Bellman

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Skeleton

Abstract

Bellman — ProofEnv: DP value bounds, MDP Bellman operator, contraction lemmas.

This paper presents 95 machine-verified theorems building on 0 established facts and 1 hypotheses. All results are formally verified in the Platonic proof kernel (232 verification units, 95 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (dp_value_nonneg). *Dp Value Nonneg.* [Platonic: dp_value_nonneg, domain: bellman]

Theorem (discounted_le_continuation). *Discounted Le Continuation.* [Platonic: discounted_le_continuation, domain: bellman]

Theorem (value_nonneg_via_continuation). *Value Nonneg Via Continuation.* [Platonic: value_nonneg_via_continuation, domain: bellman]

Theorem (zero_discount_myopic). *Zero Discount Myopic.* [Platonic: zero_discount_myopic, domain: bellman]

Theorem (discount_monotone). *Discount Monotone.* [Platonic: discount_monotone, domain: bellman]

Theorem (bellman_monotone). *Bellman Monotone.* [Platonic: bellman_monotone, domain: bellman]

Theorem (bellman_self_diff). *Bellman Self Diff.* [Platonic: bellman_self_diff, domain: bellman]

Theorem (reward_shift). *Reward Shift.* [Platonic: reward_shift, domain: bellman]

Theorem (double_discount). *Double Discount.* [Platonic: double_discount, domain: bellman]

Theorem (bellman_decomposition). *Bellman Decomposition.* [Platonic: bellman_decomposition, domain: bellman]

Theorem (value_le_trans). *Value Le Trans.* [Platonic: value_le_trans, domain: bellman]

Theorem (gamma_sq_lt_one). *Gamma Sq Lt One.* [Platonic: gamma_sq_lt_one, domain: bellman]

Theorem (contraction_forces_zero). *Contraction Forces Zero.* [Platonic: contraction_forces_zero, domain: bellman]

Theorem (contraction_inductive_step). *Contraction Inductive Step.* [Platonic: contraction_inductive_step, domain: bellman]

Theorem (discount_essential). *Discount Essential.* [Platonic: discount_essential, domain: bellman]

Theorem (shift_contracts). *Shift Contracts.* [Platonic: shift_contracts, domain: bellman]

Theorem (vf_diff_nonneg). *Vf Diff Nonneg.* [Platonic: vf_diff_nonneg, domain: bellman]

Theorem (vf_diff_nonpos). *Vf Diff Nonpos.* [Platonic: vf_diff_nonpos, domain: bellman]

Theorem (bellman_Q_gap). *Bellman Q Gap.* [Platonic: bellman_Q_gap, domain: bellman]

Theorem (Q_residual_gamma_V). *Q Residual Gamma V.* [Platonic: Q_residual_gamma_V, domain: bellman]

Theorem (bellman_slack_nonneg). *Bellman Slack Nonneg.* [Platonic: bellman_slack_nonneg, domain: bellman]

Theorem (gamma_mul_sq_x_nonneg). *Gamma Mul Sq X Nonneg.* [Platonic: gamma_mul_sq_x_nonneg, domain: bellman]

Theorem (shift_sub_cancels). *Shift Sub Cancels.* [Platonic: shift_sub_cancels, domain: bellman]

Theorem (discount_times_diff_nonneg). *Discount Times Diff Nonneg.* [Platonic: discount_times_diff_nonneg, domain: bellman]

Theorem (bellman_gap_ordered). *Bellman Gap Ordered.* [Platonic: bellman_gap_ordered, domain: bellman]

Theorem (sqrt_nonneg_any). *Sqrt Nonneg Any.* [Platonic: sqrt_nonneg_any, domain: bellman]

Theorem (sqrt_zero_eq). *Sqrt Zero Eq.* [Platonic: sqrt_zero_eq, domain: bellman]

Theorem (sqrt_one_eq). *Sqrt One Eq.* [Platonic: sqrt_one_eq, domain: bellman]

Theorem (sq_sqrt_eq). *Sq Sqrt Eq.* [Platonic: sq_sqrt_eq, domain: bellman]

Theorem (sqrt_sq_eq). *Sqrt Sq Eq.* [Platonic: sqrt_sq_eq, domain: bellman]

Theorem (le_max_left_scalar). *Le Max Left Scalar.* [Platonic: le_max_left_scalar, domain: bellman]

Theorem (le_max_right_scalar). *Le Max Right Scalar.* [Platonic: le_max_right_scalar, domain: bellman]

Theorem (min_le_left_scalar). *Min Le Left Scalar.* [Platonic: min_le_left_scalar, domain: bellman]

Theorem (min_le_right_scalar). *Min Le Right Scalar.* [Platonic: min_le_right_scalar, domain: bellman]

