Bitcoin as a Thermodynamic System: Phase Transitions at Halving Events

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Abstract

We model Bitcoin as a thermodynamic system where the hash rate plays the role of temperature, the price plays the role of pressure, and the halving events are phase transitions. The block reward $R(t) = 50 \cdot 2^{-\lfloor t/T_H \rfloor}$ defines a deterministic energy input schedule; the hash rate $H(t)$ measures the physical energy expenditure of the network; and the price $P(t)$ reflects the market's valuation of the resulting output. We derive an equation of state relating these quantities and show that halving events satisfy the Ehrenfest classification of a second-order phase transition: the first derivative of the miner free energy with respect to the block reward is continuous, while the second derivative — capturing the price-volatility response — is discontinuous across the halving boundary. The spectral representation of the return density (Nagy, 2026a) provides a candidate order parameter: the zeroth Fourier coefficient A_0 of the return density undergoes a structural shift at the halving, analogous to the susceptibility divergence in a magnetic transition. The equation of state, halving dynamics, and the 21M supply cap are formally verified in Lean 4; the Ehrenfest classification and spectral order parameter remain analytical results supported by empirical observation from four historical halvings (2012, 2016, 2020, 2024).

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1. Introduction

Bitcoin is unique among financial assets: it has a measurable, physical energy input. The hash rate $H(t)$ (in EH/s) directly corresponds to electricity consumption ($ $0.1 TWh per EH/s annually). This energy expenditure is not incidental — it is the core mechanism by which the network achieves consensus (Nakamoto, 2008). This makes Bitcoin the only financial asset with a well-defined thermodynamic interpretation.

The idea of applying thermodynamic reasoning to economic systems has a long lineage. Georgescu-Roegen (1971) argued that entropy is the fundamental constraint on economic processes, and Mirowski (1989) traced the deep structural parallels between neoclassical economics and 19th-century field physics. In the cryptocurrency domain, several authors have studied Bitcoin's energy consumption (de Vries, 2018; Stoll et al., 2019) and the economic effects of halving cycles (Tung & Cachanosky, 2022), but none have formalized the halving as a thermodynamic phase transition using Ehrenfest's classification (Ehrenfest, 1933; Jaeger, 1998).

We propose the following correspondence between thermodynamic quantities and Bitcoin observables:

Thermodynamic quantity	Bitcoin analogue	Observable
Temperature T	Hash rate $H(t)$	Blockchain data (EH/s)
Pressure P	Price $P(t)$	Exchange data (USD)
Energy input E	Block reward $\times$ price	$R(t) \cdot P(t)$
Phase transition	Halving event	$R(t) \to R(t)/2$

We note that block space supply (the analogue of volume) is approximately fixed at 1 MB per block under the original protocol, though SegWit and layer-2 solutions have introduced effective variability. Network decentralization (as measured by the Nakamoto coefficient) could serve as an entropy analogue, but formalizing this requires a microstate-counting argument beyond the scope of this paper.

The remainder of this paper is organized as follows. Section 2 derives the equation of state. Section 3 classifies the halving as a second-order phase transition under the Ehrenfest scheme. Section 4 introduces the spectral order parameter. Section 5 develops the hash rate thermodynamics. Section 6 presents empirical evidence from the four historical halvings. Section 7 describes formal verification. Section 8 discusses limitations. Section 9 concludes.

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2. The Equation of State

2.1 Miner Equilibrium

At equilibrium, miners operate at the break-even condition:

$$R(t) \cdot P(t) = c \cdot H(t)$$

where $R(t)$ is the block reward (BTC/block), $P(t)$ is the price (USD/BTC), c is the marginal cost per unit hash rate (USD/EH), and $H(t)$ is the network hash rate (EH/s). This is the **equation of state** for the Bitcoin thermodynamic system.

This equation is well-known in mining economics under various names — the "break-even hash price" (Carter, 2019), the "mining profitability condition" — but we emphasize its structural role as an equation of state analogous to $PV = nRT$.

2.2 The "Free Energy"

Definition 1 (Miner Free Energy). *The miner free energy (profitability per hash) is:*

$$F(t) = R(t) \cdot P(t) - c \cdot H(t)$$

At equilibrium, $F = 0$. When $F > 0$, miners enter (hash rate rises). When $F < 0$, miners exit (hash rate falls). This self-correcting mechanism is Bitcoin's analogue of Le Chatelier's principle: the system responds to perturbations by restoring equilibrium.

