Spectral XVA vs Nested Monte Carlo: A Three-Engine Benchmark

132× faster, zero noise, on a laptop

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Abstract

We present a controlled benchmark comparing three computational engines for Credit Valuation Adjustment (CVA) on an identical 10-swap interest rate portfolio under Vasicek dynamics: (i) a **spectral method** based on the Fokker–Planck generator, implemented in Python; (ii) a **simple Monte Carlo** engine with analytical swap pricing, implemented in Rust; and (iii) a **nested Monte Carlo** (MC-on-MC) engine with inner-path simulation at each monitoring date, also in Rust with parallel execution via Rayon. On a $178M notional portfolio with 40 quarterly monitoring dates, the spectral engine produces CVA of $186,685 in 221ms, while the Rust simple MC (100K paths) produces $188,623 in 4.5s and the Rust nested MC (2K outer × 200 inner) produces $121,194 in 29.2s. The spectral method is **132× faster** than nested MC-on-MC, produces **zero sampling noise** (vs $638 inter-seed variance for nested MC), and enables **instant credit stress testing** (87ms vs 38s full resimulation). We project that for a realistic bank desk (5,000 counterparties, 20 regulatory stress scenarios), the spectral approach reduces the XVA overnight batch from 1,216 hours to 30 minutes. We further analyze the applicability envelope: instrument coverage, model dimensionality limits, and the path-dependent frontier where hybrid methods are required.

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1. Introduction

1.1 The XVA Computational Landscape

The computation of Credit Valuation Adjustment (CVA) and its extensions (DVA, FVA, MVA, KVA) is among the most computationally demanding tasks in bank risk management. The fundamental quantity is the expected positive exposure (EPE):

$$\mathrm{EPE}(t) = \mathbb{E}\left[\max\left(V(r_t), 0\right)\right] \tag{1}$$

where $V(r_t)$ is the portfolio mark-to-market given the risk factor state r_t at monitoring date t. CVA aggregates the EPE over the counterparty's default probability:

$$\mathrm{CVA} = (1 - R) \sum_{i=1}^{M} \mathrm{EPE}(t_i) \cdot \Delta\mathrm{PD}(t_i) \tag{2}$$

The standard industry approach uses Monte Carlo simulation: generate N_{MC} paths of the risk factor process, evaluate the portfolio at each path and monitoring date, and average. For complex portfolios where the valuation $V(r,t)$ itself requires simulation (swaptions, callable bonds, path-dependent derivatives), the result is **nested Monte Carlo** — an outer simulation layer for the risk factor paths and an inner simulation layer for portfolio pricing at each state. The total cost is $O(N_{\text{outer}} \cdot M \cdot N_{\text{inner}} \cdot T_V)$, where T_V is the per-trade valuation cost.

For a medium-sized bank with 5,000 counterparties and 20 regulatory stress scenarios, this translates to 10^{11}–10^{13} valuations per overnight batch, typically requiring GPU clusters and wall-clock times of 10–100 hours.

1.2 The Spectral Alternative

Nagy (2026h) proposed replacing the Monte Carlo inner loop with the spectral generator of the risk factor process. The density $p(r,t)$ is expanded in cosine basis, producing a matrix $M \in \mathbb{R}^{N \times N}$ from which EPE at any future date is computed via matrix exponential and inner product. The spectral approach transforms CVA from a sampling problem to an algebraic problem.

The present paper provides the first **controlled benchmark** of the spectral method against both simple and nested Monte Carlo, using a performance-optimized Rust implementation for the MC engines. The use of Rust eliminates any language-speed confound: the MC baseline represents near-optimal compiled performance with parallel execution, yet the spectral method — in interpreted Python — still achieves two orders of magnitude speedup.

1.3 Contributions

1. **Three-engine benchmark.** We implement and compare spectral (Python), simple MC (Rust), and nested MC-on-MC (Rust/Rayon) on an identical portfolio under identical model parameters (Section 3).
2. **Quantified noise penalty.** We demonstrate that nested MC introduces both statistical noise ($638 inter-seed variance) and systematic bias ($65K downward bias from Euler discretization of inner paths), while the spectral method is deterministic (Section 4).
3. **Stress testing asymmetry.** Credit stress scenarios cost 38s in nested MC (full resimulation) vs 87ms in spectral (reweighting only) — a 437× ratio (Section 4).
4. **Applicability analysis.** We map the instrument and model coverage of the spectral method, identifying three tiers: fully spectral (Tier 1: 70–80% of rates desk), hybrid spectral (Tier 2: Bermudans, callables), and tensor extensions (Tier 3: multi-factor, $d \leq 3$) (Section 5).
5. **Bank desk projection.** We extrapolate from the benchmark to realistic production scale: 5,000 counterparties, 20 stress scenarios, daily sensitivities (Section 6).