Theorem (max_comm_scalar). *Max Comm Scalar.* [Platonic: max_comm_scalar, domain: bellman]

Theorem (min_comm_scalar). *Min Comm Scalar.* [Platonic: min_comm_scalar, domain: bellman]

Theorem (vec_norm_sq_mdp_nonneg). *Vec Norm Sq Mdp Nonneg.* [Platonic: vec_norm_sq_mdp_nonneg, domain: bellman]

Theorem (vec_dot_mdp_comm). *Vec Dot Mdp Comm.* [Platonic: vec_dot_mdp_comm, domain: bellman]

Theorem (finset_sum_mdp_nonneg). *Finset Sum Mdp Nonneg.* [Platonic: finset_sum_mdp_nonneg, domain: bellman]

Theorem (finset_sum_mdp_mono). *Finset Sum Mdp Mono.* [Platonic: finset_sum_mdp_mono, domain: bellman]

Theorem (finset_prod_mdp_nonneg). *Finset Prod Mdp Nonneg.* [Platonic: finset_prod_mdp_nonneg, domain: bellman]

Theorem (value_tendsto_nhds). *Value Tendsto Nhds.* [Platonic: value_tendsto_nhds, domain: bellman]

Theorem (exp_tendsto_nhds). *Exp Tendsto Nhds.* [Platonic: exp_tendsto_nhds, domain: bellman]

Theorem (sqrt_le_sqrt_scalar). *Sqrt Le Sqrt Scalar.* [Platonic: sqrt_le_sqrt_scalar, domain: bellman]

Theorem (sqrt_mul_nn_scalar). *Sqrt Mul Nn Scalar.* [Platonic: sqrt_mul_nn_scalar, domain: bellman]

Theorem (bellman_Q_under_max_reward). *Bellman Q Under Max Reward.* [Platonic: bellman_Q_under_max_reward, domain: bellman]

Theorem (discounted_vf_le_max_vf). *Discounted Vf Le Max Vf.* [Platonic: discounted_vf_le_max_vf, domain: bellman]

Theorem (reward_sqrt_nonneg). *Reward Sqrt Nonneg.* [Platonic: reward_sqrt_nonneg, domain: bellman]

Theorem (gamma_min_id). *Gamma Min Id.* [Platonic: gamma_min_id, domain: bellman]

Theorem (sqrt_pos_of_pos_scalar). *Sqrt Pos Of Pos Scalar.* [Platonic: sqrt_pos_of_pos_scalar, domain: bellman]

Theorem (vec_norm_sq_def_mdp). *Vec Norm Sq Def Mdp.* [Platonic: vec_norm_sq_def_mdp, domain: bellman]

Theorem (bellman_gap_ordered_W_ge_V). *Bellman Gap Ordered W Ge V.* [Platonic: bellman_gap_ordered_W_ge_V, domain: bellman]

Theorem (gamma_mul_le_self_of_lt_one). *Gamma Mul Le Self Of Lt One.* [Platonic: gamma_mul_le_self_of_lt_one, domain: bellman]

Theorem (Q_shift_max_assoc). *Q Shift Max Assoc.* [Platonic: Q_shift_max_assoc, domain: bellman]

Theorem (min_reward_upper). *Min Reward Upper.* [Platonic: min_reward_upper, domain: bellman]

Theorem (contraction_forces_zero). *Contraction Forces Zero.* [Platonic: contraction_forces_zero, domain: bellman]

Theorem (contraction_inductive). *Contraction Inductive.* [Platonic: contraction_inductive, domain: bellman]

Theorem (discount_essential). *Discount Essential.* [Platonic: discount_essential, domain: bellman]

Theorem (discount_first_order). *Discount First Order.* [Platonic: discount_first_order, domain: bellman]

Theorem (kkt_complementary_slackness). *Kkt Complementary Slackness.* [Platonic: kkt_complementary_slackness, domain: bellman]

Theorem (kkt_dual_feasible). *Kkt Dual Feasible.* [Platonic: kkt_dual_feasible, domain: bellman]

Theorem (kkt_optimality_gap_nonneg). *Kkt Optimality Gap Nonneg.* [Platonic: kkt_optimality_gap_nonneg, domain: bellman]

Theorem (lp_feasibility_iff_slack). *Lp Feasibility Iff Slack.* [Platonic: lp_feasibility_iff_slack , domain: bellman]

Theorem (lp_objective_monotone). *Lp Objective Monotone.* [Platonic: lp_objective_monotone, domain: bellman]

Theorem (weak_duality_gap_nonneg). *Weak Duality Gap Nonneg.* [Platonic: weak_duality_gap_nonneg, domain: bellman]

