

Bayesian Risk

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Skeleton

Abstract

Bayesian Risk — clean ProofEnv proof.

This paper presents 51 machine-verified theorems. All results are formally verified in the Platonic proof kernel (55 verification units, 51 proved statements) and exportable to Lean 4.

1. Introduction

2. Further Results

Theorem (data_reduces_risk). *Data Reduces Risk*. [Platonic: data_reduces_risk, domain: bayesian_risk]

Theorem (info_monotone). *Info Monotone*. [Platonic: info_monotone, domain: bayesian_risk]

Theorem (no_info_no_reduction). *No Info No Reduction*. [Platonic: no_info_no_reduction, domain: bayesian_risk]

Theorem (posterior_nonneg). *Posterior Nonneg*. [Platonic: posterior_nonneg, domain: bayesian_risk]

Theorem (risk_drop_is_info_gain). *Risk Drop Is Info Gain*. [Platonic: risk_drop_is_info_gain, domain: bayesian_risk]

Theorem (risk_order_trans). *Risk Order Trans*. [Platonic: risk_order_trans, domain: bayesian_risk]

Theorem (posterior_le_prior_nonneg). *Posterior Le Prior Nonneg*. [Platonic: posterior_le_prior_nonneg, domain: bayesian_risk]

Theorem (br_w2_sub_add_cancel). *Br W2 Sub Add Cancel*. [Platonic: br_w2_sub_add_cancel, domain: bayesian_risk]

Theorem (br_w2_add_sub_cancel). *Br W2 Add Sub Cancel*. [Platonic: br_w2_add_sub_cancel, domain: bayesian_risk]

Theorem (br_w2_posterior_lt_prior_strict). *Br W2 Posterior Lt Prior Strict*. [Platonic: br_w2_posterior_lt_prior_strict, domain: bayesian_risk]

Theorem (br_w2_sub_nonneg_iff_prior_ge_post). *Br W2 Sub Nonneg Iff Prior Ge Post*. [Platonic: br_w2_sub_nonneg_iff_prior_ge_post, domain: bayesian_risk]

Theorem (br_w2_prior_minus_post_is_gain). *Br W2 Prior Minus Post Is Gain.* [Platonic: br_w2_prior_minus_post_is_gain, domain: bayesian_risk]

Theorem (br_w2_sum_nonneg). *Br W2 Sum Nonneg.* [Platonic: br_w2_sum_nonneg, domain: bayesian_risk]

Theorem (br_w2_scaled_prior_nonneg). *Br W2 Scaled Prior Nonneg.* [Platonic: br_w2_scaled_prior_nonneg, domain: bayesian_risk]

Theorem (br_w2_info_gain_le_prior_implies_post_nonneg). *Br W2 Info Gain Le Prior Implies Post Nonneg.* [Platonic: br_w2_info_gain_le_prior_implies_post_nonneg, domain: bayesian_risk]

Theorem (br_w2_add_preserves_le). *Br W2 Add Preserves Le.* [Platonic: br_w2_add_preserves_le, domain: bayesian_risk]

Theorem (br_w2_mul_nonneg). *Br W2 Mul Nonneg.* [Platonic: br_w2_mul_nonneg, domain: bayesian_risk]

Theorem (br_w2_unit_interval_mul_le_left). *Br W2 Unit Interval Mul Le Left.* [Platonic: br_w2_unit_interval_mul_le_left, domain: bayesian_risk]

Theorem (br_w2_double_prior_nonneg). *Br W2 Double Prior Nonneg.* [Platonic: br_w2_double_prior_nonneg, domain: bayesian_risk]

Theorem (br_w2_post_plus_gain_eq_prior). *Br W2 Post Plus Gain Eq Prior.* [Platonic: br_w2_post_plus_gain_eq_prior, domain: bayesian_risk]

Theorem (br_w2_sub_self_zero). *Br W2 Sub Self Zero.* [Platonic: br_w2_sub_self_zero, domain: bayesian_risk]

Theorem (br_w2_le_trans_risk). *Br W2 Le Trans Risk.* [Platonic: br_w2_le_trans_risk, domain: bayesian_risk]

Theorem (br_w2_gain_le_prior_from_post_nonneg). *Br W2 Gain Le Prior From Post Nonneg.* [Platonic: br_w2_gain_le_prior_from_post_nonneg, domain: bayesian_risk]

Theorem (br_w2_mul_unit_interval_nonneg). *Br W2 Mul Unit Interval Nonneg.* [Platonic: br_w2_mul_unit_interval_nonneg, domain: bayesian_risk]

