VaR computation in Nagy (2026a) — makes the per-scenario basket sufficiently lognormal that the Black-Scholes formula, rather than Fourier inversion, is the appropriate pricing tool.

1.5 Contributions

1. **Eigenvalue-conditional BS pricing formula** (Section 2) with algorithmic pseudocode.
2. **Formal convergence analysis** (Section 3): we prove the conditional CV reduction (Proposition 1) and the total pricing error bound $|V - \hat{V}| \leq \varepsilon_{\text{FW}}(K) + \varepsilon_{\text{GH}}$ (Theorem 1), paralleling the error decomposition of Nagy (2026a).
3. **Explicit Delta formula** (Section 4) per scenario, derived from the chain rule through the conditional BS formula.
4. **Bachelier fallback** for spread options (Section 6.1) in the phase-cancellation regime (Nagy, 2026a, Section 3.6).
5. **Unified risk-pricing framework** (Section 6.4): VaR, ES, spectral risk measures, and option prices from a single eigendecomposition.

2. The Pricing Formula

2.1 Setup

Consider a basket of n assets with log-returns $Y_i \sim N(\mu_i, \sigma_i^2)$ under the risk-neutral measure, correlated via $\text{Corr}(Y_i, Y_j) = C_{ij}$. The terminal basket value is:

$$S_T = \sum_{i=1}^n w_i \exp(Y_i)$$

Eigendecompose the correlation matrix $C = V\Lambda V^T$. By the structure-scale separation (Nagy, 2026a, Proposition 5; Lean structure-verified — the eigenrelation is axiomatized, and the independence from marginal parameters follows by construction), the eigenvectors V and eigenvalues Λ depend only on the dependency structure, not on the marginal parameters (σ, w, μ) . The covariance matrix is recovered as $\Sigma = \text{diag}(\sigma) \cdot C \cdot \text{diag}(\sigma)$.

2.2 Eigenvalue Conditioning

Select the top K eigenvalues $\lambda_1 \geq \dots \geq \lambda_K$ of C using the portfolio-aware τ -variance rule (Nagy, 2026a, Section 3.2): $K = 1$ if $\lambda_1 / \sum_j \lambda_j > 0.45$, else $K = 2$. Define the factor loading matrix $A = \text{diag}(\sigma) \cdot V_{:,1:K} \cdot \text{diag}(\sqrt{\lambda_{1:K}})$ and residual covariance $\Sigma_{\text{res}} = \Sigma - AA^T$.

Conditioned on factor realizations $Z = z$, the log-returns become:

$$Y_i | Z = z \sim N\left(\mu_i + \sigma_i \left(V_{i,1:K} \sqrt{\Lambda_K} z\right), (\Sigma_{\text{res}})_{ii}\right)$$

with residual correlations near zero (absorbed by the dominant factors).

2.3 Per-Scenario Black-Scholes Pricing

For each factor realization z_q , the conditional basket $S_T | Z = z_q$ is a sum of nearly independent lognormals. Apply the Fenton-Wilkinson approximation to obtain conditional forward and variance:

$$M_q = \sum_i w_i \exp\left(\mu_i^{(q)} + \frac{(\sigma_{\text{res},i})^2}{2}\right)$$

$$V_q = \sum_{i,j} w_i w_j \exp\left(\mu_i^{(q)} + \mu_j^{(q)} + \frac{(\sigma_{\text{res},i})^2 + (\sigma_{\text{res},j})^2}{2} + (\Sigma_{\text{res}})_{ij}\right) - M_q^2$$

where $\mu_i^{(q)} = \mu_i + A_{i,:} z_q = \mu_i + \sigma_i (V_{i,1:K} \sqrt{\Lambda_K} z_q)$. The FW lognormal volatility and conditional call price are:

$$\sigma_{\text{FW},q} = \sqrt{\ln(1 + V_q/M_q^2)}$$

$$C_q = M_q \Phi(d_{1,q}) - K \Phi(d_{2,q}), \quad d_{1,q} = \frac{\ln(M_q/K) + \sigma_{\text{FW},q}^2/2}{\sigma_{\text{FW},q}}, \quad d_{2,q} = d_{1,q} - \sigma_{\text{FW},q}$$

2.4 Quadrature Mixing

The unconditional call price is the Gauss-Hermite weighted average:

$$V_{\text{call}} = e^{-rT} \sum_{q=1}^Q w_q C_q$$

where the factor realizations z_q and weights w_q come from K -dimensional Gauss-Hermite quadrature ($Q = n_q$ for $K = 1$; $Q = n_q^2$ for $K = 2$; with $n_q = 32$ or $n_q = 16$ respectively). Put prices follow from put-call parity: $P = V_{\text{call}} - e^{-rT}(F - K)$, where $F = \sum_i w_i e^{\mu_i + \sigma_i^2/2}$ is the basket forward.

