

# Bayes Decision

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Skeleton

## Abstract

Bayes decision — ProofEnv encoding of BayesClassifier / MAPClassifier (Lean BayesDecision).

This paper presents 58 machine-verified theorems. All results are formally verified in the Platonic proof kernel (71 verification units, 59 proved statements) and exportable to Lean 4.

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## 1. Introduction

## 2. Further Results

**Theorem** (risk\_nonneg). *Risk Nonneg.* [Platonic: risk\_nonneg, domain: bayes\_decision]

**Theorem** (risk\_le\_one). *Risk Le One.* [Platonic: risk\_le\_one, domain: bayes\_decision]

**Theorem** (risk\_in\_unit). *Risk In Unit.* [Platonic: risk\_in\_unit, domain: bayes\_decision]

**Theorem** (map\_risk). *Map Risk.* [Platonic: map\_risk, domain: bayes\_decision]

**Theorem** (risk\_ge\_map\_risk). *Risk Ge Map Risk.* [Platonic: risk\_ge\_map\_risk, domain: bayes\_decision]

**Theorem** (bayes\_error\_le\_half\_binary). *Bayes Error Le Half Binary.* [Platonic: bayes\_error\_le\_half\_binary, domain: bayes\_decision]

**Theorem** (better\_posterior\_lower\_error). *Better Posterior Lower Error.* [Platonic: better\_posterior\_lower\_error, domain: bayes\_decision]

**Theorem** (bayes\_error\_zero\_forward). *Bayes Error Zero Forward.* [Platonic: bayes\_error\_zero\_forward, domain: bayes\_decision]

**Theorem** (bayes\_error\_zero\_reverse). *Bayes Error Zero Reverse.* [Platonic: bayes\_error\_zero\_reverse, domain: bayes\_decision]

**Theorem** (bayes\_risk\_identity). *Bayes Risk Identity.* [Platonic: bayes\_risk\_identity, domain: bayes\_decision]

**Theorem** (map\_optimality). *Map Optimality.* [Platonic: map\_optimality, domain: bayes\_decision]

**Theorem** (risk\_gap\_nonneg). *Risk Gap Nonneg.* [Platonic: risk\_gap\_nonneg, domain: bayes\_decision]

**Theorem** (excess\_risk\_formula). *Excess Risk Formula*. [Platonic: excess\_risk\_formula, domain: bayes\_decision]

**Theorem** (excess\_risk\_upper). *Excess Risk Upper*. [Platonic: excess\_risk\_upper, domain: bayes\_decision]

**Theorem** (bd\_w4\_one\_minus\_risk\_eq\_posterior). *Bd W4 One Minus Risk Eq Posterior*. [Platonic: bd\_w4\_one\_minus\_risk\_eq\_posterior, domain: bayes\_decision]

**Theorem** (bd\_w4\_risk\_add\_posterior\_eq\_one). *Bd W4 Risk Add Posterior Eq One*. [Platonic: bd\_w4\_risk\_add\_posterior\_eq\_one, domain: bayes\_decision]

**Theorem** (bd\_w4\_posterior\_nonneg\_if\_risk\_le\_one). *Bd W4 Posterior Nonneg If Risk Le One*. [Platonic: bd\_w4\_posterior\_nonneg\_if\_risk\_le\_one, domain: bayes\_decision]

**Theorem** (bd\_w4\_risk\_pos\_if\_posterior\_lt\_one). *Bd W4 Risk Pos If Posterior Lt One*. [Platonic: bd\_w4\_risk\_pos\_if\_posterior\_lt\_one, domain: bayes\_decision]

**Theorem** (bd\_w4\_map\_error\_swap\_classes). *Bd W4 Map Error Swap Classes*. [Platonic: bd\_w4\_map\_error\_swap\_classes, domain: bayes\_decision]

**Theorem** (bd\_excess\_risk\_nonneg). *Bd Excess Risk Nonneg*. [Platonic: bd\_excess\_risk\_nonneg, domain: bayes\_decision]

**Theorem** (bd\_excess\_risk\_decomp). *Bd Excess Risk Decomp*. [Platonic: bd\_excess\_risk\_decomp, domain: bayes\_decision]

**Theorem** (bd\_regret\_nonneg). *Bd Regret Nonneg*. [Platonic: bd\_regret\_nonneg, domain: bayes\_decision]

**Theorem** (bd\_cumulative\_regret\_monotone). *Bd Cumulative Regret Monotone*. [Platonic: bd\_cumulative\_regret\_monotone, domain: bayes\_decision]

**Theorem** (bd\_w10\_sub\_as\_add\_neg). *Bd W10 Sub As Add Neg*. [Platonic: bd\_w10\_sub\_as\_add\_neg, domain: bayes\_decision]

**Theorem** (bd\_w10\_residual\_nonneg). *Bd W10 Residual Nonneg*. [Platonic: bd\_w10\_residual\_nonneg, domain: bayes\_decision]

**Theorem** (bd\_w10\_residual\_le\_one). *Bd W10 Residual Le One*. [Platonic: bd\_w10\_residual\_le\_one, domain: bayes\_decision]

**Theorem** (bd\_w10\_posterior\_conservation). *Bd W10 Posterior Conservation*. [Platonic: bd\_w10\_posterior\_conservation, domain: bayes\_decision]

**Theorem** (bd\_w10\_residual\_zero\_iff). *Bd W10 Residual Zero Iff*. [Platonic: bd\_w10\_residual\_zero\_iff, domain: bayes\_decision]

**Theorem** (bd\_w10\_excess\_risk\_zero\_iff). *Bd W10 Excess Risk Zero Iff*. [Platonic: bd\_w10\_excess\_risk\_zero\_iff, domain: bayes\_decision]

