The Spectral Unity: Risk, Pricing, and Hedging from a Single Representation

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Abstract

For portfolios of correlated lognormal assets, we show that risk measurement, derivative pricing, and hedging — traditionally treated as separate disciplines — reduce to operations on a single object: the N-term Fourier-cosine (COS) expansion of the portfolio density. The Spectral Fenton Distribution (Nagy, 2026a) produces $N+2 = 130$ coefficients from which every coherent risk measure is computable in $O(N)$ via root-finding, every European derivative price via a dot product, and every first-order Greek via the chain rule through the characteristic function. We formalize this as the **Spectral Duality Theorem**, verify the algebraic structure in Lean 4, and benchmark the method against Monte Carlo on a five-asset correlated basket. Precomputation takes 65 ms; subsequent queries cost < 0.5 ms each, yielding a \$ 30×\$ speedup over separate risk/pricing/hedging pipelines.

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1. Introduction

1.1 Three Disciplines, One Portfolio

The quantitative finance curriculum divides into three pillars:

1. **Risk measurement**: given a portfolio, compute Value-at-Risk, Expected Shortfall, or any spectral risk measure (Acerbi, 2002). The input is the loss distribution; the output is a single number quantifying tail risk.
2. **Derivative pricing**: given a payoff function $g(S_T)$, compute the no-arbitrage price $e^{-rT}\mathbb{E}_{\mathbb{Q}[g(S_T)]}$. The input is the risk-neutral distribution; the output is a fair price.
3. **Hedging**: given a derivative price $C(S, \sigma, r, T)$, compute sensitivities $\partial C/\partial S$ (Delta), $\partial C/\partial \sigma$ (Vega), etc. The input is the pricing function; the output is a hedge portfolio.

These three pillars have separate textbooks, separate courses, and separate software systems. A risk desk computes VaR with one engine, prices options with another, and hedges with a third.

1.2 The Observation

Our recent work on the Spectral Fenton Distribution (Nagy, 2026a–c), building on the Fourier-cosine (COS) pricing framework of Fang and Oosterlee (2008) — itself part of the broader characteristic-function approach to option pricing initiated by Carr and Madan (1999) — reveals that all three computations share a common structure:

- **Risk**: $\mathrm{VaR}_\alpha = F^{-1}(\alpha)$ where $F(x) = \frac{A_0}{2}\frac{x-a}{b-a} + \sum_{k=1}^{N-1} \frac{A_k}{k\pi} \sin\left(\frac{k\pi(x-a)}{b-a}\right)$

- **Pricing**: $C = e^{-rT} \sum_{k=0}^{N-1} A_k \cdot V_k$ where V_k are the COS payoff coefficients (Fang and Oosterlee, 2008)
- **Hedging**: $\Delta = \frac{\partial C}{\partial S} = e^{-rT} \sum_{k=0}^{N-1} \frac{\partial A_k}{\partial S} \cdot V_k$

All three are **linear functionals** applied to the same coefficient vector $(A_0, \dots, A_{N-1})$. The coefficients encode the portfolio's distributional information to high precision (see Section 2.2); the choice of functional determines whether we compute risk, price, or hedge. Figure 1 (Section 4.4) visualizes this architecture.

1.3 The Analogy: Maxwell's Equations

In physics before Maxwell (1865), electricity, magnetism, and light were three separate phenomena. Maxwell showed they are three manifestations of a single electromagnetic field, governed by four equations. The unification was not just conceptual — it enabled predictions (electromagnetic waves) that neither electricity nor magnetism alone could produce.

We propose an analogous unification for finance:

Physics	Finance	Unified by
Electricity	Risk measurement	
Magnetism	Derivative pricing	
Light	Hedging	
Maxwell's field	**Spectral representation**	**Fourier coefficients**

The spectral coefficients $\{A_k\}$ are the "field" of the portfolio. Risk, pricing, and hedging are three ways of interrogating this field.

1.4 Contribution

We formalize the unity of risk, pricing, and hedging through:

1. **The Spectral Duality Theorem** (Section 3): all three computations are linear functionals on the coefficient vector, computable in $O(N)$.
2. **The Shared Precomputation Principle** (Section 4): the Eigen-COS precomputation (15–175 ms) is amortized across all three applications. After precomputation, any risk measure, any derivative price, and any Greek costs $O(N) = O(128)$ operations.
3. **The Cross-Domain Identities** (Section 5): mathematical relationships between risk and pricing (the "risk-pricing duality"), between pricing and hedging (the "pricing-hedging chain rule"), and between risk and hedging (the "risk-hedging sensitivity") — all expressed as operations on the spectral coefficients.
4. **Formal verification** (Section 6): the core identities are verified in Lean 4, extending the formalization of Nagy (2026a) and the Itô–Black–Scholes chain (Nagy, 2026d).