2.5 Algorithm

ALGORITHM: Eigenvalue-Conditional BS Basket Pricing

INPUT: w (n), μ (n), σ (n), C (n × n), K_{strike} , r , T
 OUTPUT: V_{call} , $\Delta(n)$

1. EIGENDECOMPOSE $C \rightarrow$ eigenvalues lam , eigenvectors V [Prop 5, Nagy 2026a]
2. SELECT K : $K = 1$ if $\text{lam}_1/\text{sum}(\text{lam}) > 0.45$, else $K = 2$
3. SET n_q : $n_q = 32$ ($K=1$) or $n_q = 16$ ($K=2$)
4. COMPUTE $A = \text{diag}(\text{sig}) * V(:, 1:K) * \text{diag}(\text{sqrt}(\text{lam}(1:K)))$
5. COMPUTE $\text{Sig}_{\text{res}} = \text{diag}(\text{sig}) * C * \text{diag}(\text{sig}) - A * A'$
6. FOR $q = 1, \dots, Q = n_q \hat{K}$:

- a. $\mu_i(q) = \mu_i + \text{sig}_i * (V(i, 1:K) * \text{sqrt}(\text{Lam}) * z_q)$
- b. $M_q, V_q = \text{FW_moments}(\mu(q), \text{Sig_res})$
- c. $\text{sig}_q = \text{sqrt}(\ln(1 + V_q / M_q^2))$
- d. IF $M_q > 0$: $C_q = \text{BS_call}(M_q, K_strike, \text{sig}_q)$
ELSE: $C_q = \text{Bachelier_call}(M_q, K_strike, \text{sqrt}(V_q))$
- e. $\text{delta}_i(q) = \text{Phi}(d1_q) * w_i * F_i(q) / S_i(0)$

- 7. $V_call = \exp(-rT) * \text{sum}_q w_q * C_q$
- 8. $\text{Delta}_i = \exp(-rT) * \text{sum}_q w_q * \text{delta}_i(q)$

COST: $O(n^3)$ eigen + $O(Q*n^2)$ moments + $O(Q)$ BS evals
TYPICAL: < 50 ms per price ($n \leq 50, K \leq 2$)

3. Convergence Analysis

3.1 Conditional Variance Reduction

The FW lognormal approximation is accurate when the coefficient of variation $CV = \sqrt{V}/M$ is small. Without conditioning, CV can exceed 100% for volatile baskets. We show that eigenvalue conditioning reduces it systematically.

Proposition 1 (Conditional CV Reduction). *Let $S_T \sim \text{Fenton}(w, \mu, \sigma, C)$ with unconditional coefficient of variation $CV_0 = \sqrt{\text{Var}(S_T)}/\mathbb{E}[S_T]$. After conditioning on K factors capturing fraction $\tau_K = w^T \Sigma_K w / (w^T \Sigma w)$ of portfolio variance, the conditional coefficient of variation at quadrature point q satisfies:*

$$CV_q^2 \leq (1 - \tau_K) \cdot CV_0^2 + O(\sigma_{\text{res}}^4)$$

In particular, $CV_q \leq \sqrt{1 - \tau_K} \cdot CV_0$ to leading order.

Proof. The portfolio variance decomposes as $\text{Var}(S_T) = \text{Var}(\mathbb{E}[S_T|Z]) + \mathbb{E}[\text{Var}(S_T|Z)]$ by the law of total variance. The first term captures the explained variance from the K conditioning factors; the second is the residual. By construction, $\text{Var}(\mathbb{E}[S_T|Z])/\text{Var}(S_T) = \tau_K$. Hence $\mathbb{E}[\text{Var}(S_T|Z)] = (1 - \tau_K) \cdot \text{Var}(S_T)$. The conditional variance at any particular z_q deviates from the expectation by $O(\sigma_{\text{res}}^4)$ due to the multiplicative structure of the lognormal. Since $M_q \approx M_0$ up to factor-shift corrections that preserve the ratio V_q/M_q^2 , the CV bound follows. \square

Remark (Lean-verified). The variance decomposition underlying Proposition 1 — specifically, the identity $w^T \Sigma w = w^T \Sigma_K w + w^T \Sigma_{\text{res}} w$ and the well-definedness of $\tau_K \in [0, 1]$ — has been formally verified in Lean 4 (ConditionalVariance.lean; see Nagy, 2026a, Appendix D).

Table 1. Empirical conditional volatility reduction across representative portfolios.

Scenario	Portfolio σ	FW σ (unconditional)	FW σ (conditional)	τ_K
2 asset, low vol	0.20	0.14	0.04–0.08	0.85

Scenario	Portfolio σ	FW σ (unconditional)	FW σ (conditional)	τ_K
4 asset, crypto	0.70	0.37	0.11	0.91
10 asset, high vol	0.50	0.24	0.06–0.12	0.88
50 asset, flat	0.15	0.05	0.01–0.03	0.78

At conditional $\sigma_{\text{FW}} \leq 0.12$, the FW approximation — and therefore the BS formula — achieves sub-percent accuracy on the conditional basket. The unconditional pricing error is the quadrature-weighted average of small conditional errors.

3.2 Pricing Error Bound

Theorem 1 (Pricing Error Decomposition). *Let $V = e^{-rT} \mathbb{E}[\max(S_T - K, 0)]$ be the true basket option price and \hat{V} the eigenvalue-conditional BS estimate from Section 2. Then:*

$$|V - \hat{V}| \leq \varepsilon_{\text{FW}}(K) + \varepsilon_{\text{GH}}$$

where:

Component	Source	Bound
$\varepsilon_{\text{FW}}(K)$	Per-scenario FW lognormal approximation	$\leq \Psi_{\text{pr}} \cdot (1 - \tau_K) \cdot \text{CV}_0^2$
ε_{GH}	Gauss-Hermite quadrature over factor space	$\leq K \cdot \frac{n_q! \sqrt{\pi}}{2^{n_q} (2n_q)!} \cdot D_{2n_q}$

The sensitivity constant Ψ_{pr} depends on the payoff: for call options, $\Psi_{\text{pr}} \leq (F/\sqrt{2\pi}) \cdot e^{-d_1^2/2}$ where F is the basket forward and d_1 is the ATM d -parameter. For $n_q = 32$ and $K \leq 2$, $\varepsilon_{\text{GH}} < 10^{-9}$, leaving $\varepsilon_{\text{FW}}(K)$ as the sole effective error component.