**Theorem** (bd\_w10\_excess\_risk\_le\_one). *Bd W10 Excess Risk Le One*. [Platonic: bd\_w10\_excess\_risk\_le\_one, domain: bayes\_decision]

**Theorem** (bd\_w10\_learning\_gap\_nonneg). *Bd W10 Learning Gap Nonneg*. [Platonic: bd\_w10\_learning\_gap\_nonneg, domain: bayes\_decision]

**Theorem** (bd\_w10\_perturb\_max\_diff\_rewrites). *Bd W10 Perturb Max Diff Rewrites*. [Platonic: bd\_w10\_perturb\_max\_diff\_rewrites, domain: bayes\_decision]

**Theorem** (risk\_gap\_formula). *Risk Gap Formula*. [Platonic: risk\_gap\_formula, domain: bayes\_decision]

**Theorem** (higher\_max\_lower\_residual). *Higher Max Lower Residual*. [Platonic: higher\_max\_lower\_residual, domain: bayes\_decision]

**Theorem** (dimension\_free\_classification). *Dimension Free Classification*. [Platonic: dimension\_free\_classification, domain: bayes\_decision]

**Theorem** (high\_keff\_fast\_classification). *High Keff Fast Classification*. [Platonic: high\_keff\_fast\_classification, domain: bayes\_decision]

**Theorem** (uniform\_prior\_baseline). *Uniform Prior Baseline*. [Platonic: uniform\_prior\_baseline, domain: bayes\_decision]

**Theorem** (more\_classes\_higher\_baseline\_risk). *More Classes Higher Baseline Risk*. [Platonic: more\_classes\_higher\_baseline\_risk, domain: bayes\_decision]

**Theorem** (bias\_variance\_decomposition\_nonneg). *Bias Variance Decomposition Nonneg*. [Platonic: bias\_variance\_decomposition\_nonneg, domain: bayes\_decision]

**Theorem** (reducing\_bias\_reduces\_total). *Reducing Bias Reduces Total*. [Platonic: reducing\_bias\_reduces\_total, domain: bayes\_decision]

**Theorem** (variance\_dominates\_when\_bias\_zero). *Variance Dominates When Bias Zero*. [Platonic: variance\_dominates\_when\_bias\_zero, domain: bayes\_decision]

**Theorem** (calibration\_error\_nonneg). *Calibration Error Nonneg*. [Platonic: calibration\_error\_nonneg, domain: bayes\_decision]

**Theorem** (perfect\_calibration\_zero\_gap). *Perfect Calibration Zero Gap*. [Platonic: perfect\_calibration\_zero\_gap, domain: bayes\_decision]

**Theorem** (minimax\_achieves\_upper). *Minimax Achieves Upper*. [Platonic: minimax\_achieves\_upper, domain: bayes\_decision]

**Theorem** (ensemble\_avg\_le\_max). *Ensemble Avg Le Max*. [Platonic: ensemble\_avg\_le\_max, domain: bayes\_decision]

**Theorem** (ensemble\_diversity\_reduces\_risk). *Ensemble Diversity Reduces Risk*. [Platonic: ensemble\_diversity\_reduces\_risk, domain: bayes\_decision]

### 3. Bounds and Estimates

**Theorem** (risk\_lower\_bound). *Risk Lower Bound*. [Platonic: risk\_lower\_bound, domain: bayes\_decision]

**Theorem** (bd\_w4\_half\_complement\_bound). *Bd W4 Half Complement Bound*. [Platonic: bd\_w4\_half\_complement\_bound, domain: bayes\_decision]

**Theorem** (bd\_excess\_risk\_bound). *Bd Excess Risk Bound*. [Platonic: bd\_excess\_risk\_bound, domain: bayes\_decision]

**Theorem** (bd\_more\_data\_tighter\_bound). *Bd More Data Tighter Bound.* [Platonic: bd\_more\_data\_tighter\_bound, domain: bayes\_decision]

**Theorem** (multiclass\_risk\_bounded\_by\_one\_minus\_inv\_K). *Multiclass Risk Bounded By One Minus Inv K.* [Platonic: multiclass\_risk\_bounded\_by\_one\_minus\_inv\_K, domain: bayes\_decision]

**Theorem** (minimax\_risk\_lower\_bounds\_bayes). *Minimax Risk Lower Bounds Bayes.* [Platonic: minimax\_risk\_lower\_bounds\_bayes, domain: bayes\_decision]

**Theorem** (ensemble\_avg\_bounded). *Ensemble Avg Bounded.* [Platonic: ensemble\_avg\_bounded, domain: bayes\_decision]

## 4. Existence and Uniqueness

**Theorem** (bd\_w10\_map\_uniqueness\_gap). *Bd W10 Map Uniqueness Gap.* [Platonic: bd\_w10\_map\_uniqueness\_gap, domain: bayes\_decision]

**Theorem** (map\_uniqueness). *Map Uniqueness.* [Platonic: map\_uniqueness, domain: bayes\_decision]

## 5. Spectral Theory

**Theorem** (keff\_one\_worst\_spectral). *Keff One Worst Spectral.* [Platonic: keff\_one\_worst\_spectral, domain: bayes\_decision]

**Theorem** (spectral\_rate\_nonneg). *Spectral Rate Nonneg.* [Platonic: spectral\_rate\_nonneg, domain: bayes\_decision]

## 6. Convergence Results

**Theorem** (bd\_convergence\_monotone). *Bd Convergence Monotone.* [Platonic: bd\_convergence\_monotone, domain: bayes\_decision]

## 7. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/bayes\_decision/).

File	Role
bayes_decision_proof.py	

## 8. Discussion

## References