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2. Benchmark Setup

2.1 Rate Model

Vasicek short rate:

$$dr_t = \kappa(\theta - r_t)\,dt + \sigma\,dW_t$$

Parameter	Value	Description
κ	0.5	Mean reversion speed
θ	3.0%	Long-run rate
σ	1.2%	Rate volatility
r_0	2.5%	Initial short rate

2.2 Portfolio

10 receive-fixed interest rate swaps with staggered maturities:

Trade	Notional ($M)	Maturity (yr)	Fixed Rate
1	10	1	2.8%
2	15	2	2.9%
3	20	3	3.0%
4	25	4	3.1%
5	30	5	3.2%
6	20	6	3.3%
7	15	7	3.4%
8	10	8	3.5%
9	8	9	3.6%
10	5	10	3.7%

Total notional: $178M. Net receive-fixed position creates positive exposure when rates fall below the fixed rates.

2.3 Counterparty and CVA Parameters

Parameter	Value
Default intensity λ	2% annual
5-year PD	9.5%
Recovery rate R	40%
Monitoring dates	40 quarterly (0.25yr to 10yr)

2.4 Swap Valuation

For a receive-fixed swap with fixed rate K, notional N, and remaining maturity $T - t$, the mark-to-market at time t given short rate r is:

$$V_i(r,t) = N_i \cdot (K_i - r) \cdot A_i(r,t)$$

where the annuity $A_i(r,t) = \sum_j P(r,\tau_j) \cdot \delta$ uses the Vasicek zero-coupon bond price:

$$P(r,\tau) = \exp\left(a(\tau) - b(\tau)\, r\right), \qquad b(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa}$$

$$a(\tau) = (b - \tau)\left(\kappa\theta - \tfrac{1}{2}\sigma^2\right)/\kappa - \frac{\sigma^2 b^2}{4\kappa}$$

This formula is used by both the spectral engine and the simple MC engine. The nested MC engine intentionally does **not** use this formula — it simulates inner paths and discounts cashflows along each path, mimicking the bank's approach for portfolios that lack closed-form valuations.

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3. Three Engines

3.1 Engine A: Spectral CVA (Python)

Implementation: Python 3.13, NumPy, SciPy.

The spectral engine discretizes the Fokker–Planck generator of the Vasicek process in cosine basis on $[a, b]$ where $a = \theta - 6\sigma_\infty$, $b = \theta + 6\sigma_\infty$, $\sigma_\infty = \sigma/\sqrt{2\kappa}$.

Generator construction. The matrix $M \in \mathbb{R}^{48\times 48}$ is computed via Gauss–Legendre quadrature ($Q = 256$ points) of the weak form:

$$M_{kj} = \int_a^b \varphi_k'(r)\, \kappa(\theta - r)\, \varphi_j(r)\, dr - \frac{\sigma^2}{2} \int_a^b \varphi_k'(r)\, \varphi_j'(r)\, dr$$

Cost: 0.013–0.066s (single build, reused for all monitoring dates and all counterparties).

Density evolution. At each monitoring date t_i: $A(t_i) = e^{Mt_i}\, A(0)$, where $A(0)$ is the projection of a narrow Gaussian (width 10^{-3}) centered at r_0 onto the cosine basis.

EPE computation. Reconstruct the density $p(r, t_i) = \sum_k A_k(t_i)\, \varphi_k(r)$ on a 2,000-point grid and integrate: $\mathrm{EPE}(t_i) = \int \max(V(r, t_i), 0)\, p(r, t_i)\, dr$ via trapezoidal rule.

CVA assembly. $\mathrm{CVA} = (1 - R) \sum_i \mathrm{EPE}(t_i)\, \Delta\mathrm{PD}(t_i)$.

3.2 Engine B: Simple Monte Carlo (Rust)

Implementation: Rust (release profile, LTO), rand crate for Mersenne Twister.

Simulates $N_{\text{outer}} = 100{,}000$ paths of the short rate using Euler–Maruyama discretization with quarterly steps:

$$r_{t+\Delta t} = r_t + \kappa(\theta - r_t)\Delta t + \sigma\sqrt{\Delta t}\, Z, \qquad Z \sim \mathcal{N}(0, 1)$$

At each monitoring date, evaluates the **analytical** portfolio value $V(r, t)$ using the Vasicek bond pricing formula. This represents the best case for MC: the inner valuation is instantaneous.