Proof. Decompose the error via the law of iterated expectations:

$$\begin{aligned}
|V - \hat{V}| &= e^{-rT} \left| \mathbb{E}_Z[\mathbb{E}[\max(S_T - K, 0)|Z]] - \sum_q w_q C_q^{\text{BS}} \right| \\
&\leq e^{-rT} \underbrace{\left| \mathbb{E}_Z[C^{\text{true}}(Z)] - \sum_q w_q C^{\text{true}}(z_q) \right|}_{\varepsilon_{\text{GH}}} + e^{-rT} \underbrace{\sum_q w_q |C^{\text{true}}(z_q) - C_q^{\text{BS}}|}_{\varepsilon_{\text{FW}}(K)}
\end{aligned}$$

The first term is Gauss-Hermite quadrature error for the smooth function $z \mapsto C^{\text{true}}(z)$, bounded by the classical GH error formula (see Nagy, 2026a, Lemma 3). The second term is bounded by the FW pricing error per scenario, which depends on CV_q via Proposition 1. \square

Remark. This two-component decomposition parallels the six-component decomposition of the CDF error in Nagy (2026a, Theorem 2). Four of the six CDF-error components (ε_N , $\varepsilon_{\text{GH,inner}}$, $\varepsilon_{[a,b]}$, ε_{fp}) are absent here because the Eigen-Cond BS method bypasses the Fourier inversion entirely — no COS terms, no domain truncation, no inner GH integral per frequency. The remaining two map to $\varepsilon_{\text{FW}}(K)$ (analogous to ε_{res}) and ε_{GH} (the outer quadrature error).

3.3 FW Pricing Error per Scenario

Proposition 2 (FW Pricing Error per Scenario; Lean structure-verified — the FW error bound is taken as standard; the aggregation structure is fully verified). *Let C_q^{true} be the exact conditional call price and C_q^{BS} the Black-Scholes price using FW moments $(M_q, \sigma_{\text{FW},q})$. Then:*

$$|C_q^{\text{true}} - C_q^{\text{BS}}| \leq \frac{M_q}{6\sqrt{2\pi}} \cdot |\gamma_{3,q}| \cdot \sigma_{\text{FW},q}^2$$

where $\gamma_{3,q}$ is the conditional skewness residual (the third standardized moment of the conditional basket distribution minus the lognormal third moment). This is $O(\text{CV}_q^3)$.

Proof. The FW approximation matches the first two moments exactly. By a Taylor expansion of the call payoff around the lognormal density, the leading-order error is proportional to the third-moment discrepancy. The density bound at the strike is $f(K) \leq (M_q \sigma_{\text{FW},q} \sqrt{2\pi})^{-1}$ for the lognormal. The third-moment effect scales as $\gamma_{3,q} \cdot \sigma_{\text{FW},q}^3$. Combining: the pricing error is bounded by $M_q / (6\sqrt{2\pi}) \cdot |\gamma_{3,q}| \cdot \sigma_{\text{FW},q}^2$. Since $\gamma_{3,q} \sim \text{CV}_q$ and $\sigma_{\text{FW},q} \sim \text{CV}_q$ for small CV, the bound is $O(\text{CV}_q^3)$. \square

Remark (Lean-verified). The algebraic structure of this bound — moment-matched pricing error bounded by density times skewness times variance — has been verified in Lean 4 (FWPricingError.lean). The quadrature-weighted aggregation $|V - \hat{V}| \leq \sum_q w_q \varepsilon_q$ with non-negative weights is also verified (weighted_pricing_error).

Remark. Combining Propositions 1 and 2: conditioning on K factors with $\tau_K = 0.90$ reduces CV_q by $\sqrt{0.10} \approx 0.32$, which reduces the cubic pricing error by a factor of $0.32^3 \approx 0.03$ — a $30\times$ error reduction. This explains the empirical $9\times$ improvement ($16.5\% \rightarrow 1.76\%$) observed in Table 3: the cubic scaling applies to the systematic FW bias, not to the total error which includes quadrature noise.

3.4 Comparison with COS Pricing Error

The COS payoff projection on the wide FW domain produces a systematic error:

$$\varepsilon_{\text{COS}} \approx \frac{1}{2} \left(\sqrt{1 + \delta_{\text{var}}} - 1 \right) \cdot V_{\text{ATM}}$$

where δ_{var} is the fractional variance overestimate in the A_k coefficients. For the L1 test case: $\delta_{\text{var}} = 0.175$, giving $\varepsilon_{\text{COS}} \approx 8.4\%$ — matching the observed 8.0% error. This is a bias, not a random error, and cannot be reduced by increasing N on the same domain (confirmed empirically: Richardson extrapolation fails).

The Eigen-Cond method avoids this entirely: no Fourier inversion, no wide domain, no spectral coefficients for pricing. The pricing-relevant information — conditional forward M_q and conditional volatility σ_q — is computed directly from portfolio parameters.

4. Greeks

4.1 Delta

The basket Delta with respect to asset i follows from the chain rule through the conditional BS formula:

$$\Delta_i = \frac{\partial V}{\partial S_i(0)} = e^{-rT} \sum_{q=1}^Q w_q \Phi(d_{1,q}) \cdot \frac{w_i F_i^{(q)}}{S_i(0)}$$

where $F_i^{(q)} = \exp(\mu_i^{(q)} + \sigma_{\text{res},i}^2/2)$ is the conditional forward contribution of asset i in scenario q .

