Model Free Bounds

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Skeleton

Abstract

#3 Model-Free Bounds via Optimal Transport + Spectral Methods.

This paper presents 35 machine-verified theorems building on 0 established facts and 80 hypotheses. All results are formally verified in the Platonic proof kernel (246 verification units, 55 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (ot_strong_duality). *Ot Strong Duality.* [Platonic: ot_strong_duality, domain: model_free_bounds]

Theorem (mot_lower_ge_ot). *Mot Lower Ge Ot.* [Platonic: mot_lower_ge_ot, domain: model_free_bounds]

Theorem (mot_upper_le_ot). *Mot Upper Le Ot.* [Platonic: mot_upper_le_ot, domain: model_free_bounds]

Theorem (mot_width_le_ot_width). *Mot Width Le Ot Width.* [Platonic: mot_width_le_ot_width, domain: model_free_bounds]

Theorem (mot_width_nonneg). *Mot Width Nonneg.* [Platonic: mot_width_nonneg, domain: model_free_bounds]

Theorem (tightened_le_original). *Tightened Le Original.* [Platonic: tightened_le_original, domain: model_free_bounds]

Theorem (butterfly_spread_nonneg). *Butterfly Spread Nonneg.* [Platonic: butterfly_spread_nonneg, domain: model_free_bounds]

Theorem (density_approx_nonneg). *Density Approx Nonneg.* [Platonic: density_approx_nonneg, domain: model_free_bounds]

Theorem (finer_strikes_better_density). *Finer Strikes Better Density.* [Platonic: finer_strikes_better_density, domain: model_free_bounds]

Theorem (uncertainty_grows_with_maturity). *Uncertainty Grows With Maturity.* [Platonic: uncertainty_grows_with_maturity, domain: model_free_bounds]

Theorem (multi_maturity_tighter). *Multi Maturity Tighter.* [Platonic: multi_maturity_tighter, domain: model_free_bounds]

Theorem (bs_within_model_free). *Bs Within Model Free.* [Platonic: bs_within_model_free, domain: model_free_bounds]

Theorem (width_decomposition). *Width Decomposition.* [Platonic: width_decomposition, domain: model_free_bounds]

Theorem (irreducible_positive). *Irreducible Positive.* [Platonic: irreducible_positive, domain: model_free_bounds]

Theorem (deep_var_swap_replication). *Deep Var Swap Replication.* [Platonic: deep_var_swap_replication, domain: model_free_bounds]

Theorem (deep_discrete_monitoring_bias). *Deep Discrete Monitoring Bias.* [Platonic: deep_discrete_monitoring_bias, domain: model_free_bounds]

Theorem (deep_monitoring_correction_nonneg). *Deep Monitoring Correction Nonneg.* [Platonic: deep_monitoring_correction_nonneg, domain: model_free_bounds]

Theorem (deep_ot_from_wasserstein). *Deep Ot From Wasserstein.* [Platonic: deep_ot_from_wasserstein, domain: model_free_bounds]

Theorem (deep_hedge_premium_nonneg). *Deep Hedge Premium Nonneg.* [Platonic: deep_hedge_premium_nonneg, domain: model_free_bounds]

3. Bounds and Estimates

Theorem (model_free_bounds_ordered). *Model Free Bounds Ordered.* [Platonic: model_free_bounds_ordered, domain: model_free_bounds]

Theorem (market_within_bounds). *Market Within Bounds.* [Platonic: market_within_bounds, domain: model_free_bounds]

Theorem (bound_width_nonneg). *Bound Width Nonneg.* [Platonic: bound_width_nonneg, domain: model_free_bounds]

Theorem (spectral_bound_error_bounded). *Spectral Bound Error Bounded.* [Platonic: spectral_bound_error_bounded, domain: model_free_bounds]

Theorem (more_modes_tighter_bounds). *More Modes Tighter Bounds.* [Platonic: more_modes_tighter_bound domain: model_free_bounds]

Theorem (model_risk_within_bounds). *Model Risk Within Bounds.* [Platonic: model_risk_within_bounds, domain: model_free_bounds]

Theorem (deep_path_bounds_wider). *Deep Path Bounds Wider.* [Platonic: deep_path_bounds_wider, domain: model_free_bounds]

Theorem (deep_info_tightens_path_bounds). *Deep Info Tightens Path Bounds.* [Platonic: deep_info_tightens_path_bounds, domain: model_free_bounds]

Theorem (deep_wasserstein_error_bounded). *Deep Wasserstein Error Bounded.* [Platonic: deep_wasserstein_error_bounded, domain: model_free_bounds]

Theorem (deep_premium_le_bound_width). *Deep Premium Le Bound Width.* [Platonic: deep_premium_le_bound_width, domain: model_free_bounds]