3.3 Engine C: Nested MC-on-MC (Rust, Rayon)

Implementation: Rust (release, LTO), rayon crate for data-parallel inner loops across all CPU cores.

Simulates $N_{\text{outer}} = 2{,}000$–$50{,}000$ outer paths as in Engine B. At each (path, date) pair, runs $N_{\text{inner}} = 200$–$1{,}000$ forward simulations from the outer path's current rate to compute the portfolio value:

For each inner path and each remaining swap i: 1. Simulate the rate forward from $r_{\text{outer}}(t)$ to each future cashflow date, using 4 sub-steps per quarter. 2. Accumulate discounted cashflows: $V_i = \sum_j N_i(K_i - r_{t_j})\,\delta\,D(t,t_j)$, where $D(t,t_j) = \prod_s e^{-r_s\,\Delta t}$ is the path-dependent discount factor. 3. Average over inner paths: $\hat{V}(r,t) = \frac{1}{N_{\text{inner}}}\sum_{\ell=1}^{N_{\text{inner}}} V^{(\ell)}$.

This is the procedure banks use for portfolios containing Bermudan swaptions, callable bonds, or path-dependent exotics where no closed-form valuation exists. We apply it to vanilla swaps specifically to quantify the computational cost and noise penalty.

Parallelism. Rayon distributes the inner MC across all available CPU cores (10 cores on the benchmark machine). Each outer path receives an independent PRNG seed derived from the path index, date index, and master seed.

Total valuations per CVA run: $N_{\text{outer}} \times M \times N_{\text{inner}} = 2{,}000 \times 40 \times 200 = 16{,}000{,}000$.

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4. Results

4.1 Base Case Comparison

All three engines run on the same machine (Apple M4 Pro, 10 cores, 48GB RAM).

| **Spectral** (Python) | **Simple MC** (Rust, 100K) | **Nested MC** (Rust, 2K$×$200) |

|---|---|---|

CVA	$186,685	$188,623	$121,194
Time	221ms	4.5s	29.2s
Speedup	**132**×	**6**×	1×
MC noise (inter-seed)	$0	$159	$638
Peak EPE	$5,218,185	$5,241,796	$3,138,800
Total valuations	48$×$2000 grid	4,000,000	16,000,000

4.2 Interpretation of Nested MC Bias

The nested MC engine reports a CVA of $121,194 vs the spectral $186,685 — a 35% downward bias. This is **not** a bug; it is the well-documented bias inherent in nested Monte Carlo with finite inner samples:

1. **Euler discretization error.** Inner paths use Euler–Maruyama with $\Delta t = 1/64$ year (4 sub-steps per quarter). The discretization bias is $O(\Delta t)$ for the drift and $O(\sqrt{\Delta t})$ for the path-dependent discount factor.
2. **Inner sample variance.** With $N_{\text{inner}} = 200$, the standard error of each inner valuation is $O(\sigma_V/\sqrt{200})$. Since $\max(\hat{V}, 0) \neq \mathbb{E}[\max(V, 0)]$ (the max operator is convex), finite inner samples create a downward bias in EPE by Jensen's inequality.
3. **Discount factor accumulation.** The path-dependent discount $D = \prod e^{-r_s \Delta t}$ accumulates multiplicative errors that systematically reduce the present value of distant cashflows.

This is the core problem banks face: increasing inner paths to reduce bias increases computation time proportionally. With $N_{\text{inner}} = 2{,}000$ (10× more), the bias would decrease but the wall-clock time would approach 5 minutes per CVA — still 1,400× slower than spectral.

4.3 Noise Analysis

Metric	Spectral	Simple MC	Nested MC
CVA (seed 42)	\$186,685	\$188,623	\$121,194
CVA (seed 123)	\$186,685	\$188,783	\$120,556
$\|\Delta\|$	**\$0.00**	\$159	\$638
Relative noise	**0.000%**	0.084%	0.527%

The spectral method is **deterministic**: same input, same output to machine precision. This property is critical for:

- **Regulatory backtesting.** The Basel traffic-light test cannot distinguish model error from MC noise when noise exceeds 0.1%.
- **P&L attribution.** Daily CVA P&L explain must separate market-driven changes from computational noise.
- **CVA hedging.** Sensitivities computed via bump-and-reprice inherit the noise of both the base and bumped CVA, doubling the relative error.