Derivation. The conditional call price $C_q = M_q \Phi(d_{1,q}) - K \Phi(d_{2,q})$ depends on $S_i(0)$ through $\mu_i = \ln S_i(0) + (r - \sigma_i^2/2)T$, which enters $\mu_i^{(q)}$ and hence $M_q, V_q, \sigma_{\text{FW},q}$. By the standard BS identity $\partial C/\partial M = \Phi(d_1)$ (where the $\phi(d)$ terms from the d_1 and d_2 derivatives cancel by the lognormal relationship $M\phi(d_1) = K\phi(d_2)$), the leading-order contribution is $\Phi(d_{1,q}) \cdot \partial M_q/\partial S_i(0)$. Since $M_q = \sum_j w_j F_j^{(q)}$ and $\partial F_i^{(q)}/\partial S_i(0) = F_i^{(q)}/S_i(0)$, the stated formula follows. The Vega correction from $\sigma_{\text{FW},q}$ dependence on $S_i(0)$ is $O(\sigma_{\text{FW},q}^2)$ and subdominant after conditioning (Table 1).

4.2 Gamma

$$\Gamma_{ij} = \frac{\partial^2 V}{\partial S_i \partial S_j} = e^{-rT} \sum_q w_q \left[\frac{\phi(d_{1,q})}{M_q \sigma_{\text{FW},q}} \cdot \frac{w_i F_i^{(q)}}{S_i(0)} \cdot \frac{w_j F_j^{(q)}}{S_j(0)} + \delta_{ij} \Phi(d_{1,q}) \cdot \frac{w_i F_i^{(q)}}{S_i(0)^2} \right]$$

where ϕ is the standard normal density. The first term is the BS Gamma of the conditional basket; the second captures the asset-level contribution to the forward.

Derivation. Differentiating the Delta formula (Section 4.1) with respect to $S_j(0)$: the $\Phi(d_{1,q})$ term contributes the conditional BS Gamma via $\partial \Phi(d_1)/\partial M \cdot \partial M/\partial S_j = \phi(d_1)/(M_q \sigma_{\text{FW},q}) \cdot w_j F_j^{(q)}/S_j(0)$, which multiplies the existing $w_i F_i^{(q)}/S_i(0)$ factor from Delta. The diagonal term ($i = j$) captures the second derivative $\partial^2 F_i^{(q)}/\partial S_i(0)^2 = 0$ of the conditional forward, but the product rule on $\Phi(d_1) \cdot F_i^{(q)}/S_i(0)$ yields the δ_{ij} correction from $\partial(F_i/S_i(0))/\partial S_i(0)$.

4.3 Correlation Sensitivity (Cross-Vega)

Changing ρ_{ij} modifies the eigendecomposition, factor loadings A , and residual variances. The sensitivity $\partial V/\partial \rho_{ij}$ is computed by finite difference: one additional eigendecomposition + quadrature pass at $C + \epsilon e_{ij} e_{ij}^T$. Cost: one pricing call per correlation pair.

4.4 Vega

The basket Vega with respect to asset i 's volatility has two channels — unlike Delta, where the volatility channel is subdominant:

$$\text{Vega}_i = \frac{\partial V}{\partial \sigma_i} = e^{-rT} \sum_{q=1}^Q w_q \left[\Phi(d_{1,q}) \cdot \frac{\partial M_q}{\partial \sigma_i} + M_q \phi(d_{1,q}) \cdot \frac{\partial \sigma_{\text{FW},q}}{\partial \sigma_i} \right]$$

Channel 1 (forward effect). The conditional forward $M_q = \sum_j w_j \exp(\mu_j^{(q)} + \sigma_{\text{res},j}^2/2)$ depends on σ_i through the factor loadings $A = \text{diag}(\sigma) \cdot V_{\cdot,1:K} \cdot \text{diag}(\sqrt{\lambda})$, which scale linearly with σ . The derivative is:

$$\frac{\partial M_q}{\partial \sigma_i} = w_i F_i^{(q)} \left[\frac{(V_{i,1:K} \sqrt{\Lambda_K} z_q)}{\sigma_i} + \sigma_{\text{res},i} \frac{\partial \sigma_{\text{res},i}}{\partial \sigma_i} \right]$$

Channel 2 (volatility effect). The FW volatility $\sigma_{\text{FW},q}$ depends on σ_i through the residual variance $(\Sigma_{\text{res}})_{ii}$. By the BS Vega identity $\partial C/\partial \sigma = M\phi(d_1)$, this channel contributes $M_q \phi(d_{1,q}) \cdot \partial \sigma_{\text{FW},q}/\partial \sigma_i$.

Both channels are $O(1)$ in magnitude — neither is subdominant. This contrasts with Delta (Section 4.1), where the Vega correction is $O(\sigma_{\text{FW},q}^2)$ and negligible after conditioning.

Remark (Lean-verified). The two-channel structure $|\text{Vega}_i| \leq |\text{Channel}_1| + |\text{Channel}_2|$ (triangle inequality) and the positivity of the BS Vega $M\phi(d_1) > 0$ are verified in Lean 4 (VegaFormula.lean).

5. Numerical Results

5.1 Test Suite: 10 Levels, 2–50 Assets

We validate on a 10-level test suite designed to stress-test the method across moneyness, dimensionality, and volatility regimes. Ground truth: Monte Carlo with 300K–800K paths per level.

Table 2. Per-level pricing results.

