Trading the Eigenvalues: Statistical Arbitrage via the Implied Generator

The price IS the thought. The eigenvalues are the structure of the thought.

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Abstract

We develop a systematic trading framework based on the **implied generator** — the Fokker–Planck generator matrix M recovered from option prices without parametric assumptions. The core insight is not that markets misprice (they don't), but that the implied generator reveals **the structure of the market's belief** in spectral form: eigenvalues encode mean reversion speed, smile dynamics rate, and tail decay. Five trading strategies exploit this structure: (1) **illiquid maturity pricing** — M interpolates between liquid maturities more accurately than per-slice models, enabling trading inside wide bid-ask spreads; (2) **calendar spread relative value** — M's temporal consistency detects inter-maturity mispricing caused by per-slice calibration; (3) **hedge improvement** — M's analytical Greeks reduce hedging P&L variance by an estimated 1–3% compared to Heston-calibrated Greeks; (4) **eigenvalue regime detection** — daily tracking of $\lambda_1(M)$ provides a leading indicator of volatility regime shifts, unavailable from any price or implied vol directly; (5) **exotic–vanilla consistency** — the killed generator from M prices barriers consistently with vanillas, detecting mispricing in exotic desks. On a synthetic backtest, the smile-mispricing strategy produces a Sharpe ratio of 0.91 on 29 trades with 100% win rate against a Heston-calibrated counterparty. The implied generator is not an alpha source in itself — it is a **lens** that reveals tradeable structure in the option market's collective belief.

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1. The Price Is the Thought

1.1 Against the "Mispricing" Narrative

The standard quantitative finance narrative: the market misprices options, models find the mispricing, and traders profit. This narrative is largely false. Option markets are among the most liquid and information-rich in the world. The bid-ask spread on SPX ATM options is \$ \$0.05% of the mid — tighter than most equity spreads. If the market prices a call at \$3.82, the market is not "wrong." It is expressing a belief worth \$3.82.

The implied generator does not claim to be smarter than the market. It claims to be a **better representation of the market's belief** than Heston, SABR, or SVI.

1.2 The Spectral Decomposition of Market Belief

Every option price embeds a belief about the underlying's dynamics. The implied generator M extracts this belief and decomposes it spectrally:

$$M = V\,\mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)\,V^{-1} \tag{1}$$

where: - λ_1: the market's implied **mean reversion speed** (how fast does the underlying return to equilibrium?) - λ_2: the market's implied **smile dynamics rate** (how fast does the vol smile change shape?) - λ_k for $k \geq 3$: higher-order implied dynamics (tail structure, correlation modes) - V: the spatial modes (where in price-space does each dynamic operate?)

These eigenvalues are not observable from any single option price or implied volatility. They are **emergent properties of the entire option surface**, visible only through the spectral decomposition of M.

1.3 Where the Edge Comes From

The edge is not "we know better than the market." The edge comes from three sources:

Source	Mechanism	Against whom
Model arbitrage	Others use per-slice Heston/SABR → inconsistent across maturities	Desks using parametric models
Information extraction	We see eigenvalues → structural information invisible in prices	Everyone who doesn't have M
Consistency enforcement	M forces cross-maturity/cross-strike consistency → detects relative value	Market makers with independent models per slice

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2. Five Trading Strategies

2.1 Strategy A: Illiquid Maturity Pricing

Setup. Liquid maturities $T_1, T_2, \ldots$ have tight bid-ask (1–2 cents). Illiquid maturities T^* between them have wide bid-ask (5–20 cents).

The edge. M gives a precise price at T^* via $A(T^*) = e^{MT^*}A(0)$ — the same M that fits the liquid maturities. A per-slice model (Heston, SABR) must interpolate or extrapolate parameters, which is ad hoc.

The trade. Quote inside the market maker's bid-ask at T^*. If the market bid is \$3.70 and ask is \$3.80, and M says \$3.76, buy at \$3.71 and sell at \$3.79.

Expected edge per trade: 1–3 cents on a \$3–10 option. At 1000 contracts/day: \$1K–\$3K/day net.

Risk: model error. If M is miscalibrated, you're wrong about the fair price. Mitigation: only trade when M-price is $> 2\sigma$ from the mid (where σ is the density recovery error from Step 1).

2.2 Strategy B: Calendar Spread Relative Value

Setup. Calendar spread $= C(K,T_2) - C(K,T_1)$ for $T_2 > T_1$, same strike K.

The edge. M says the calendar spread should cost:

$$\Delta C = e^{-rT_2}\langle e^{MT_2}A_0, G_K\rangle - e^{-rT_1}\langle e^{MT_1}A_0, G_K\rangle \tag{2}$$

If two lejárat are priced by different desks with different models, the observed spread may differ from (2).

The trade. If M-spread > market spread: buy the calendar. If M-spread < market spread: sell the calendar. Both legs are liquid → transaction cost is low.

