

# Spectral Portfolio

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Skeleton

## Abstract

Spectral Portfolio — ProofEnv proofs (covariance / PC risk split).

This paper presents 93 machine-verified theorems. All results are formally verified in the Platonic proof kernel (89 verification units, 93 proved statements) and exportable to Lean 4.

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## 1. Introduction

## 2. Further Results

**Theorem** (risk\_decomposition). *Risk Decomposition*. [Platonic: risk\_decomposition, domain: spectral\_portfolio]

**Theorem** (systematic\_le\_total). *Systematic Le Total*. [Platonic: systematic\_le\_total, domain: spectral\_portfolio]

**Theorem** (idiosyncratic\_le\_total). *Idiosyncratic Le Total*. [Platonic: idiosyncratic\_le\_total, domain: spectral\_portfolio]

**Theorem** (diversification\_helps). *Diversification Helps*. [Platonic: diversification\_helps, domain: spectral\_portfolio]

**Theorem** (zero\_idio\_pure\_systematic). *Zero Idio Pure Systematic*. [Platonic: zero\_idio\_pure\_systematic, domain: spectral\_portfolio]

**Theorem** (variance\_split\_comm). *Variance Split Comm*. [Platonic: variance\_split\_comm, domain: spectral\_portfolio]

**Theorem** (portfolio\_variance\_nonneg). *Portfolio Variance Nonneg*. [Platonic: portfolio\_variance\_nonneg, domain: spectral\_portfolio]

**Theorem** (systematic\_nonneg\_of\_split). *Systematic Nonneg Of Split*. [Platonic: systematic\_nonneg\_of\_split, domain: spectral\_portfolio]

**Theorem** (idiosyncratic\_nonneg\_of\_split). *Idiosyncratic Nonneg Of Split*. [Platonic: idiosyncratic\_nonneg\_of\_split, domain: spectral\_portfolio]

**Theorem** (split\_refl). *Split Refl*. [Platonic: split\_refl, domain: spectral\_portfolio]

**Theorem** (w2\_var\_sum\_nonneg). *W2 Var Sum Nonneg*. [Platonic: w2\_var\_sum\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_portfolio\_ge\_each\_nonneg). *W2 Portfolio Ge Each Nonneg.* [Platonic: w2\_portfolio\_ge\_each\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_idio\_share\_nonneg). *W2 Idio Share Nonneg.* [Platonic: w2\_idio\_share\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_weighted\_variance\_nonneg). *W2 Weighted Variance Nonneg.* [Platonic: w2\_weighted\_variance\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_pc1\_scale\_order). *W2 Pc1 Scale Order.* [Platonic: w2\_pc1\_scale\_order, domain: spectral\_portfolio]

**Theorem** (w2\_split\_swap\_eq). *W2 Split Swap Eq.* [Platonic: w2\_split\_swap\_eq, domain: spectral\_portfolio]

**Theorem** (w2\_variance\_idempotent\_add\_zero). *W2 Variance Idempotent Add Zero.* [Platonic: w2\_variance\_idempotent\_add\_zero, domain: spectral\_portfolio]

**Theorem** (w2\_diversification\_monotone\_idio). *W2 Diversification Monotone Idio.* [Platonic: w2\_diversification\_monotone\_idio, domain: spectral\_portfolio]

**Theorem** (w2\_beta\_exposure\_nonneg\_scale). *W2 Beta Exposure Nonneg Scale.* [Platonic: w2\_beta\_exposure\_nonneg\_scale, domain: spectral\_portfolio]

**Theorem** (w2\_residual\_nonneg\_implies\_port). *W2 Residual Nonneg Implies Port.* [Platonic: w2\_residual\_nonneg\_implies\_port, domain: spectral\_portfolio]

**Theorem** (w2\_total\_ge\_systematic\_alt). *W2 Total Ge Systematic Alt.* [Platonic: w2\_total\_ge\_systematic\_alt, domain: spectral\_portfolio]

**Theorem** (w2\_subadd\_variance\_upper). *W2 Subadd Variance Upper.* [Platonic: w2\_subadd\_variance\_upper, domain: spectral\_portfolio]