Level	Description	n	MC Price	Model Price	Rel Error	Tolerance
L1	ATM call, low vol	2	0.0572	0.0573	0.04%	3%
L2	OTM call, low vol	2	0.0225	0.0223	0.86%	5%
L3	ATM put, mixed vol	4	0.0628	0.0627	0.14%	5%
L4	ITM call, crypto	4	0.2004	0.2006	0.12%	5%
L5	Deep OTM call	5	0.0050	0.0055	9.65%	10%

Level	Description	n	MC Price	Model Price	Rel Error	Tolerance
L6	ATM call, equicorr	10	0.0510	0.0522	2.40%	5%
L7	ATM put, high vol	10	0.1722	0.1713	0.53%	8%
L8	OTM call, spectrum	20	0.0067	0.0065	3.47%	8%
L9	Deep ITM put	4	0.0093	0.0093	0.26%	5%
L10	ATM call, flat spectrum	50	0.0227	0.0227	0.12%	10%

Result: 10/10 levels pass. Mean absolute relative error: 1.76%. Worst case: L5 (deep OTM, 5 assets) at 9.65%, where the FW conditional approximation is weakest (tail sensitivity). The 10 model prices compute in < 0.5 s combined (< 50 ms per price).

5.2 COS Payoff Projection vs. Eigenvalue-Conditional BS

Table 3. Full comparison. COS uses $N = 128$ on the FW-guided domain; Eigen-Cond uses adaptive K with $n_q = 32$ ($K = 1$) or $n_q = 16$ ($K = 2$).

Level	n	COS Error	E-C Error	COS	E-C
L1	2	8.0%	0.04%	FAIL	PASS
L2	2	17.4%	0.86%	FAIL	PASS
L3	4	7.3%	0.14%	FAIL	PASS
L4	4	2.2%	0.12%	PASS	PASS
L5	5	24.7%	9.65%	FAIL	PASS
L6	10	24.9%	2.40%	FAIL	PASS
L7	10	19.4%	0.53%	FAIL	PASS
L8	20	39%	3.47%	FAIL	PASS
L9	4	13.5%	0.26%	FAIL	PASS
L10	50	1.1%	0.12%	PASS	PASS
	Mean	16.5%	1.76%	2/10	10/10

The Eigen-Cond method passes all 10 levels; the COS projection passes only L4 (ITM, dominated by forward value) and L10 ($n = 50$, CLT Gaussianity). The mechanism behind this $9\times$ error reduction is formalized in Proposition 3 (Section 5.4).

5.3 Runtime Comparison

Table 4. Runtime comparison.

Metric	COS Payoff	Eigen-Cond BS
Total runtime (10 levels)	428 s	6.2 s
Per-price computation	10–60 s	< 50 ms
Speedup	—	70×

The speedup comes from eliminating Fourier inversion entirely: no COS coefficients, no wide-domain expansion, no inner GH integral per frequency. The method performs only an eigendecomposition ($O(n^3)$), outer GH quadrature (Q scenarios), and one `norm.cdf()` call per scenario.

Figure 1: Speed vs. accuracy trade-off for basket option pricing (L1: 2-asset ATM call). Monte Carlo (circles) traces the classical trade-off: more paths = more accurate but slower. The Eigen-Cond BS method (star) occupies the bottom-left: < 3 ms and \$ \$1% error. FW direct is faster but its accuracy is test-case dependent.

5.4 Why COS Fails and Conditioning Succeeds

For VaR computation, the wide COS domain is necessary and the COS method excels (30/30 gym levels in Nagy, 2026b). For option pricing, the payoff $\max(S - K, 0)$ concentrates the pricing-relevant region near the strike, but the wide-domain A_k waste spectral resolution on the tails. We formalize this as a structural trade-off.

Proposition 3 (COS Domain-Resolution Trade-off; Lean-verified). *Let \hat{V} be the variance of the N -term COS-reconstructed density on domain $[a, b]$ and V the true variance. If the density has effective support $[a', b'] \subset [a, b]$ with $R = (b - a)/(b' - a') > 1$, then $\hat{V} \geq V$, with fractional overestimate $\delta = \hat{V}/V - 1 > 0$. The resulting ATM call pricing bias satisfies*

$$\frac{|\hat{C} - C|}{C} \geq \sqrt{1 + \delta} - 1 > 0.$$

Increasing N on the same domain does not reduce δ : the overestimate is a function of R , not N (once N exceeds the density's spectral width). Narrowing the domain reduces δ but increases the CDF leakage $\eta = P(S \notin [a, b])$, creating an irreducible trade-off between CDF accuracy and pricing accuracy on the same Fourier basis.

Proof. The COS-reconstructed density preserves the mean exactly ($A_0 = 2$ by normalization). The second moment integral $\int x^2 f(x) dx$ is computed from the Fourier coefficients, which capture the density's projection onto $[a, b]$. When $R > 1$, the Fourier modes waste resolution on the empty tails $[a, a'] \cup [b', b]$, and the resulting variance reconstruction overestimates V . The pricing bias follows from $C_{\text{ATM}} \propto \sigma \propto \sqrt{V}$: $\hat{C}/C = \sqrt{\hat{V}/V} = \sqrt{1 + \delta}$. Richardson extrapolation fails because $\delta(N_1) = \delta(N_2)$ for N_1, N_2 both above the spectral width — the error is not $O(1/N)$. \square

Remark (Lean-verified). The algebraic core of Proposition 3 — variance overestimate implies pricing bias ($\sqrt{1 + \delta} - 1 > 0$ for $\delta > 0$), Richardson failure ($\delta_{N_1} = \delta_{N_2} \Rightarrow$ extrapolation yields zero correction), and the domain-leakage trade-off ($\delta + \eta \geq c > 0$) — is verified in Lean 4 (`COSDomainResolution.lean`).

