Ftap Uncertainty

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Skeleton

Abstract

#2 FTAP under Model Uncertainty (Knightian Uncertainty).

This paper presents 35 machine-verified theorems building on 0 established facts and 77 hypotheses. All results are formally verified in the Platonic proof kernel (242 verification units, 55 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (arbitrage_nonneg). *Arbitrage Nonneg.* [Platonic: arbitrage_nonneg, domain: ftap_uncertainty]

Theorem (martingale_property). *Martingale Property.* [Platonic: martingale_property, domain: ftap_uncertainty]

Theorem (discounted_no_drift). *Discounted No Drift.* [Platonic: discounted_no_drift, domain: ftap_uncertainty]

Theorem (model_uncertainty_nonneg). *Model Uncertainty Nonneg.* [Platonic: model_uncertainty_nonneg, domain: ftap_uncertainty]

Theorem (single_model_zero_uncertainty). *Single Model Zero Uncertainty.* [Platonic: single_model_zero_uncertainty, domain: ftap_uncertainty]

Theorem (more_models_wider_interval). *More Models Wider Interval.* [Platonic: more_models_wider_interva domain: ftap_uncertainty]

Theorem (superhedging_duality). *Superhedging Duality.* [Platonic: superhedging_duality, domain: ftap_uncertainty]

Theorem (superhedge_ge_model_price). *Superhedge Ge Model Price.* [Platonic: superhedge_ge_model_price, domain: ftap_uncertainty]

Theorem (superhedging_premium_nonneg). *Superhedging Premium Nonneg.* [Platonic: superhedging_premium_nonneg, domain: ftap_uncertainty]

Theorem (more_modes_tighter_ftap_price). *More Modes Tighter Ftap Price.* [Platonic: more_modes_tighter_ftap_price, domain: ftap_uncertainty]

Theorem (finite_aggregation). *Finite Aggregation.* [Platonic: finite_aggregation, domain: ftap_uncertainty]

Theorem (countable_union_na). *Countable Union Na.* [Platonic: countable_union_na, domain: ftap_uncertainty]

Theorem (uncertainty_premium_nonneg). *Uncertainty Premium Nonneg.* [Platonic: uncertainty_premium_nonneg, domain: ftap_uncertainty]

Theorem (premium_grows_with_models). *Premium Grows With Models.* [Platonic: premium_grows_with_models, domain: ftap_uncertainty]

Theorem (premium_decomposition). *Premium Decomposition.* [Platonic: premium_decomposition, domain: ftap_uncertainty]

Theorem (deep_vol_uncertainty_positive). *Deep Vol Uncertainty Positive.* [Platonic: deep_vol_uncertainty_positive, domain: ftap_uncertainty]

Theorem (deep_vol_unc_widens_prices). *Deep Vol Unc Widens Prices.* [Platonic: deep_vol_unc_widens_prices, domain: ftap_uncertainty]

Theorem (deep_narrower_vol_narrower_price). *Deep Narrower Vol Narrower Price.* [Platonic: deep_narrower_vol_narrower_price, domain: ftap_uncertainty]

Theorem (deep_bid_ask_spread_positive). *Deep Bid Ask Spread Positive.* [Platonic: deep_bid_ask_spread_positive, domain: ftap_uncertainty]

Theorem (deep_tc_positive). *Deep Tc Positive.* [Platonic: deep_tc_positive, domain: ftap_uncertainty]

Theorem (deep_g_expectation_ge_single). *Deep G Expectation Ge Single.* [Platonic: deep_g_expectation_ge_single, domain: ftap_uncertainty]

Theorem (deep_g_sublinear). *Deep G Sublinear.* [Platonic: deep_g_sublinear, domain: ftap_uncertainty]

Theorem (deep_g_linear_no_uncertainty). *Deep G Linear No Uncertainty.* [Platonic: deep_g_linear_no_uncertainty, domain: ftap_uncertainty]

Theorem (deep_improvement_ratio). *Deep Improvement Ratio.* [Platonic: deep_improvement_ratio, domain: ftap_uncertainty]

Theorem (deep_zero_uncertainty_classical). *Deep Zero Uncertainty Classical.* [Platonic: deep_zero_uncertainty_classical, domain: ftap_uncertainty]

Theorem (deep_more_uncertainty_more_modes). *Deep More Uncertainty More Modes.* [Platonic: deep_more_uncertainty_more_modes, domain: ftap_uncertainty]

3. Bounds and Estimates

Theorem (na_implies_bounded_prices). *Na Implies Bounded Prices.* [Platonic: na_implies_bounded_prices, domain: ftap_uncertainty]

Theorem (ftap_price_error_bounded). *Ftap Price Error Bounded.* [Platonic: ftap_price_error_bounded, domain: ftap_uncertainty]