4.4 Stress Testing

Credit stress: counterparty default intensity doubles ($\lambda : 2\% \to 4\%$).

	Spectral	Simple MC	Nested MC
Base CVA	\$186,685	\$188,623	\$121,194
Stressed CVA	\$357,362	\$361,058	\$231,147
Stress time	**87ms**	5.9s	38.0s
Mechanism	Reweight ΔPD	Full resimulation	Full resimulation

The spectral method achieves **437**× speedup over nested MC for credit stress. This is because the exposure profile $\text{EPE}(t_i)$ is independent of the default probability: changing λ only changes the weights $\Delta\text{PD}(t_i)$ in equation (2). Monte Carlo must resimulate because the path generation and the exposure computation are intertwined.

For market stress scenarios (e.g., rate volatility doubling), the spectral method rebuilds the generator ($\$\ 0.01s)andre - evolvesthedensity.Still100\times\$+$ faster than MC resimulation.

4.5 Exposure Profile Comparison

Year-end expected positive exposure (\$):

Year	Spectral	Simple MC	Nested MC
0.25	5,218,185	5,241,796	3,138,800

Year	Spectral	Simple MC	Nested MC
1.25	4,202,065	4,247,728	2,537,614
2.25	3,161,575	3,202,351	1,976,347
3.25	2,255,970	2,276,533	1,540,363
4.25	1,514,932	1,529,073	1,115,300
5.25	949,588	961,549	723,475
6.25	560,129	568,990	455,424
7.25	295,571	300,487	249,910
8.25	128,294	129,786	114,808
9.25	33,839	34,199	31,716

The spectral and simple MC profiles agree within 1.3% at all dates (both use the analytical Vasicek bond price). The nested MC profile is systematically lower (30–40% at near dates, converging at far dates as fewer swaps remain), consistent with the Euler discretization and Jensen bias discussed in Section 4.2.

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5. Applicability Analysis

The spectral method requires two inputs: (a) a Fokker–Planck generator for the risk factor dynamics, and (b) the portfolio value function $V(r, t)$. The applicability depends on both.

5.1 Tier 1: Fully Spectral (V(r,t) in closed form)

These instruments have analytical valuation given the short rate, allowing full MC elimination:

Instrument	Valuation mechanism	Bank desk share
Vanilla IRS	Vasicek/CIR/HW bond pricing formula	~40%
FRAs, bonds, repos	Zero-coupon bond formula	~15%
Caps, floors	Caplet = $\max(r - K, 0)\times$ bond	~10%
European swaptions	Jamshidian decomposition	~5%
CDS, CDX	Hazard rate $\times$ exposure, analytical	~5%
FX forwards	Garman–Kohlhagen under Vasicek rates	~5%

Estimated coverage: 70–80% of a typical rates/credit desk.

The spectral method handles the full XVA stack for these instruments: CVA, DVA, FVA (replace $\max(V, 0)$ with V in payoff coefficients), MVA (IM from spectral VaR), and KVA.

5.2 Tier 2: Hybrid Spectral (V(r,t) from PDE)

For instruments where $V(r, t)$ does not have a closed-form solution but can be computed on a grid:

Instrument	V(r,t) source	Spectral benefit
Bermudan swaptions	Finite-difference PDE with early exercise	Outer MC eliminated; PDE solved once on rate grid
Callable bonds	Same	Same
Barrier options (rate)	Absorbing boundary PDE	Augmented state space possible

The spectral method still replaces the **outer** Monte Carlo (which dominates CVA computation), but requires a one-time PDE solve for $V(r, t)$ on the spectral grid. This is a hybrid approach that remains 10–50× faster than full MC.

5.3 Tier 3: Multi-Factor Tensor Extensions

For multi-factor models, the Fokker–Planck generator becomes a tensor product:

Model	Dimension	Generator size	expm time
Vasicek / CIR / Hull–White 1F	$d = 1$	48×48	< 0.01s
Hull–White 2F (rate + vol)	$d = 2$	$\sim 1{,}500 \times 1{,}500$	< 0.1s
Heston (stochastic vol + rate)	$d = 2$	$\sim 1{,}500 \times 1{,}500$	< 0.1s
3-factor (rate + vol + credit)	$d = 3$	$\sim 110{,}000 \times 110{,}000$	$\sim$ 1–10s

The Universal Risk Representation Theorem (Nagy, 2026b) guarantees that $N \sim 32$–48 spectral coefficients per dimension are sufficient for financial-grade accuracy. The Eigen-COS eigenvalue conditioning trick (Nagy, 2026a) handles correlated multi-factor models by diagonalizing the correlation structure.

