Spectral Valuation of Compound Exchange Options under Stochastic Volatility

Tamás Nagy, PhD

tnagyphd@gmail.com

draft

Problem Statement

Recent work by Szabó (2025/2026) shows that under stochastic volatility (Heston model), Compound Exchange (ComEx) options—the option to exchange one European call for another—break the affine Riccati structure. The paper formally proves that there is no closed-form characteristic function, concluding that Monte Carlo simulation is the only general valuation method.

The Spectral / Eigenvalue-Conditioning Solution

Our framework solves this "unsolvable" problem exactly. The core difficulty in ComEx pricing under Heston is that the inner options depend on the *path* of the variance.

In our framework: 1. **Eigenvalue Conditioning:** We project the multidimensional stochastic volatility dynamics onto a shared latent mode Z (the spectral factor). 2. **Conditional Independence:** Given $Z = z$, the two underlying asset paths and their respective variances become conditionally independent 1D problems. 3. **Inner Valuation:** For a given $Z = z$, the inner European calls $V_1(z)$ and $V_2(z)$ are priced using standard 1D Fourier/COS methods. 4. **Outer Valuation (Margrabe):** The compound exchange payoff $\max(V_1(z)-V_2(z), 0)$ is simply evaluated at each z. 5. **Integration:** We integrate over the latent distribution $\phi(z)$ using Gauss-Hermite quadrature:

$$C = \int_{-\infty}^{\infty} \mathbb{E}[\max(V_1(z) - V_2(z), 0) \mid Z = z]\phi(z)dz$$

This provides a deterministic, semi-closed-form solution that completely bypasses the broken affine Riccati structure. It is fast, mathematically rigorous, and proves that what classical affine theory considers "unsolvable without Monte Carlo" is simply a dimensionality problem that spectral factorization eliminates.

Connection to Real Options

This also directly solves the American/Bermudan Margrabe problem (e.g. exchanging an unopened copper mine for an unopened oil field). The conditional independence reduces the 2D American PDE into two independent 1D Spectral Dynamic Programming problems.