2.3 Halving as Phase Transition

At a halving event ($t = t_H$), $R(t)$ drops by 50% instantaneously. If P and H are continuous:

$$F(t_H^+) = \frac{1}{2}R(t_H^-) \cdot P(t_H) - c \cdot H(t_H) = \frac{1}{2}F(t_H^-) - \frac{1}{2}c \cdot H(t_H)$$

The free energy jumps negative: miners are instantly unprofitable. This triggers:

1. **Short-term**: hash rate drops (least efficient miners shut down)
2. **Medium-term**: supply reduction drives price up
3. **Long-term**: new equilibrium at higher P, lower H, $F = 0$

This three-phase relaxation has been observed empirically at every halving event to date, with adjustment periods ranging from approximately 2 to 8 months.

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3. Ehrenfest Classification

The Ehrenfest classification (Ehrenfest, 1933; Landau & Lifshitz, 1980) categorizes phase transitions by the lowest-order derivative of the thermodynamic potential that is discontinuous across the transition. A first-order transition has a discontinuous first derivative (latent heat); a second-order transition has a continuous first derivative but a discontinuous second derivative (e.g., the lambda transition in helium).

Theorem 1 (Halving is a Second-Order Phase Transition). *At a halving event:*

(i) The first derivative of the free energy with respect to the block reward is continuous:

$$\left.\frac{\partial F}{\partial R}\right|_{t_H^-} = P(t_H) = \left.\frac{\partial F}{\partial R}\right|_{t_H^+}$$

Since $F = R \cdot P - c \cdot H$ and $P(t)$ is continuous at t_H (no instantaneous price jump), both one-sided derivatives equal $P(t_H)$.

(ii) The second derivative of the free energy exhibits a discontinuity:

$$\left.\frac{\partial^2 F}{\partial R^2}\right|_{t_H^-} \neq \left.\frac{\partial^2 F}{\partial R^2}\right|_{t_H^+}$$

This discontinuity arises because the market's price elasticity with respect to supply changes differs fundamentally between the pre-halving regime (where miners anticipate the halving and price adjusts gradually) and the post-halving regime (where the supply shock has materialized and volatility spikes). Formally, $\partial P/\partial R$ changes slope across the halving because the marginal miner's cost curve shifts discontinuously.

Remark (First-Order Analogy). Although the Ehrenfest classification places this as second-order, the halving shares a feature with first-order transitions: the free energy F itself is discontinuous (it jumps from $F = 0$ to $F < 0$). In equilibrium thermodynamics, F is minimized and continuous; here, the miner system is driven *out* of equilibrium by the external quench. The profitability drop $\Delta F = -\frac{1}{2}c \cdot H(t_H)$ plays the role of latent heat that must be "absorbed" through hash rate reduction and price appreciation. This mixed character — second-order in the Ehrenfest sense, but with a latent-heat analogue — reflects the fact that Bitcoin is an open, driven system rather than a closed thermodynamic one.

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4. The Spectral Order Parameter

4.1 Fourier Coefficients as Order Parameter

In statistical mechanics, the order parameter is the quantity that distinguishes the two phases (Goldenfeld, 1992). For a ferromagnet, it is the magnetization M; for a superconductor, the gap function Δ. For Bitcoin, we propose that the spectral representation of the return density provides the natural order parameter.

Let $\rho(\xi, t)$ denote the probability density of log-returns at time t, and let $A_k(t)$ denote its k-th Fourier coefficient in the Spectral Fenton representation (Nagy, 2026a):

$$\rho(\xi, t) = \sum_{k=0}^{\infty} A_k(t)\, \phi_k(\xi)$$

where $\{\phi_k\}$ is the orthonormal spectral basis. The zeroth coefficient $A_0(t)$ captures the mean level of the return density.

Definition (Spectral Order Parameter). The spectral order parameter for the halving transition is:

$$\Psi(\tau) = A_0(\tau) - A_0^{\text{base}}$$

where $\tau = t - t_H$ is the time relative to the halving and A_0^{base} is the pre-halving baseline.

Pre-halving ($\tau < 0$): $\Psi \approx 0$ (baseline returns). Post-halving ($\tau > 0$): $\Psi = \Delta A_0 > 0$ (shifted returns due to supply contraction).

The discontinuity $\Delta A_0 > 0$ is the spectral signature of the phase transition.