**Theorem** (w2\_eq\_trans\_port). *W2 Eq Trans Port.* [Platonic: w2\_eq\_trans\_port, domain: spectral\_portfolio]

**Theorem** (w2\_nonneg\_minus\_nonpos). *W2 Nonneg Minus Nonpos.* [Platonic: w2\_nonneg\_minus\_nonpos, domain: spectral\_portfolio]

**Theorem** (w2\_square\_nonneg\_refl). *W2 Square Nonneg Refl.* [Platonic: w2\_square\_nonneg\_refl, domain: spectral\_portfolio]

**Theorem** (w2\_portfolio\_refl). *W2 Portfolio Refl.* [Platonic: w2\_portfolio\_refl, domain: spectral\_portfolio]

**Theorem** (w2\_scalar\_sum\_linearity). *W2 Scalar Sum Linearity.* [Platonic: w2\_scalar\_sum\_linearity, domain: spectral\_portfolio]

**Theorem** (w2\_risk\_budget\_nonneg). *W2 Risk Budget Nonneg.* [Platonic: w2\_risk\_budget\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_le\_add\_nonneg\_tail). *W2 Le Add Nonneg Tail.* [Platonic: w2\_le\_add\_nonneg\_tail, domain: spectral\_portfolio]

**Theorem** (w2\_tobin\_scale\_nonneg). *W2 Tobin Scale Nonneg.* [Platonic: w2\_tobin\_scale\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_mul\_comm\_port). *W2 Mul Comm Port*. [Platonic: w2\_mul\_comm\_port, domain: spectral\_portfolio]

**Theorem** (w2\_add\_assoc\_port). *W2 Add Assoc Port*. [Platonic: w2\_add\_assoc\_port, domain: spectral\_portfolio]

**Theorem** (w2\_split\_symmetric\_nonneg). *W2 Split Symmetric Nonneg*. [Platonic: w2\_split\_symmetric\_nonneg, domain: spectral\_portfolio]

**Theorem** (w2\_idio\_le\_total\_repeat). *W2 Idio Le Total Repeat*. [Platonic: w2\_idio\_le\_total\_repeat, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_loading\_norm\_sq\_nonneg). *W3 Sp Loading Norm Sq Nonneg*. [Platonic: w3\_sp\_loading\_norm\_sq\_nonneg, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_sigma\_sqrt\_nonneg). *W3 Sp Sigma Sqrt Nonneg*. [Platonic: w3\_sp\_sigma\_sqrt\_nonneg, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_loading\_dot\_comm). *W3 Sp Loading Dot Comm*. [Platonic: w3\_sp\_loading\_dot\_comm, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_loading\_norm\_sq\_def). *W3 Sp Loading Norm Sq Def*. [Platonic: w3\_sp\_loading\_norm\_sq\_def, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_basket\_sum\_weights\_nonneg). *W3 Sp Basket Sum Weights Nonneg*. [Platonic: w3\_sp\_basket\_sum\_weights\_nonneg, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_basket\_prod\_factors\_nonneg). *W3 Sp Basket Prod Factors Nonneg*. [Platonic: w3\_sp\_basket\_prod\_factors\_nonneg, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_basket\_prod\_monotone). *W3 Sp Basket Prod Monotone*. [Platonic: w3\_sp\_basket\_prod\_monotone, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_sigma\_sq\_sqrt). *W3 Sp Sigma Sq Sqrt*. [Platonic: w3\_sp\_sigma\_sq\_sqrt, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_sqrt\_sigma\_sq). *W3 Sp Sqrt Sigma Sq*. [Platonic: w3\_sp\_sqrt\_sigma\_sq, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_sigma\_sqrt\_mul). *W3 Sp Sigma Sqrt Mul*. [Platonic: w3\_sp\_sigma\_sqrt\_mul, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_pc\_cov\_sqrt\_mono). *W3 Sp Pc Cov Sqrt Mono*. [Platonic: w3\_sp\_pc\_cov\_sqrt\_mono, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_dense\_factor\_sum\_pos). *W3 Sp Dense Factor Sum Pos*. [Platonic: w3\_sp\_dense\_factor\_sum\_pos, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_identity\_det). *W3 Sp Identity Det*. [Platonic: w3\_sp\_identity\_det, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_idio\_norm\_energy\_augment\_nonneg). *W3 Sp Idio Norm Energy Augment Nonneg*. [Platonic: w3\_sp\_idio\_norm\_energy\_augment\_nonneg, domain: spectral\_portfolio]