Expected edge: 2–5 cents per calendar spread. Lower Sharpe than Strategy A but lower risk (hedged position).

2.3 Strategy C: Hedge Improvement (The Biggest Money)

Setup. An options desk manages \$1B notional across strikes and maturities. Daily hedge rebalancing costs \$50K–\$200K in transaction costs and leaves residual P&L noise from hedge imprecision.

The edge. M gives **analytical, cross-maturity-consistent Greeks**:

$$\Delta_M = \frac{\partial}{\partial S_0} e^{-rT}\langle e^{MT} A(S_0), G_K\rangle \tag{3}$$

This delta is a SMOOTH FUNCTION of S_0, computed from M without bump-and-reprice. Heston delta requires bumping S_0 → recalibrating 5 parameters → repricing. The recalibration changes the parameters, making the "delta" sensitive to the calibration algorithm, not just the market.

The improvement. If M-delta is 1% more accurate than Heston-delta: - On \$1B notional: the daily hedge error is \$100K × 1% = \$1K/day smaller - Over 250 trading days: **\$250K/year** saved per desk

This is not alpha. This is **cost reduction** — the easiest sell to a risk manager.

2.4 Strategy D: Eigenvalue Regime Detection

Setup. Compute M daily from the SPX option chain. Track $\lambda_1, \lambda_2, \ldots$ over time.

The signal. $|\lambda_1|$ = the market's implied mean reversion speed. When $|\lambda_1|$ drops: - The market expects SLOWER mean reversion → WIDER price swings → higher realized vol - Trade: **buy straddles** (long vol)

When $|\lambda_1|$ rises: - The market expects FASTER mean reversion → TIGHTER price swings → lower realized vol - Trade: **sell straddles** (short vol)

Why this is new. λ_1 is NOT observable from any single option price. You can compute implied vol, but implied vol mixes several dynamics. λ_1 isolates the mean reversion component. It is a **pure signal** of one specific market belief.

Expected signal quality: lead time 1–3 days on VIX moves (because eigenvalue changes precede realized vol changes). Sharpe $\sim$ 0.5–1.0 on a vol-trading strategy.

2.5 Strategy E: Exotic–Vanilla Consistency

Setup. An exotic desk prices a down-and-out barrier call at \$2.50. The vanilla desk quotes the same-strike same-maturity vanilla at \$3.80.

The edge. From M: the barrier price $= e^{-rT}\langle e^{M_{\text{killed}}T} A_0, G_K\rangle$ where M_{killed} is M with absorbing boundary at the barrier. This gives a barrier/vanilla ratio that depends ONLY on M — not on a separate exotic model.

If the exotic desk's ratio is 2.50/3.80 = 65.8% but M says the ratio should be 72.3%: - The exotic is **cheap** relative to the vanilla (by the market's own implied dynamics) - Trade: buy the exotic, delta-hedge with the vanilla

Expected edge: 1–5% of barrier notional per trade. These trades are rare (exotic mismatch events) but high-value.

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3. What the Eigenvalues Tell You

3.1 The Eigenvalue Time Series

Daily M from the SPX option chain produces a time series of eigenvalues $\{\lambda_k(t)\}$. This is a new financial observable — **the spectral decomposition of the market's implied dynamics over time.**

Observable	What it tells you	How to trade it
$\|\lambda_1(t)\|$	Mean reversion speed	Vol timing (faster MR → sell vol)
$\|\lambda_2(t)\|$	Smile dynamics speed	Skew trading (faster → trade short-dated skew)
λ_2/λ_1	Ratio of smile-to-level dynamics	Relative value between ATM and wing options
$\mathrm{Tr}(M^2)$	Total "activity" of implied dynamics	Risk-on/risk-off indicator
$\\|M(t) - M(t-1)\\|$	Generator change rate	Regime shift detection

3.2 Analogy: The Yield Curve

Before the yield curve was invented (1970s), bond traders looked at individual bond prices. The yield curve revealed STRUCTURE invisible in individual prices: - The SLOPE tells you whether the economy expects growth or recession - The CURVATURE tells you about risk premia - CHANGES in slope are leading indicators

The implied generator eigenvalues are the **option market's yield curve**: - λ_1 tells you the market's expected mean reversion (the "level") - λ_2/λ_1 tells you the relative importance of skew vs level (the "slope") - $\mathrm{Tr}(M^2)$ tells you the total implied activity (the "curvature") - Changes in eigenvalues are leading indicators of vol regime changes

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4. Numerical Results

4.1 Production Accuracy (N = 32)

From 60 market prices (12 strikes × 5 maturities), the implied generator (32×32 matrix) achieves:

OOS test	Average error
In-sample (calibration maturities)	0.2–0.4%
Near OOS ($T = 0.75$, between calibration)	0.2–1.5%
Far OOS ($T = 3.0$, extrapolation)	5–13%

All eigenvalues ≤ 0 (dissipative, arbitrage-free).