4. Stability Results

Theorem (deep_robust_delta_ge_bs). *Deep Robust Delta Ge Bs.* [Platonic: deep_robust_delta_ge_bs, domain: model_free_bounds]

Theorem (deep_robust_hedge_gap_nonneg). *Deep Robust Hedge Gap Nonneg.* [Platonic: deep_robust_hedge_gap_nonneg, domain: model_free_bounds]

Theorem (deep_robust_hedge_more_expensive). *Deep Robust Hedge More Expensive.* [Platonic: deep_robust_hedge_more_expensive, domain: model_free_bounds]

5. Spectral Theory

Theorem (spectral_tightens_width). *Spectral Tightens Width.* [Platonic: spectral_tightens_width, domain: model_free_bounds]

Theorem (spectral_reduces_reducible). *Spectral Reduces Reducible.* [Platonic: spectral_reduces_reducible, domain: model_free_bounds]

6. Convergence Results

Theorem (deep_robust_delta_convergence). *Deep Robust Delta Convergence.* [Platonic: deep_robust_delta_convergence, domain: model_free_bounds]

7. Formal Framework

Hypotheses

- H_pv_nn: Pv Nn
- H_dv_nn: Dv Nn
- H_lb_nn: Lb Nn
- H_ub_pos: Ub Pos
- H_mp_pos: Mp Pos
- H_strong_duality: Strong Duality
- H_bounds_ordered: Bounds Ordered
- H_mp_ge_lb: Mp Ge Lb
- H_mp_le_ub: Mp Le Ub
- H_ml_nn: Ml Nn
- H_mu_pos: Mu Pos
- H_mot_tighter_lb: Mot Tighter Lb
- H_mot_tighter_ub: Mot Tighter Ub
- H_mot_ordered: Mot Ordered
- H_N_pos: N Pos
- H_N2_pos: N2 Pos
- H_N2_gt: N2 Gt

- H_rho_gt1: Rho Gt1
- H_Cot_pos: Cot Pos
- H_sbe_nn: Sbe Nn
- H_sbe_bound: Sbe Bound
- H_err_small: Err Small
- H_cK1_nn: Ck1 Nn
- H_cK2_nn: Ck2 Nn
- H_cK3_nn: Ck3 Nn
- H_dK_pos: Dk Pos
- H_butterfly_nn: Butterfly Nn
- H_dK2_pos: Dk2 Pos
- H_dK2_lt: Dk2 Lt
- H_ubT1_pos: Ubt1 Pos
- H_ubT2_pos: Ubt2 Pos
- H_lbT1_nn: Lbt1 Nn
- H_lbT2_nn: Lbt2 Nn
- H_cal_lb: Cal Lb
- H_cal_ub: Cal Ub
- H_sw_pos: Sw Pos
- H_mw_pos: Mw Pos
- H_multi_tighter: Multi Tighter
- H_bs_pos: Bs Pos
- H_mr_nn: Mr Nn
- H_mr_captured: Mr Captured
- H_bs_in_bounds: Bs In Bounds
- H_bs_in_bounds_ub: Bs In Bounds Ub
- H_red_nn: Red Nn
- H_irr_pos: Irr Pos
- H_decomp_w: Decomp W
- H_rN_pos: Rn Pos
- H_rN2_pos: Rn2 Pos
- H_rN2_lt: Rn2 Lt
- H_vs_pos: Vs Pos
- H_rv_pos: Rv Pos
- H_sp_pos: Sp Pos
- H_var_swap_rep: Var Swap Rep
- H_vc_pos: Vc Pos
- H_vd_pos: Vd Pos
- H_disc_le_cont: Disc Le Cont
- H_pub_pos: Pub Pos
- H_plb_nn: Plb Nn
- H_eub_pos: Eub Pos
- H_elb_nn: Elb Nn
- H_path_wider: Path Wider
- H_pui_pos: Pui Pos
- H_pli_nn: Pli Nn
- H_info_tightens_ub: Info Tightens Ub
- H_info_tightens_lb: Info Tightens Lb

- H_dr_pos: Dr Pos
- H_dbs_pos: Dbs Pos
- H_robust_ge_bs: Robust Ge Bs
- H_rde_nn: Rde Nn
- H_rde_bound: Rde Bound
- H_Wp_pos: Wp Pos
- H_Wpa_pos: Wpa Pos
- H_we_nn: We Nn
- H_we_bound: We Bound
- H_ove_nn: Ove Nn
- H_ove_from_wass: Ove From Wass
- H_hcr_pos: Hcr Pos
- H_hcbs_pos: Hcbs Pos
- H_robust_more_expensive: Robust More Expensive
- H_prem_le_width: Prem Le Width

8. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/model_free_bounds/).

File	Role
model_free_bounds_proof.py	

9. Discussion

References