**Theorem** (w3\_sp\_sys\_vol\_sqrt\_augment\_nonneg). *W3 Sp Sys Vol Sqrt Augment Nonneg*. [Platonic: w3\_sp\_sys\_vol\_sqrt\_augment\_nonneg, domain: spectral\_portfolio]

**Theorem** (rebalance\_drift\_factor\_nonneg). *Rebalance Drift Factor Nonneg.* [Platonic: rebalance\_drift\_factor\_nonneg, domain: spectral\_portfolio]

**Theorem** (tracking\_error\_quadratic\_nonneg). *Tracking Error Quadratic Nonneg.* [Platonic: tracking\_error\_quadratic\_nonneg, domain: spectral\_portfolio]

**Theorem** (mode\_capacity\_pos). *Mode Capacity Pos.* [Platonic: mode\_capacity\_pos, domain: spectral\_portfolio]

**Theorem** (slow\_modes\_more\_capacity). *Slow Modes More Capacity.* [Platonic: slow\_modes\_more\_capacity, domain: spectral\_portfolio]

**Theorem** (marginal\_diversification\_value\_nonneg). *Marginal Diversification Value Nonneg.* [Platonic: marginal\_diversification\_value\_nonneg, domain: spectral\_portfolio]

**Theorem** (portfolio\_sharpe\_from\_modes). *Portfolio Sharpe From Modes.* [Platonic: portfolio\_sharpe\_from\_modes, domain: spectral\_portfolio]

**Theorem** (risk\_budget\_sum\_is\_total). *Risk Budget Sum Is Total.* [Platonic: risk\_budget\_sum\_is\_total, domain: spectral\_portfolio]

**Theorem** (risk\_contribution\_nonneg). *Risk Contribution Nonneg.* [Platonic: risk\_contribution\_nonneg, domain: spectral\_portfolio]

**Theorem** (equal\_risk\_contribution). *Equal Risk Contribution.* [Platonic: equal\_risk\_contribution, domain: spectral\_portfolio]

**Theorem** (marginal\_value\_zero\_iff). *Marginal Value Zero Iff.* [Platonic: marginal\_value\_zero\_iff, domain: spectral\_portfolio]

**Theorem** (marginal\_value\_pos\_iff). *Marginal Value Pos Iff.* [Platonic: marginal\_value\_pos\_iff, domain: spectral\_portfolio]

**Theorem** (bl\_scalar\_convex\_recombine). *Bl Scalar Convex Recombine.* [Platonic: bl\_scalar\_convex\_recombine, domain: spectral\_portfolio]

**Theorem** (portfolio\_return\_linear). *Portfolio Return Linear.* [Platonic: portfolio\_return\_linear, domain: spectral\_portfolio]

**Theorem** (variance\_nonneg\_from\_weights). *Variance Nonneg From Weights.* [Platonic: variance\_nonneg\_from\_weights, domain: spectral\_portfolio]

**Theorem** (efficient\_frontier\_monotone). *Efficient Frontier Monotone.* [Platonic: efficient\_frontier\_monotone, domain: spectral\_portfolio]

**Theorem** (dominated\_portfolio). *Dominated Portfolio.* [Platonic: dominated\_portfolio, domain: spectral\_portfolio]

**Theorem** (sharpe\_ratio\_positive). *Sharpe Ratio Positive.* [Platonic: sharpe\_ratio\_positive, domain: spectral\_portfolio]

**Theorem** (higher\_return\_higher\_sharpe). *Higher Return Higher Sharpe.* [Platonic: higher\_return\_higher\_sharpe, domain: spectral\_portfolio]

**Theorem** (lower\_vol\_higher\_sharpe). *Lower Vol Higher Sharpe.* [Platonic: lower\_vol\_higher\_sharpe, domain: spectral\_portfolio]

**Theorem** (hhi\_single\_asset). *Hhi Single Asset*. [Platonic: hhi\_single\_asset, domain: spectral\_portfolio]

**Theorem** (weight\_square\_le\_weight). *Weight Square Le Weight*. [Platonic: weight\_square\_le\_weight, domain: spectral\_portfolio]

**Theorem** (factor\_exposure\_contribution). *Factor Exposure Contribution*. [Platonic: factor\_exposure\_contribution, domain: spectral\_portfolio]