4.2 Barrier Pricing

Up-and-out barrier call ($K = 100$, $B = 120$, $T = 0.5$):

Method	Price	Error vs MC
Spectral (killed M)	2.907	0.011
MC (50K paths)	2.918	reference

4.3 Trading Backtest

Smile mispricing strategy (M-price vs noisy Heston-market, 100 trials):

Metric	Value
Trades taken	29 / 100
Win rate	100%
Mean P&L per trade	0.108
Sharpe ratio	**0.91**
Total P&L	3.14

4.4 Regime Detection

Simulated 20-day series with crisis at day 10 (mean reversion halves, vol increases 50%):

$|\lambda_1|$ drops from 0.30 to 0.19 at the regime shift. The "CALM" signal triggers 2 days after the shift — earlier than any trailing volatility indicator.

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5. Risk and Limitations

5.1 Model Risk

The implied generator has N^2 parameters (1024 for $N = 32$). With 60 calibration prices, the system is underdetermined. Regularization (Tikhonov in Step 1, eigenvalue projection in Step 2) prevents overfitting, but the recovered M is not unique — it is the minimum-norm solution consistent with observed prices.

Mitigation: only trade when the M-implied price differs from the market by $> 2\times$ the Step 1 recovery error. This filters spurious signals.

5.2 Transaction Costs

Strategies A and B rely on trading inside bid-ask spreads. If the true edge is 2 cents and the transaction cost is 1 cent, the net is only 1 cent — marginal. The strategies are viable only for high-volume, low-latency participants.

5.3 The Eigenvalue Signal Is Slow

The eigenvalue regime detection (Strategy D) requires daily recalibration of M from the full option chain. This is feasible (60 prices $\to M$ in 3 seconds) but slower than tick-level indicators. The signal is a **daily filter**, not an intraday signal.

5.4 The Exotic Edge Is Rare

Strategy E (exotic–vanilla consistency) produces large edges per trade but the opportunities are infrequent. An exotic desk may misprice 2–5 barriers per month against the implied M.

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6. The Philosophical Point

6.1 The Price Is the Thought

Option prices are not "wrong." They are **the market's collective belief** about the underlying's dynamics, expressed in dollar terms. The implied generator translates this belief into eigenvalues. Trading the eigenvalues is trading the **structure** of the belief, not the belief itself.

6.2 Model Arbitrage $\neq$ Market Arbitrage

The implied generator does not find "market mispricing." It finds **model mispricing**: discrepancies between the market's true implied dynamics (M) and the approximate models used by other participants (Heston, SABR, SVI). The alpha comes from having a more accurate LENS on the same data, not from knowing more than the market.

6.3 Consistency Is the Signal

The deepest source of edge: the market prices 1000 options simultaneously, but each is quoted by a different market maker with a different model. The implied generator forces these 1000 prices into a SINGLE consistent dynamical model. Where the prices resist consistency (the fitting residuals are large), there is either: - Genuine information (the market is pricing in something M doesn't capture), or - Relative mispricing (the market makers are inconsistent with each other)

Distinguishing the two is the art. The implied generator provides the quantitative framework.

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7. Conclusion

The implied generator is not an oracle. It does not predict where the market will go. It reveals the **structure of where the market THINKS it will go**, decomposed into eigenvalues that encode specific dynamics. Trading this structure produces edge through five mechanisms: illiquid maturity pricing, calendar spread relative value, hedge improvement, eigenvalue regime detection, and exotic–vanilla consistency.

The analogy is exact: the yield curve did not predict bond prices, but it revealed the TERM STRUCTURE of interest rates, enabling trillion-dollar markets in rates derivatives. The implied generator reveals the **spectral structure of option dynamics**, enabling a new class of trades: trades on the eigenvalues of the market's belief.

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References

- Dupire, B (1994). Pricing with a smile. *Risk*, 7(1), 18-20.
- Gatheral, J (2006). The Volatility Surface. *The Volatility Surface*. DOI: 10.1002/9781119202073
- Hagan, P. S., Kumar, D., Lesniewski, A. S., & Woodward, D. E (2002). Managing smile risk. *Wilmott Magazine*, 84-108.
- Heston, S.L (1993). A closed-form solution for options with stochastic volatility. *Heston, S.L.*, 6(2).
- Nagy, T. (2026). Lean 4 Formal Verification of the Spectral Fenton Distribution and Related Financial Mathematics. *Working paper.*
- Nagy, T. (2026). The Quantum Spectral Representation Theorem: What Can and Cannot Be Compressed. *Working paper.*
- Nagy, T. (2026). The Pythagorean Gap: A Mode-Decomposition Indicator for Financial Crises. *Working paper.*

Appendix: Reproducibility

```
python3 examples/implied_generator_trading.py
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Runtime: 11 seconds. Self-contained (NumPy + SciPy). Includes: production accuracy (N=32), barrier pricing, 5 trading strategies, P&L backtest, regime detection.