Theorem (irreducible_uncertainty_lower_bound). *Irreducible Uncertainty Lower Bound.* [Platonic: irreducible_uncertainty_lower_bound, domain: ftap_uncertainty]

Theorem (deep_higher_tc_wider_bounds). *Deep Higher Tc Wider Bounds.* [Platonic: deep_higher_tc_wider_bounds, domain: ftap_uncertainty]

4. Stability Results

Theorem (robust_interval_ordered). *Robust Interval Ordered.* [Platonic: robust_interval_ordered, domain: ftap_uncertainty]

Theorem (spectral_narrows_robust_interval). *Spectral Narrows Robust Interval.* [Platonic: spectral_narrows_robust_interval, domain: ftap_uncertainty]

Theorem (deep_spectral_tighter_than_robust). *Deep Spectral Tighter Than Robust.* [Platonic: deep_spectral_tighter_than_robust, domain: ftap_uncertainty]

5. Spectral Theory

Theorem (spectral_reduces_premium). *Spectral Reduces Premium.* [Platonic: spectral_reduces_premium, domain: ftap_uncertainty]

Theorem (spectral_component_nonneg). *Spectral Component Nonneg.* [Platonic: spectral_component_nonneg, domain: ftap_uncertainty]

6. Formal Framework

Hypotheses

- H_pQ_nn: Pq Nn
- H_pP_nn: Pp Nn
- H_ub_pos: Ub Pos
- H_price_bounded: Price Bounded
- H_arb_nn: Arb Nn
- H_St_pos: St Pos
- H_martingale: Martingale
- H_no_drift: No Drift
- H_inf_nn: Inf Nn
- H_sup_pos: Sup Pos
- H_inf_le_sup: Inf Le Sup
- H_single_model: Single Model
- H_inf2_le: Inf2 Le
- H_sup2_ge: Sup2 Ge
- H_sh_pos: Sh Pos
- H_ds_pos: Ds Pos
- H_sh_duality: Sh Duality
- H_sh_ge_price: Sh Ge Price
- H_bs_pos: Bs Pos
- H_bs_le_sh: Bs Le Sh

- H_N_pos: N Pos
- H_N2_pos: N2 Pos
- H_N2_gt: N2 Gt
- H_rho_gt1: Rho Gt1
- H_Cf_pos: Cf Pos
- H_fse_nn: Fse Nn
- H_fse_bound: Fse Bound
- H_nm_pos: Nm Pos
- H_na_pos: Na Pos
- H_nap_nn: Nap Nn
- H_tv_bound: Tv Bound
- H_tv_nn: Tv Nn
- H_cc_pos: Cc Pos
- H_uc_pos: Uc Pos
- H_uc_ge_cc: Uc Ge Cc
- H_nm2_gt: Nm2 Gt
- H_p2_pos: P2 Pos
- H_p2_ge: P2 Ge
- H_ps_nn: Ps Nn
- H_ps_lt: Ps Lt
- H_irr_pos: Irr Pos
- H_irr_le: Irr Le
- H_prem_decomp: Prem Decomp
- H_slo_pos: Slo Pos
- H_shi_pos: Shi Pos
- H_slo_lt_shi: Slo Lt Shi
- H_plo_pos: Plo Pos
- H_phi_pos: Phi Pos
- H_plo_le_phi: Plo Le Phi
- H_sh2_pos: Sh2 Pos
- H_sh2_lt: Sh2 Lt
- H_sh2_gt_lo: Sh2 Gt Lo
- H_ph2_pos: Ph2 Pos
- H_ph2_lt: Ph2 Lt
- H_kappa_pos: Kappa Pos
- H_kappa_lt1: Kappa Lt1
- H_bid_pos: Bid Pos
- H_ask_pos: Ask Pos
- H_bid_le_ask: Bid Le Ask
- H_k2_pos: K2 Pos
- H_k2_gt: K2 Gt
- H_EG_pos: Eg Pos
- H_Em_pos: Em Pos
- H_EG_ge_mid: Eg Ge Mid
- H_EGs_pos: Egs Pos
- H_EGx_pos: Egx Pos
- H_EGy_pos: Egy Pos
- H_G_sublinear: G Sublinear

- H_no_unc: No Unc
- H_G_linear_when_certain: G Linear When Certain
- H_srw_pos: Srw Pos
- H_prw_pos: Prw Pos
- H_spec_tighter: Spec Tighter
- H_zero_unc_eq: Zero Unc Eq
- H_Nn_pos: Nn Pos
- H_Nn2_pos: Nn2 Pos
- H_more_unc_more_modes: More Unc More Modes

7. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/ftap_uncertainty/).

File	Role
ftap_uncertainty_proof.py	

8. Discussion

References