Theorem (theorem_D_lp_iff_kkt). *Theorem D Lp Iff Kkt.* [Platonic: theorem_D_lp_iff_kkt, domain: bellman]

Theorem (theorem_F_american_continuation_nonneg). *Theorem F American Continuation Nonneg.* [Platonic: theorem_F_american_continuation_nonneg, domain: bellman]

Theorem (banach_iteration_monotone). *Banach Iteration Monotone.* [Platonic: banach_iteration_monotone, domain: bellman]

Theorem (banach_two_step_contraction). *Banach Two Step Contraction.* [Platonic: banach_two_step_contraction, domain: bellman]

Theorem (call_intrinsic_nonneg). *Call Intrinsic Nonneg.* [Platonic: call_intrinsic_nonneg, domain: bellman]

Theorem (time_value_nonneg_from_superreplication). *Time Value Nonneg From Superreplication.* [Platonic: time_value_nonneg_from_superreplication, domain: bellman]

Theorem (discount_preserves_ordering). *Discount Preserves Ordering.* [Platonic: discount_preserves_ordering, domain: bellman]

Theorem (relative_error_positive). *Relative Error Positive.* [Platonic: relative_error_positive, domain: bellman]

Theorem (policy_improvement_nonneg). *Policy Improvement Nonneg.* [Platonic: policy_improvement_nonneg, domain: bellman]

Theorem (bellman_residual_nonneg). *Bellman Residual Nonneg.* [Platonic: bellman_residual_nonneg, domain: bellman]

Theorem (discount_product_contraction). *Discount Product Contraction.* [Platonic: discount_product_contraction, domain: bellman]

Theorem (two_step_discount_contraction). *Two Step Discount Contraction.* [Platonic: two_step_discount_contraction, domain: bellman]

Theorem (reward_discounted_sum_nonneg). *Reward Discounted Sum Nonneg.* [Platonic: reward_discounted_sum_nonneg, domain: bellman]

Theorem (advantage_function_zero_at_optimal). *Advantage Function Zero At Optimal.* [Platonic: advantage_function_zero_at_optimal, domain: bellman]

Theorem (advantage_nonneg_for_optimal_action). *Advantage Nonneg For Optimal Action.* [Platonic: advantage_nonneg_for_optimal_action, domain: bellman]

Theorem (td_error_decomposition). *Td Error Decomposition.* [Platonic: td_error_decomposition, domain: bellman]

Theorem (value_iteration_monotone). *Value Iteration Monotone.* [Platonic: value_iteration_monotone, domain: bellman]

Theorem (reward_shaping_preserves_optimal). *Reward Shaping Preserves Optimal.* [Platonic: reward_shaping_preserves_optimal, domain: bellman]

Theorem (epsilon_greedy_explores). *Epsilon Greedy Explores.* [Platonic: epsilon_greedy_explores, domain: bellman]

3. Bounds and Estimates

Theorem (bellman_contraction_bound). *Bellman Contraction Bound.* [Platonic: bellman_contraction_bound, domain: bellman]

Theorem (banach_error_bound_from_first_step). *Banach Error Bound From First Step.* [Platonic: banach_error_bound_from_first_step, domain: bellman]

Theorem (early_exercise_boundary). *Early Exercise Boundary.* [Platonic: early_exercise_boundary, domain: bellman]

Theorem (soft_value_lower_bounded). *Soft Value Lower Bounded.* [Platonic: soft_value_lower_bounded, domain: bellman]

4. Convergence Results

Theorem (hjb_limit_residual). *Hjb Limit Residual.* [Platonic: hjb_limit_residual, domain: bellman]

Theorem (error_halving_convergence). *Error Halving Convergence.* [Platonic: error_halving_convergence, domain: bellman]

Theorem (quadratic_convergence_faster). *Quadratic Convergence Faster.* [Platonic: quadratic_convergence_faster, domain: bellman]

5. Cross-Domain Bridges

Theorem (max_le_bridge). *Max Le Bridge.* [Platonic: max_le_bridge, domain: bellman]

Theorem (le_min_bridge). *Le Min Bridge.* [Platonic: le_min_bridge, domain: bellman]

6. Existence and Uniqueness

Theorem (theorem_E_uniqueness). *Theorem E Uniqueness.* [Platonic: theorem_E_uniqueness, domain: bellman]

7. Stability Results

Theorem (theorem_H_robustness_contraction). *Theorem H Robustness Contraction.* [Platonic: theorem_H_robustness_contraction, domain: bellman]

8. Formal Framework

Hypotheses

- gamma_nonneg: Gamma Nonneg

9. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/bellman/).

File	Role
bellman_proof.py	

10. Discussion

References