**Theorem** (residual\_variance\_nonneg). *Residual Variance Nonneg*. [Platonic: residual\_variance\_nonneg, domain: spectral\_portfolio]

**Theorem** (equal\_risk\_implies\_equal\_contribution). *Equal Risk Implies Equal Contribution*. [Platonic: equal\_risk\_implies\_equal\_contribution, domain: spectral\_portfolio]

**Theorem** (risk\_parity\_budget\_exhaustive). *Risk Parity Budget Exhaustive*. [Platonic: risk\_parity\_budget\_exhaustive, domain: spectral\_portfolio]

**Theorem** (tobin\_two\_fund\_separation). *Tobin Two Fund Separation*. [Platonic: tobin\_two\_fund\_separation, domain: spectral\_portfolio]

**Theorem** (leverage\_amplifies\_return). *Leverage Amplifies Return*. [Platonic: leverage\_amplifies\_return, domain: spectral\_portfolio]

**Theorem** (leverage\_amplifies\_risk). *Leverage Amplifies Risk*. [Platonic: leverage\_amplifies\_risk, domain: spectral\_portfolio]

**Theorem** (perfect\_correlation\_no\_diversification). *Perfect Correlation No Diversification*. [Platonic: perfect\_correlation\_no\_diversification, domain: spectral\_portfolio]

**Theorem** (diversification\_benefit\_nonneg). *Diversification Benefit Nonneg*. [Platonic: diversification\_benefit\_nonneg, domain: spectral\_portfolio]

**Theorem** (diagonal\_variance\_nonneg). *Diagonal Variance Nonneg*. [Platonic: diagonal\_variance\_nonneg, domain: spectral\_portfolio]

**Theorem** (truncation\_error\_nonneg). *Truncation Error Nonneg*. [Platonic: truncation\_error\_nonneg, domain: spectral\_portfolio]

**Theorem** (explained\_variance\_monotone). *Explained Variance Monotone*. [Platonic: explained\_variance\_monotone, domain: spectral\_portfolio]

### 3. Bounds and Estimates

**Theorem** (pc1\_contribution\_bounded). *Pc1 Contribution Bounded*. [Platonic: pc1\_contribution\_bounded, domain: spectral\_portfolio]

**Theorem** (w2\_cov\_bound\_abs\_triangle). *W2 Cov Bound Abs Triangle*. [Platonic: w2\_cov\_bound\_abs\_triangle, domain: spectral\_portfolio]

**Theorem** (w2\_one\_bound). *W2 One Bound*. [Platonic: w2\_one\_bound, domain: spectral\_portfolio]

**Theorem** (w2\_stress\_delta\_bound). *W2 Stress Delta Bound*. [Platonic: w2\_stress\_delta\_bound, domain: spectral\_portfolio]

**Theorem** (minimum\_variance\_bounded). *Minimum Variance Bounded.* [Platonic: minimum\_variance\_bounded, domain: spectral\_portfolio]

**Theorem** (r\_squared\_bounded). *R Squared Bounded.* [Platonic: r\_squared\_bounded, domain: spectral\_portfolio]

**Theorem** (correlation\_bounded\_product). *Correlation Bounded Product.* [Platonic: correlation\_bounded\_product, domain: spectral\_portfolio]

## 4. Spectral Theory

**Theorem** (w3\_sp\_cov\_operator\_mul\_id). *W3 Sp Cov Operator Mul Id.* [Platonic: w3\_sp\_cov\_operator\_mul\_id, domain: spectral\_portfolio]

**Theorem** (spectral\_ratio\_div\_nonneg). *Spectral Ratio Div Nonneg.* [Platonic: spectral\_ratio\_div\_nonneg, domain: spectral\_portfolio]

**Theorem** (spectral\_ratio\_div\_le\_one). *Spectral Ratio Div Le One.* [Platonic: spectral\_ratio\_div\_le\_one, domain: spectral\_portfolio]

**Theorem** (eigenvalue\_contribution\_nonneg). *Eigenvalue Contribution Nonneg.* [Platonic: eigenvalue\_contribution\_nonneg, domain: spectral\_portfolio]

## 5. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/spectral\_portfolio/).

File	Role
spectral_portfolio_proof.py	

## 6. Discussion

## References