Theorem (br_positive_homogeneity). *Br Positive Homogeneity.* [Platonic: br_positive_homogeneity, domain: bayesian_risk]

Theorem (br_translation_invariance). *Br Translation Invariance.* [Platonic: br_translation_invariance, domain: bayesian_risk]

Theorem (br_monotonicity). *Br Monotonicity.* [Platonic: br_monotonicity, domain: bayesian_risk]

Theorem (br_subadditivity). *Br Subadditivity.* [Platonic: br_subadditivity, domain: bayesian_risk]

Theorem (br_memory_nonneg). *Br Memory Nonneg.* [Platonic: br_memory_nonneg, domain: bayesian_risk]

Theorem (br_capital_buffer_monotone). *Br Capital Buffer Monotone.* [Platonic: br_capital_buffer_monotone, domain: bayesian_risk]

Theorem (lean_bayesian_update_decreases_kl). *Lean Bayesian Update Decreases Kl.* [Platonic: lean_bayesian_update_decreases_kl, domain: bayesian_risk]

Theorem (lean_kl_decrease_is_regret). *Lean Kl Decrease Is Regret.* [Platonic: lean_kl_decrease_is_regret, domain: bayesian_risk]

Theorem (lean_posterior_width_nonneg). *Lean Posterior Width Nonneg.* [Platonic: lean_posterior_width_nonneg, domain: bayesian_risk]

Theorem (lean_memory_state_nonneg). *Lean Memory State Nonneg.* [Platonic: lean_memory_state_nonneg, domain: bayesian_risk]

Theorem (lean_mirror_descent_product_pos). *Lean Mirror Descent Product Pos.* [Platonic: lean_mirror_descent_product_pos, domain: bayesian_risk]

Theorem (lean_regret_div_nonneg). *Lean Regret Div Nonneg.* [Platonic: lean_regret_div_nonneg, domain: bayesian_risk]

Theorem (sequential_update_risk_decreasing). *Sequential Update Risk Decreasing.* [Platonic: sequential_update_risk_decreasing, domain: bayesian_risk]

Theorem (incremental_info_gain_nonneg). *Incremental Info Gain Nonneg.* [Platonic: incremental_info_gain_nonneg, domain: bayesian_risk]

Theorem (two_step_risk_reduction). *Two Step Risk Reduction.* [Platonic: two_step_risk_reduction, domain: bayesian_risk]

Theorem (kl_divergence_nonneg). *Kl Divergence Nonneg.* [Platonic: kl_divergence_nonneg, domain: bayesian_risk]

Theorem (minimax_bayes_gap_nonneg). *Minimax Bayes Gap Nonneg.* [Platonic: minimax_bayes_gap_nonneg, domain: bayesian_risk]

Theorem (posterior_precision_increases). *Posterior Precision Increases.* [Platonic: posterior_precision_increases, domain: bayesian_risk]

Theorem (posterior_variance_decreases). *Posterior Variance Decreases.* [Platonic: posterior_variance_decreases, domain: bayesian_risk]

Theorem (more_data_tighter_concentration). *More Data Tighter Concentration.* [Platonic: more_data_tighter_concentration, domain: bayesian_risk]

3. Bounds and Estimates

Theorem (br_memory_bounded). *Br Memory Bounded.* [Platonic: br_memory_bounded, domain: bayesian_risk]

Theorem (lean_cumulative_regret_bound). *Lean Cumulative Regret Bound.* [Platonic: lean_cumulative_regret_bound, domain: bayesian_risk]

Theorem (lean_width_perturbation_bound). *Lean Width Perturbation Bound.* [Platonic: lean_width_perturbation_bound, domain: bayesian_risk]

Theorem (risk_bounded_by_kl_plus_bayes). *Risk Bounded By Kl Plus Bayes.* [Platonic: risk_bounded_by_kl_plus_bayes, domain: bayesian_risk]

Theorem (worst_case_prior_bounded). *Worst Case Prior Bounded*. [Platonic: worst_case_prior_bounded, domain: bayesian_risk]

Theorem (risk_concentration_bound). *Risk Concentration Bound*. [Platonic: risk_concentration_bound, domain: bayesian_risk]

Theorem (sample_avg_risk_bounded). *Sample Avg Risk Bounded*. [Platonic: sample_avg_risk_bounded, domain: bayesian_risk]

4. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/bayesian_risk/).

File	Role
bayesian_risk_proof.py	

5. Discussion

References