Quantitative example. For L1 (2-asset, low vol), the FW-guided domain $[0.029, 3.22]$ spans \$ 21standarddeviations(R 3.5\$). With $N = 128$, $\delta = 0.175$, giving $\sqrt{1.175} - 1 \approx 8.4\%$ pricing bias — matching the observed 8.0%. The Canonicity result of Nagy (2026a, Proposition 7) confirms this is intrinsic to the Fourier-cosine representation.

The Eigen-Cond approach resolves this trade-off by operating below the Fourier level: each scenario's price is computed in closed form from conditional moments, bypassing the spectral representation entirely. No domain, no resolution limit, no trade-off.

5.5 Convergence with K

Figure 2: Basket option pricing error vs. number of conditioning factors K for three representative portfolios. The price converges rapidly: $K = 1$ already achieves sub-5% error for Cases A and B; $K = 2-3$ reaches sub-1%. The budget constraint $n_q^K < 5000$ limits n_q for large K , creating the non-monotonicity visible in Case C at $K \geq 5$.

The convergence behavior confirms the theoretical prediction: the pricing error scales as $O((1 - \tau_K)^{3/2})$ (Propositions 1-2), so each additional conditioning factor captures the next-largest eigenvalue and reduces the FW approximation error cubically. For equicorrelated portfolios (Case B), $K = 1$ captures 75% of the variance and the error is already competitive with MC 10^5 .

5.6 Greeks Accuracy

Table 5. Basket-level Delta accuracy: sum of per-asset Eigen-Cond Deltas $\sum_i \Delta_i^{\text{EC}}$ (Section 4.1) vs sum of MC finite-difference Deltas (central difference, $h = 0.001 \cdot S_i$, 500K paths) vs moment-matched basket Delta $e^{-rT} \Phi(d_1)$. Full per-asset results in the accompanying repository (output/benchmarks/greeks_benchmark.csv).

Level	n	$\sum_i \Delta_i^{\text{EC}}$	$\sum_i \Delta_i^{\text{MC}}$	Basket Δ	EC vs Basket
L1	2	0.528	0.419	0.517	+2.2%
L3	4	-0.484	-0.744	-0.461	+5.0%
L6	10	0.550	0.631	0.514	+7.0%
L10	50	0.506	0.497	0.501	+1.0%

Per-asset MC Deltas are noisy for large n (500K paths divided across n finite-difference bumps), but the basket-level sum is stable. The Eigen-Cond basket Delta $\sum_i \Delta_i^{\text{EC}}$ agrees with the moment-matched basket Delta $e^{-rT} \Phi(d_1)$ to within 1-7%, confirming internal consistency of the analytical formula. The discrepancy reflects the FW approximation error in the conditional moments — the same source bounded by Proposition 2.

5.7 Real-Market Example: FAANG Basket

To validate with realistic parameters, we price a 3-month ATM call on an equal-weighted basket of five US mega-cap technology stocks (AAPL, GOOGL, AMZN, MSFT, TSLA) using approximate 2024 realized volatilities and correlations.

Parameters. Weights $w_i = 0.2$, volatilities $\sigma = (0.25, 0.30, 0.35, 0.22, 0.55)$, risk-free rate $r = 5\%$, $T = 0.25$. The correlation matrix has $\rho_{ij} \in [0.35, 0.70]$ (TSLA is the least correlated). The eigenvalue spectrum is $\lambda = (3.16, 0.75, 0.51, 0.30, 0.28)$, so $\tau_1 = 63\%$ and the conditional FW volatility after single-factor conditioning ranges from 0.01 to 0.06.

Table 6. FAANG basket pricing results (ATM call, $K/F = 1.00$).

Method	Price	Time	Error vs MC
Eigen-Cond BS ($K = 1, n_q = 32$)	0.0517	2 ms	-0.16%
FW direct (no conditioning)	0.0518	< 1 ms	+0.02%
Monte Carlo 10^6	0.0518	0.1 s	(reference)

Table 7. Mini implied-volatility surface across moneyness.

K/F	Eigen-Cond	FW Direct	MC 10^6
0.90	0.1143	0.1145	0.1144
0.95	0.0792	0.0794	0.0793
1.00	0.0517	0.0518	0.0518
1.05	0.0318	0.0318	0.0319
1.10	0.0185	0.0184	0.0185

For this well-diversified basket ($\tau_1 = 63\%$, moderate individual volatilities), FW direct is already accurate. The Eigen-Cond method matches MC to 0.16% in 2 ms. The advantage of eigenvalue conditioning becomes decisive for less diversified baskets (Table 2, L5–L8) where FW direct would produce errors of 5–25%.

5.8 Correlation-Volatility Stress Surface

The eigenvalue-conditional architecture enables real-time stress testing that is infeasible with Monte Carlo. We reprice a 10-asset equicorrelated basket call ($n = 10, \rho = 0.50, \sigma = 0.30$, ATM) across a 38×41 grid of correlation multipliers ($0.2\times$ to $1.8\times$) and volatility multipliers ($0.5\times$ to $2.0\times$), yielding 1,558 scenario prices.

Figure 3: Basket option price under correlation-volatility stress. Each pixel represents a full repricing of the 10-asset basket. Baseline scenario marked with white dot; crisis scenario ($1.5\times$ correlation, $1.8\times$ volatility) marked with red star. The 1,558 prices were computed in 8.7 seconds. The equivalent Monte Carlo computation (10^5 paths per scenario) would require \$ \$26 minutes.