4.2 Critical Exponents (Conjectured)

Near the halving ($\tau \to 0$), we conjecture that the coefficient dynamics follow power laws:

$$|A_k(\tau) - A_k(0)| \sim |\tau|^{\beta_k}$$

where β_k is the critical exponent for mode k. Preliminary estimates from the 2016 and 2020 halvings suggest:

Mode	β_k (estimate)	Interpretation
$k = 0$ (mean)	≈ 0.5	Square-root approach (diffusive)
$k = 1$ (skewness)	≈ 0.3	Faster than diffusive
$k > 10$ (high-frequency)	≈ 1.0	Linear approach (jump-like)

These estimates should be treated as conjectures pending rigorous statistical validation with confidence intervals; see Section 6 for methodology.

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5. Hash Rate Thermodynamics

5.1 The "Temperature" of Bitcoin

Definition 2 (Network Temperature). *The Bitcoin network temperature is:*

$$T_{\text{BTC}}(t) = H(t) \cdot c / P(t)$$

Measured in BTC per block. At equilibrium, $T_{BTC} = R(t)$.

When $T_{\text{BTC}} > R(t)$: the network is "overheated" (hash rate too high for the reward). When $T_{\text{BTC}} < R(t)$: the network is "undercooled" (hash rate too low).

5.2 The Halving as "Quench"

In condensed matter physics, a quench is a sudden change in an external parameter that drives the system out of equilibrium (Goldenfeld, 1992). The halving is a quench of the reward parameter $R \to R/2$, driving the system from $T_{\text{BTC}} = R$ to $T_{\text{BTC}} = 2R$ (suddenly overheated by a factor of two).

The relaxation dynamics are governed by two competing mechanisms: 1. **Hash rate adjustment** (~weeks): inefficient miners shut down, reducing $H(t)$. 2. **Price adjustment** (~months): reduced supply flow drives $P(t)$ upward.

The combined relaxation time for the system to reach new equilibrium is typically 2–8 months, as observed across the four historical halvings.

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6. Empirical Evidence

To ground the thermodynamic model in data, we examine all four Bitcoin halvings:

Halving	Date	Block Height	Reward Before → After	Pre-Halving H (EH/s)
1st	2012-11-28	210,000	50 → 25 BTC	~0.03

Halving	Date	Block Height	Reward Before → After	Pre-Halving H (EH/s)
2nd	2016-07-09	420,000	25 → 12.5 BTC	~1.5
3rd	2020-05-11	630,000	12.5 → 6.25 BTC	~120
4th	2024-04-19	840,000	6.25 → 3.125 BTC	~600

6.1 Hash Rate and Price Dynamics Around Halvings

At each halving, the predicted thermodynamic pattern is observed:

1. **Pre-halving anticipation**: Price begins rising 3–6 months before the halving as markets price in reduced future supply (the "efficient" component). Hash rate continues climbing as miners rush to earn the higher reward before it halves.
2. **The quench**: At the halving block, R drops discontinuously. F goes negative. Within days to weeks, the least efficient miners (highest c) shut down, and hash rate drops 5–30%.
3. **Relaxation to new equilibrium**: Over 2–8 months, price appreciation (P rises) and continued hash rate shakeout restore $F \approx 0$ at a new equilibrium with higher P and (eventually) higher H as next-generation hardware arrives.

Hash rate data is sourced from blockchain.com [TODO:cite]; price data from CoinGecko [TODO:cite].

6.2 Empirical Methodology for Critical Exponents

To estimate the critical exponents β_k from Section 4.2, we propose the following methodology:

1. Compute daily log-returns $r_t = \log(P_t/P_{t-1})$ in a window of ± 365 days around each halving.
2. Estimate the return density $\hat{\rho}(\xi, t)$ using kernel density estimation with Silverman bandwidth.
3. Project onto the spectral basis to obtain $\hat{A}_k(\tau)$ for $k = 0, 1, \dots, K$.
4. Fit $\log|A_k(\tau) - A_k(0)| = \beta_k \log|\tau| + C_k$ by ordinary least squares on both sides of the halving separately.
5. Report $\hat{\beta}_k$ with 95% bootstrap confidence intervals.

This procedure has not yet been executed with full rigor across all four halvings; the estimates in Section 4.2 are preliminary. A companion computational paper will present the complete analysis [TODO:cite].

6.3 Qualitative Phase Diagram

The thermodynamic state of the Bitcoin network can be represented in the (R, T_{BTC}) plane. At equilibrium, the system lies on the curve $T_{\text{BTC}} = R$. Each halving moves $R \to R/2$ instantaneously, placing the system at $(R/2, R)$ — above the equilibrium curve by a factor of two. The subsequent relaxation traces a path back to the equilibrium curve. The phase diagram thus consists of a sequence of "jumps and relaxations," with each halving epoch defining a distinct thermodynamic phase.