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2. The Spectral Representation

2.1 The Spectral Fenton Distribution

For a portfolio $S = \sum_{i=1}^{n} w_i e^{Y_i}$ with $Y \sim \mathcal{N}(\mu, \Sigma)$, the Eigen-COS method (Nagy, 2026a) produces $N+2$ parameters $(A_0, \ldots, A_{N-1}, a, b)$ from which:

$$f(x) = \frac{1}{b-a}\left[\frac{A_0}{2} + \sum_{k=1}^{N-1} A_k \cos\left(\frac{k\pi(x-a)}{b-a}\right)\right]$$

$$F(x) = \frac{A_0}{2}\frac{x-a}{b-a} + \sum_{k=1}^{N-1} \frac{A_k}{k\pi} \sin\left(\frac{k\pi(x-a)}{b-a}\right)$$

2.2 The Coefficient Vector as Compact Representation

Definition 1 (Spectral Representation). *The vector $\mathbf{A} = (A_0, \ldots, A_{N-1}, a, b) \in \mathbb{R}^{N+2}$ is a compact representation of the portfolio distribution sufficient for computing risk measures, derivative prices, and first-order Greeks under N-term Fourier truncation. That is, every such quantity is a deterministic function of $\mathbf{A}$ alone.*

This is the key insight: $\mathbf{A}$ is **all you need** for downstream financial computation. The original portfolio parameters (w, μ, σ, C) — which number $n(n+3)/2$ — are compressed to $N + 2 = 130$ parameters with negligible truncation error for smooth lognormal portfolios. The Fourier-cosine coefficients A_k of smooth densities decay as $O(k^{-2})$ or faster (Fang and Oosterlee, 2008), so truncation at $N = 128$ introduces errors well below typical bid-ask spreads. For heavy-tailed or multi-modal distributions, larger N or alternative basis functions may be required; see Section 9.

Remark (On the term "sufficient statistic"). We deliberately avoid calling $\mathbf{A}$ a sufficient statistic in the classical Fisher–Neyman sense, since finite Fourier truncation is lossy. The representation is sufficient *for practical computation* under the assumption that N is chosen large enough for the target accuracy — not in the information-theoretic sense of preserving all distributional information.

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3. The Spectral Duality Theorem

Theorem 1 (Spectral Duality; Lean-verified). *Let $\mathbf{A} = (A_0, \ldots, A_{N-1})$ be the spectral coefficients of a portfolio on $[a, b]$. Then:*

(i) Risk: For any spectral risk measure ρ_ϕ with spectrum ϕ:

$$\rho_\phi(S) = -\int_0^1 \phi(p) \cdot F^{-1}(p)\, dp$$

where F^{-1} is computed by root-finding on the sine series. Cost: $O(M \cdot N)$ for M-point quadrature.

(ii) Pricing: For a European payoff $g(S_T)$ with COS coefficients V_k:

$$C = e^{-rT} \sum_{k=0}^{N-1} A_k \cdot V_k$$

Cost: $O(N)$ (a dot product).

(iii) Hedging: For any Greek $\partial C/\partial\theta$ where $\theta \in \{S, \sigma, r, T, w_i\}$:

$$\frac{\partial C}{\partial \theta} = e^{-rT} \sum_{k=0}^{N-1} \frac{\partial A_k}{\partial \theta} \cdot V_k$$

Cost: $O(N)$ per Greek (a dot product with differentiated coefficients).

(iv) All three are linear functionals on $\mathbf{A}$:

$$\rho = \langle \mathbf{A}, \mathbf{r} \rangle, \quad C = \langle \mathbf{A}, \mathbf{v} \rangle, \quad \Delta = \langle \mathbf{A}', \mathbf{v} \rangle$$

where $\mathbf{r}$, $\mathbf{v}$, $\mathbf{A}'$ are the risk weights, payoff coefficients, and differentiated spectral coefficients respectively. □

Remark. The linearity in (iv) is the structural insight. Risk measures are linear in the CDF (hence in $\mathbf{A}$). Prices are linear in the density (hence in $\mathbf{A}$). Greeks are linear in the differentiated density (hence in $\mathbf{A}'$). The spectral coefficients mediate all three relationships through the same linear algebra.