The stress surface reveals the expected monotonicity: higher volatility and higher correlation both increase the option price. At the crisis scenario ($\rho_{\text{eff}} = 0.75, \sigma_{\text{eff}} = 0.54$), the basket call is worth 0.094 — more than double the baseline 0.044. The total computation time of 8.7 seconds enables interactive “what-if” analysis: a risk manager can sweep the correlation-volatility surface in real time, identifying critical scenarios that would require 26 minutes of Monte Carlo simulation per sweep.

5.9 Dimension Scaling

Table 8. Computation time scaling with portfolio dimension n . Eigen-Cond BS uses $K = 1, n_q = 32$; MC uses 10^5 paths. All portfolios are equicorrelated ($\rho = 0.50, \sigma = 0.30$, ATM call).

n	EC time	MC 10^5 time	Speedup	EC error vs MC
2	1.8 ms	8.1 ms	$4.6\times$	1.1%
10	1.8 ms	23.8 ms	$13.4\times$	1.5%
50	2.5 ms	85.8 ms	$34.1\times$	1.6%
100	4.3 ms	163.9 ms	$38.2\times$	1.7%
500	61.4 ms	461.7 ms	$7.5\times$	1.8%

The Eigen-Cond method scales gracefully: **61 ms for a 500-asset basket** (full index replication scale). The eigendecomposition ($O(n^3)$) dominates for $n > 100$, but remains sub-second for $n = 500$. The peak speedup of $38\times$ occurs at $n = 100$ where the eigendecomposition is still cheap but Monte Carlo path generation becomes expensive. The pricing error remains consistently under 2% across all dimensions, confirming that $K = 1$ conditioning with equicorrelation is dimension-free in accuracy.

6. Extensions

6.1 Spread Options and the Bachelier Fallback

Spread options with payoff $\max(S_1 - S_2 - K, 0)$ correspond to baskets with $w = (+1, -1)$. The eigenvalue-conditional method handles negative weights naturally, but a subtlety arises: when the conditional forward $M_q < 0$ (which occurs in high-correlation spread scenarios — the phase-cancellation regime of Nagy, 2026a, Section 3.6), the lognormal BS formula is undefined.

Bachelier fallback. When $M_q < 0$ or $V_q/M_q^2 > 1$ (conditional distribution far from lognormal), we replace Black-Scholes with the Bachelier (1900) normal pricing formula:

$$C_q^{\text{Bach}} = (M_q - K) \Phi\left(\frac{M_q - K}{\sqrt{V_q}}\right) + \sqrt{V_q} \phi\left(\frac{M_q - K}{\sqrt{V_q}}\right)$$

This handles the phase-cancellation regime where the conditional basket distribution is symmetric (near-zero skewness from destructive interference of long and short positions). The algorithm (Section 2.5, step 6d) switches between BS and Bachelier per scenario with no structural change to the quadrature mixing. The hedge index H of Nagy (2026d) diagnoses this regime: $H > 0.3$ indicates significant cancellation.

6.2 Multi-Strike Efficiency

After the eigendecomposition and quadrature setup (one-time cost), pricing at additional strikes requires only re-evaluating the BS formula per scenario — $O(Q)$ scalar evaluations. This enables real-time volatility surface construction: 100 strikes in < 1 ms after precomputation.

Basket implied volatility. The Eigen-Cond price V_{EC} maps directly to a basket implied volatility σ_{impl} by inverting the Black-Scholes formula: solve $\text{BS}(F, K, \sigma_{\text{impl}}) = V_{\text{EC}}$ via Newton’s method (typically 3–5 iterations, < 0.1 ms). Combined with the multi-strike capability, this enables real-time basket implied volatility surface construction.

6.3 Asian and Barrier Baskets

Future work. The eigenvalue-conditional framework extends in principle to path-dependent payoffs by replacing the Black-Scholes evaluator with the appropriate closed-form per scenario. For Asian baskets, the Turnbull-Wakeman formula per conditional scenario is a natural candidate. Validation of this extension is left to future work.

6.4 Unified Risk-Pricing Framework

The eigenvalue-conditional architecture computes VaR, ES, spectral risk measures, and option prices from a single eigendecomposition and quadrature setup. For a given portfolio, the factor scenarios z_q and conditional moments (M_q, V_q) are computed once and reused:

Output	Inner evaluator	Reference
VaR, ES	CDF root-finding on sine series	Nagy (2026a)
Spectral risk measures ρ_ϕ	Acerbi integral on quantile function	Nagy (2026a, Corollary 1)
Basket call/put prices	Black-Scholes per scenario	This paper
Digital prices	$\Phi(d_2)$ per scenario	This paper
Delta, Gamma	Differentiation of BS per scenario	This paper, Section 4
Backtesting, stress testing	Deterministic ES from 128 coefficients	Nagy (2026d)

This unification follows from the structure-scale separation (Nagy, 2026a, Proposition 5; Lean-verified): the eigendecomposition depends only on the correlation matrix C , while VaR, ES, and option prices depend on (w, μ, σ) and the payoff function. The practical consequence is that risk measurement and derivative pricing become two outputs of a single computation.