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7. Formal Verification

The following structural results have been formally verified in Lean 4 with zero sorry and no custom axioms:

Result	Lean file	Key theorem
Miner equilibrium	Thermodynamic.lean	miner_equilibrium
Free energy at halving	Thermodynamic.lean	free_energy_halving_drop
Post-halving unprofitability	Thermodynamic.lean	post_halving_unprofitable
Hash rate adjustment	Thermodynamic.lean	hash_rate_adjusts
Price doubling restoration	Thermodynamic.lean	price_doubling_restores
Network temperature at eq.	Thermodynamic.lean	network_temp_at_equilibrium
Halving overheating	Thermodynamic.lean	halving_overheating
Reward halving	HalvingSchedule.lean	reward_halving
Supply formula	HalvingSchedule.lean	supply_formula
Supply cap (21M)	HalvingSchedule.lean	total_supply_bounded
Supply convergence	HalvingSchedule.lean	supply_converges
Daily supply decreasing	HalvingSchedule.lean	daily_supply_decreasing

Scope of verification. The Lean proofs cover the algebraic structure of the equation of state and the halving schedule (including the 21M supply cap via geometric series convergence). The Ehrenfest classification (Theorem 1) and the spectral order parameter (Section 4) are analytical results that depend on empirical properties of $P(t)$ and $\rho(\xi, t)$; they are not formalized in Lean.

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8. Limitations

We acknowledge several important limitations of the thermodynamic model:

1. **Homogeneous miner assumption.** The equation of state uses a single marginal cost c, but real mining is heterogeneous: ASIC generations, electricity costs, and cooling efficiency vary by orders of magnitude across miners. A more realistic model would use a cost distribution $c \sim G(c)$ and integrate over the miner population.

2. **Transaction fees ignored.** The free energy $F = R \cdot P - c \cdot H$ uses only the block reward R. As halvings continue, transaction fees become an increasingly significant fraction of miner revenue. By 2140 (after all 64 halvings), fees will be the *sole* revenue source. The model must be extended to $F = (R + f) \cdot P - c \cdot H$ where f is the average fee per block.

3. **Price continuity assumption.** Theorem 1 assumes $P(t)$ is continuous at the halving. In reality, markets may exhibit a small jump at the halving block. If P is discontinuous, the first derivative $\partial F/\partial R$ may also be discontinuous, making the transition first-order rather than second-order.

4. **Critical exponents are conjectural.** The power-law exponents β_k in Section 4.2 have not been rigorously estimated with confidence intervals across all four halvings. They should be treated as hypotheses, not established facts.

5. **Open vs. closed system.** Bitcoin is an open, driven, dissipative system — energy flows in continuously from miners. Classical Ehrenfest classification was developed for closed systems at equilibrium. The application to Bitcoin is therefore an analogy with structural content, not a strict isomorphism. Recent work on non-equilibrium phase transitions (Henkel et al., 2008) may provide a more appropriate framework.
6. **Difficulty adjustment.** Bitcoin's difficulty adjustment algorithm (every 2016 blocks, $ $2 weeks) introduces a discrete feedback mechanism that smooths the hash rate response. This is not captured in the continuous model above.

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9. Conclusion

The thermodynamic model of Bitcoin provides three insights:

1. **The halving is a second-order phase transition in the Ehrenfest sense**: the first derivative of the miner free energy is continuous across the halving boundary, but the second derivative — capturing the volatility response to supply changes — is discontinuous. The profitability drop ΔF serves as a latent-heat analogue that is absorbed through hash rate reduction and price appreciation over a 2–8 month relaxation period.
2. **Hash rate is temperature**: the network self-adjusts to maintain miner equilibrium, just as a thermodynamic system adjusts temperature to maintain free energy at zero. The halving is a "quench" that suddenly doubles the effective temperature.
3. **The spectral coefficients provide a candidate order parameter**: the zeroth Fourier coefficient A_0 of the return density captures the structural change at the halving, going beyond point statistics like mean and variance.

Bitcoin is the first financial asset amenable to genuine thermodynamic modeling — because it is the first financial asset with measurable physical energy input. The Spectral Fenton framework (Nagy, 2026a) provides the mathematical language to make this precise, and the formal verification in Lean 4 ensures that the algebraic foundations are machine-checked.

Several directions for future work are apparent: extending the model to include transaction fees and miner heterogeneity; rigorously estimating critical exponents with bootstrap confidence intervals; connecting the spectral order parameter to the non-equilibrium phase transition literature; and developing the block-space-as-volume analogy using SegWit and layer-2 throughput data.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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