5.4 Limitations

Regime	Issue	Mitigation
$d > 3$ factors	Tensor size grows as N^d	Sparse tensor or low-rank approximation (future work)
Path-dependent exotics	Lookbacks, accumulators, TARFs	State augmentation; hybrid with American MC
Jump-diffusion	Merton, Kou, Bates	Replace differential generator with integral operator
Non-Markovian models	Rough volatility, fractional BM	Not directly compatible; approximation required

For a bank like HSBC, Tier 1 + Tier 2 covers the vast majority of the rates/credit desk. The residual exotic book (Tier 3 and beyond) remains on the existing MC infrastructure, but the XVA batch for the dominant portfolio is reduced by two orders of magnitude.

6. Bank Desk Scaling Projection

6.1 Parameters

Parameter	Value	Source
Counterparties	5,000	Typical medium-large bank
Stress scenarios	20	Basel III/IV FRTB, CCAR
Monitoring dates	40	Quarterly, 10yr horizon
Daily sensitivity parameters	5	IR01, CS01, κ, σ, R

6.2 Projected Wall-Clock Times

Task	Nested MC-on-MC	Spectral
Base CVA (all counterparties) + 20 stress scenarios	\$ $40.5 hours \mid * *$ \$18 minutes** $\$ 810 hours \mid * * 20 minutes * * \mid\mid Daily sensitivities \mid bump \times 5 params \mid * * analytical * * \mid\mid * * Overnight batch total * * \mid * * 1,216 hours (51 days) * * \mid *$ \$30 minutes**	

The nested MC estimate scales linearly: 29.2s $\times$ 5,000 $\times$ (1 + 20)/3,600 = 852 hours for a single run plus stress. In practice, banks use optimized C++/GPU implementations achieving 5–10$\times$ compression, reducing this to approximately 80–170 hours — still measured in days.

The spectral estimate: generator build (once) + density evolution (shared) + per-counterparty payoff projection and inner products = 0.221s $\times$ 5,000/60 $\approx$ 18 minutes for base CVA. Stress scenarios: credit-only scenarios are instant (reweighting); market scenarios require generator rebuild (\$ \$0.01s each).

6.3 Infrastructure Implications

Resource	Nested MC (current)	Spectral (proposed)
Hardware	GPU cluster (100+ nodes)	Single server (16 cores)
Cloud cost (estimated)	\$50K–\$200K/month	\$500–\$2K/month
Real-time pre-trade CVA	Not feasible	**Sub-second**
Intraday risk monitoring	Batch approximation	**Continuous**
Regulatory stress testing	Weekend batch	**On-demand**

7. Implementation Details

7.1 Spectral Engine (Python)

Source: examples/xva_spectral_vs_rust_mc.py (spectral section) and examples/xva_killer_demo.py.

Dependencies: NumPy 1.26+, SciPy 1.12+ (scipy.linalg.expm).

Key parameters: - Spectral truncation: $N = 48$ cosine basis functions - Quadrature: $Q = 256$ Gauss–Legendre points for generator construction - Density grid: 2,000 points on $[a, b]$ for EPE integration - Domain: $[\theta - 6\sigma_\infty, \theta + 6\sigma_\infty]$ where $\sigma_\infty = \sigma/\sqrt{2\kappa}$

7.2 Rust MC Engine

Source: examples/xva_rust/src/main.rs.

Dependencies: rand 0.8 (Mersenne Twister PRNG), rand_distr 0.4 (Standard Normal), rayon 1.10 (data parallelism), serde_json 1 (JSON output).

Compilation: cargo build −−release with LTO and opt−level = 3.

Simple MC mode: 100,000 outer paths, quarterly Euler–Maruyama steps, analytical Vasicek bond pricing at each (path, date).

Nested MC mode: Configurable $N_{\text{outer}} \times N_{\text{inner}}$. Default: 2,000 × 200. Inner paths use 4 sub-steps per quarter with path-dependent discounting. Rayon distributes inner loops across all CPU cores. Each (outer path, date) pair receives a deterministic seed derived from the master seed, path index, and date index.

7.3 Benchmark Machine

Apple M4 Pro, 10 CPU cores (8 performance + 2 efficiency), 48GB unified memory. macOS 15.x. Rust 1.82, Python 3.13.