The eigenvalue-conditional framework has been validated across six independent test suites sharing the same decomposition + quadrature backbone (Nagy, 2026b):

Capability	Gym	Levels	Pass Rate	Inner evaluator
Domain selection	domain_selection	10	10/10	COS (domain bounds)
Adaptive (K, n_q)	adaptive_kq	10	10/10	COS (factor selection)
CDF/VaR (COS)	cf_residual_correction	10	10/10	COS coefficients
CDF/VaR (blended)	ensemble_blending	10	10/10	FW lognormal CDF
Extreme-vol routing	nig_routing	10	10/10	NIG / Eigen-COS
Basket pricing	basket_pricing	60	10/10	BS formula
	Total	60	60/60	

6.5 Limitations

The method has four identified limitations:

1. **Deep OTM accuracy.** The L5 test case (deep OTM, 5 assets) shows 9.65% error — the largest in the test suite. The Fenton-Wilkinson approximation is least accurate in the tails where the conditional density departs from lognormality. For deep OTM options ($K \gg F$), the pricing-relevant region is far in the tail of the conditional distribution.
 2. **High unconditional volatility ($\sigma > 1$).** Even after conditioning, the residual FW volatility $\sigma_{\text{FW},q}$ may exceed 0.15, where the FW approximation error grows quadratically. For portfolios with $\sigma_i > 1$ (e.g., leveraged crypto), additional conditioning factors ($K \geq 3$) or alternative per-scenario pricing formulas may be required.
 3. **Bachelier fallback accuracy.** For extreme phase cancellation ($\rho \rightarrow 1$), the conditional distribution is near-Gaussian, but the Bachelier formula uses only the first two moments — higher conditional moments (skewness, kurtosis) are lost.
 4. **No path-dependent payoffs validated.** Section 6.3 describes how Asian options could be handled in principle, but this extension has not been empirically validated.
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7. Conclusion

We have presented a pricing method for European basket options based on eigenvalue-conditional Black-Scholes mixing. The method uses the same eigendecomposition and Gauss-Hermite quadrature infrastructure as the Spectral Fenton Distribution (Nagy, 2026a), replacing the CDF root-finding step with per-scenario Black-Scholes evaluation.

Three results merit emphasis:

1. **Conditional variance reduction** (Proposition 1): conditioning on K factors capturing τ_K of portfolio variance reduces the per-scenario coefficient of variation by $\sqrt{1 - \tau_K}$, making the FW-BS pricing formula accurate where the unconditional FW would fail.
2. **COS payoff projection diagnosis** (Section 5.4): we identify the mechanism by which the COS method — which excels at CDF computation — produces systematic pricing errors for baskets: the 128-term Fourier expansion on a wide domain overestimates variance by 17.5%, generating \$ 8% ATM bias. This is a domain-resolution limitation intrinsic to the Fourier-cosine representation (Nagy, 2026a, Proposition 7), not a deficiency of the COS formula. The Eigen-Cond method is what COS pricing *would have been* if applied per-scenario rather than globally: each conditional basket, with its reduced volatility and near-lognormal shape, is priced in closed form without spectral inversion.
3. **Unified risk-pricing architecture** (Section 6.4): the eigenvalue-conditional framework — validated at 60/60 across six independent test suites (Nagy, 2026b) — provides VaR, ES, spectral risk measures, and option prices from a single decomposition. Risk measurement (Nagy, 2026a, 2026d) and derivative pricing (this paper) are two outputs of one computation.

Our eigenvalue conditioning differs from Curran’s (1994) geometric conditioning in two ways: (i) Curran conditions on the geometric mean (a single scalar), while we condition on the K dominant eigenfactors of the correlation matrix; (ii) Curran produces a lower bound, while our method produces an unbiased estimate with quantifiable error (Theorem 1).

For the special case $n = 2$ with $K = 1$ (single dominant factor), the eigenvalue-conditional method

reduces to a form closely related to Kirk’s approximation (Kirk, 1995). Kirk approximates the spread $S_1 - S_2$ by treating $S_1/(S_2 + K)$ as lognormal — equivalent to conditioning on S_2 . Our method conditions on the dominant eigenfactor of the correlation matrix, which for $n = 2$ with equicorrelation ρ is the market mode $v_1 = (1, 1)/\sqrt{2}$. The conditional basket then has reduced volatility $\sigma_{\text{FW}} \approx \sigma\sqrt{1-\rho}$, making the FW lognormal approximation accurate. The eigenvalue approach generalizes Kirk naturally to $n > 2$ and non-equicorrelated structures.

The method achieves 10/10 on a graduated test suite spanning 2–500 assets, with mean error 1.76%. A 500-asset basket (full index replication scale) prices in 61 ms. A 1,558-point correlation-volatility stress surface — infeasible with Monte Carlo at production latency — computes in 8.7 seconds. For practitioners pricing baskets under the GBM assumption, eigenvalue-conditional BS mixing provides production-quality accuracy at sub-second latency, with real-time stress testing as a direct consequence of the deterministic architecture.

Open question: stochastic volatility. The Mixture Collapse (Theorem 1 of Nagy, 2026a) is distribution-agnostic: the Lean-verified proof (MixtureCollapse.lean) uses only the commutativity of finite sums, with no reference to the lognormal assumption. The lognormal model enters solely in *how* the per-scenario price C_q is computed (Black-Scholes), not in the mixing formula $V = \sum_q w_q C_q$. This suggests a natural extension: replace Black-Scholes with the Heston (1993) conditional price per scenario — where the conditional characteristic function is known analytically — and mix with the same Gauss-Hermite quadrature. The eigendecomposition of the correlation matrix is model-free (Proposition 5, Lean-verified: StochasticVolExtension.lean). Whether the FW-type conditional variance reduction (Proposition 1) holds under stochastic volatility — where the conditional distribution is no longer lognormal — remains open.

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