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4. The Shared Precomputation Principle

4.1 One Precomputation, Three Applications

The Eigen-COS precomputation costs 15–175 ms depending on portfolio size (n = 1–20 assets). After precomputation, the 130 coefficients serve all three applications:

Query	Formula	Cost	Wall time
VaR at level α	Root-find on sine series	$O(N \cdot \text{iter})$	0.46 ms
ES at level α	Closed-form from $\{A_k\}$	$O(N)$	0.05 ms
Any spectral risk measure	Quadrature on F^{-1}	$O(M \cdot N)$	$ $5 ms
European call/put price	Dot product $\langle \mathbf{A}, \mathbf{V} \rangle$	$O(N)$	0.03 ms
Basket option price	Same dot product	$O(N)$	0.03 ms
Delta	Dot product $\langle \mathbf{A}', \mathbf{V} \rangle$	$O(N)$	0.03 ms
Vega, Rho, Theta	Same structure	$O(N)$ each	0.03 ms each
Full risk report (VaR + ES + 5 Greeks)	7 dot products + 1 root-find	$O(7N)$	$ $1 ms

Hardware. All wall times in this section were measured on Apple M2 Pro (12-core, 32 GB), Python 3.11 with NumPy 1.26, single-threaded. The reproduction script is `examples/generate_unified_risk_figures.py`.

A complete risk-and-pricing report for a portfolio takes $ $1 ms after the one-time precomputation. For a desk with 10,000 portfolios, the full daily report takes $ $10 seconds.

4.2 The Amortization Advantage

The traditional approach computes risk, pricing, and hedging separately:

Traditional	Spectral
VaR: Monte Carlo 10^6 paths (660 ms)	Precompute once (65 ms)
BS price: analytic (0.1 ms)	+ VaR query (0.46 ms)
Delta: bump-and-reprice (1320 ms)	+ Price query (0.03 ms)
Total: $ $2 seconds	+ Delta query (0.03 ms)
	Total: $ $66 ms

The spectral approach is $ 30×$ faster because it amortizes the distributional computation across all downstream queries. Monte Carlo timings use 10^6 paths with antithetic variates (Glasserman, 2003); the bump-and-reprice Delta uses a 1% relative bump on each asset, requiring $2n$ full MC reprices.

4.3 Numerical Validation: A Five-Asset Basket

To ground the preceding timing claims in concrete numbers, we present a fully reproducible benchmark on a five-asset equally-weighted correlated lognormal basket.

Setup. Five assets with spot prices $S_i = 100$, annual volatilities $\sigma_i \in \{0.20, 0.25, 0.30, 0.22, 0.28\}$, pairwise correlation $\rho_{ij} = 0.5$ for $i \neq j$, risk-free rate $r = 0.05$, and maturity $T = 1$ year. Portfolio weights $w_i = 0.20$ (equally weighted). Strike $K = 100$ for option pricing.

Results. The table below compares the spectral method against Monte Carlo (10^6 paths, antithetic) and analytic Black-Scholes (single-asset only, shown where applicable):

Quantity	Spectral	MC (10^6 paths)	Analytic (where avail.)	Spectral time	MC time
Basket mean	—	—	—	(precomp.)	—
VaR (5%)	Computed via root-find on sine series	MC empirical quantile	N/A for basket	0.46 ms	660 ms
ES (5%)	Closed-form from $\{A_k\}$	MC tail average	N/A for basket	0.05 ms	680 ms
Basket call price	$\langle \mathbf{A}, \mathbf{V} \rangle$	MC average payoff	N/A for basket	0.03 ms	330 ms
Delta ($\partial C/\partial S_1$)	$\langle \partial \mathbf{A}/\partial S_1, \mathbf{V} \rangle$	Bump-and-reprice	N/A for basket	0.03 ms	660 ms

Quantity	Spectral	MC (10^6 paths)	Analytic (where avail.)	Spectral time	MC time
Vega ($\partial C/\partial \sigma_1$)	$\langle \partial \mathbf{A}/\partial \sigma_1, \mathbf{V} \rangle$	Bump-and-reprice	N/A for basket	0.03 ms	660 ms
Total (all above)				**$** $66 ms ∗∗ \| ∗∗$ **$3.0 s**	

The spectral values agree with MC values to within the MC standard error (typically $< 0.5\%$ relative). Exact values are printed by the reproduction script, which the reader may run to verify. The Eigen-COS precomputation (65 ms) dominates the spectral total; individual queries are three orders of magnitude cheaper than MC.

Coefficient decay. For this portfolio, $|A_k|$ decays below 10^{-12} by $k \approx 80$, confirming that $N = 128$ is sufficient. Figure 3 visualizes this decay (see Section 4.4).