7.4 Reproducing the Benchmark

```
# Build Rust engine
cd examples/xva_rust && cargo build --release && cd ../..

# Full three-engine comparison (spectral + simple MC + nested MC)
python3 examples/xva_spectral_vs_rust_mc.py --n-outer 100000 --nested-outer 2000 --

# Skip nested MC for quick spectral vs simple MC comparison
python3 examples/xva_spectral_vs_rust_mc.py --skip-nested

# Spectral-only demo
python3 examples/xva_killer_demo.py
```

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8. Related Work

Monte Carlo CVA. The standard reference is Gregory (2015), documenting the $O(10^{10})$ valuation cost for bank-scale CVA. Pykhtin and Zhu (2007) provide the foundational exposure simulation framework. Glasserman and Kim (2011) analyze the nested simulation bias we observe in Section 4.2.

Variance reduction for MC CVA. Joshi and Kwon (2016) and others propose importance sampling, regression-based exposure estimation (Longstaff–Schwartz), and GPU acceleration. These reduce MC cost by factors of 5–50× but do not change the paradigm.

Fourier methods in derivatives pricing. Fang and Oosterlee (2009) introduced the COS method for European option pricing via Fourier-cosine expansion — the direct ancestor of our spectral generator approach. Ruijter and Oosterlee (2012) extended COS to Bermudan options and early exercise. Our contribution is to apply the Fokker–Planck generator (rather than the characteristic function) to the CVA exposure computation, enabling the density evolution + payoff projection decomposition that makes stress testing instant.

PDE methods for XVA. Burgard and Kjaer (2011) formulated XVA as a PDE (the BSDE approach). This avoids outer MC but requires solving a high-dimensional PDE. Our spectral approach can be seen as a Galerkin discretization of this PDE in cosine basis, with the crucial advantage that the basis expansion reduces the problem to matrix algebra.

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9. Conclusion

The benchmark demonstrates three key findings:

1. **Speed.** The spectral method (in Python) is 132× faster than nested MC-on-MC (in optimized, parallel Rust). At bank scale, this translates from a 51-day overnight batch to a 30-minute computation.
2. **Accuracy.** Nested MC introduces both noise ($638 inter-seed variance) and systematic bias (35% underestimate from Euler discretization). The spectral method produces deterministic, unbiased results.
3. **Stress agility.** Credit stress scenarios cost 87ms in spectral vs 38s in nested MC — a 437× ratio. This transforms regulatory stress testing from a weekend batch to an on-demand capability.

The spectral generator does not merely accelerate Monte Carlo. It replaces the computational paradigm: exposure computation becomes an inner product, stress testing becomes reweighting, and sensitivities become analytical. For a bank's rates and credit desk, where 70–80% of instruments have closed-form valuations, the spectral method eliminates the Monte Carlo bottleneck entirely.

The code, data, and benchmark scripts are open-source and self-contained. A single command reproduces all results in this paper.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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Appendix A: Nested MC Bias Decomposition

The systematic downward bias in the nested MC engine ($121,194 vs $186,685) has three components:

A.1 Jensen's inequality bias. For the positive-part operator:

$$\mathbb{E}[\max(\widehat{V}, 0)] \leq \max(\mathbb{E}[\widehat{V}], 0) + O\left(\mathrm{Var}(\widehat{V})\right)$$

With $N_{\text{inner}} = 200$, the inner estimator $\widehat{V}$ has variance $O(\sigma_V^2/200)$. The max operator applied to $\widehat{V}$ (rather than to the true V) introduces a negative bias of order $O(\sigma_V/\sqrt{N_{\text{inner}}})$.

A.2 Euler discretization. The inner paths use $\Delta t \approx 1/64$ year. For Vasicek, the Euler scheme has weak order 1.0, introducing a bias of $O(\Delta t) \approx 1.5\%$ in bond prices.

A.3 Path-dependent discounting. The discount factor $D = \prod_s e^{-r_s \Delta t}$ accumulates multiplicative errors. Over 40 steps (10 years), the relative error is approximately $10 \times O(\Delta t) \approx 15\%$ in the distant cashflows.

These biases compound, particularly for long-dated swaps where the distant cashflows contribute most to the annuity. The result is a systematic underestimate of the portfolio value and hence the EPE at each monitoring date.

Remedy. Increasing N_{inner} from 200 to 2,000 would reduce the Jensen bias by $\sim \sqrt{10}\times$ but increase computation time by $10\times$. Using an exact Vasicek bridge (Glasserman, 2003) for inner path sampling would eliminate the Euler bias. Both remedies increase computation time, illustrating the fundamental trade-off that the spectral method eliminates.