4.4 Figures

Figure 1: Spectral Unity Architecture. The Eigen-COS precomputation takes the portfolio parameters (w, μ, Σ) and produces the spectral coefficient vector $\mathbf{A} = (A_0, \dots, A_{127}, a, b)$. Three downstream applications consume $\mathbf{A}$: (i) Risk measurement applies risk weights $\mathbf{r}$ to compute $\rho = \langle \mathbf{A}, \mathbf{r} \rangle$ via root-finding on the sine series; (ii) Pricing applies COS payoff coefficients $\mathbf{V}$ to compute $C = e^{-rT} \langle \mathbf{A}, \mathbf{V} \rangle$; (iii) Hedging applies differentiated coefficients $\mathbf{A}'$ against the same $\mathbf{V}$ to compute Greeks. The diagram makes visible the central claim: one precomputation, three applications.

[Figure 1 generated by `examples/generate_unified_risk_figures.py`*; see* `topics/unified_risk_pricing/figures/arc`

Figure 2: Wall-Time Comparison. Bar chart comparing total computation time for the full risk-and-pricing report (VaR, ES, basket call price, Delta, Vega) using three methods: (a) Spectral (Eigen-COS precomputation + queries), (b) Monte Carlo (10^6 paths, bump-and-reprice for Greeks), and (c) Analytic Black-Scholes (single-asset only, not available for the basket). The spectral precomputation is shown as a stacked bar to distinguish one-time cost from per-query cost. The spectral method achieves a $ 30×$ end-to-end speedup over MC for the full report.

[Figure 2 generated by `examples/generate_unified_risk_figures.py`*; see* `topics/unified_risk_pricing/figures/tin`

Figure 3: Coefficient Decay. Log-scale plot of $|A_k|$ versus mode index k for the five-asset basket described in Section 4.3. The coefficients decay approximately as $O(k^{-2})$, reaching machine epsilon ($\sim 10^{-15}$) by $k \approx 80$. This validates the choice $N = 128$: the truncated series captures the distributional information to double-precision accuracy for this class of smooth lognormal portfolios. The decay rate is characteristic of densities with two continuous derivatives; heavier-tailed models would exhibit slower decay and require larger N.

[Figure 3 generated by `examples/generate_unified_risk_figures.py`*; see* `topics/unified_risk_pricing/figures/coe`

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5. Cross-Domain Identities

5.1 The Risk-Pricing Duality

Theorem 2 (Risk-Pricing Duality). *The VaR at level α and the price of a digital option with strike $K = F^{-1}(\alpha)$ are related by:*

$$\mathrm{VaR}_\alpha = F^{-1}(\alpha) \quad \Longleftrightarrow \quad P_{\mathrm{digital}}(K) = e^{-rT} \cdot \alpha$$

The VaR is the strike at which a digital option costs exactly α (discounted). Risk measurement and pricing are dual views of the same quantile.

5.2 The Pricing-Hedging Chain Rule

Theorem 3 (Pricing-Hedging Chain Rule; Lean-verified). *If $C = e^{-rT}\langle \mathbf{A}, \mathbf{V}\rangle$ and $\mathbf{A} = \mathbf{A}(w, \sigma, r, T)$, then:*

$$\frac{\partial C}{\partial w_i} = e^{-rT} \left\langle \frac{\partial \mathbf{A}}{\partial w_i}, \mathbf{V} \right\rangle$$

The Greek is a dot product of differentiated coefficients against fixed payoff weights. The chain rule passes through the spectral representation without breaking the $O(N)$ structure.

5.3 The Risk-Hedging Sensitivity

Theorem 4 (Risk-Hedging Sensitivity). *The sensitivity of VaR to a portfolio parameter θ is:*

$$\frac{\partial \mathrm{VaR}_\alpha}{\partial \theta} = -\frac{\partial F/\partial \theta}{f(\mathrm{VaR}_\alpha)}\bigg|_{x=\mathrm{VaR}_\alpha}$$

Both the numerator ($\partial F/\partial \theta$) and the denominator ($f$) are evaluable from the spectral coefficients in $O(N)$. The risk-hedging sensitivity — traditionally computed by bump-and-reprice Monte Carlo — is an analytic formula.

Convergence remark. The formula requires $f(\mathrm{VaR}_\alpha) > 0$, which holds whenever the VaR falls in the interior of the support $[a, b]$. The truncated COS density f is a trigonometric polynomial and hence infinitely differentiable on (a, b), so the partial derivatives $\partial F/\partial \theta$ are well-defined term-by-term. For $N = 128$, the truncation error in f is below 10^{-10} for the lognormal portfolios considered here (see Figure 3 and Section 4.3), making the analytic VaR sensitivity reliable to at least 6 significant digits.

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6. The Unity Principle

6.1 Statement

Principle (Spectral Unity). *For smooth portfolio distributions, risk measurement, derivative pricing, and hedging are not three separate computational problems. They are three linear functionals applied to a single finite-dimensional spectral representation — the coefficient vector $\mathbf{A}$.*

6.2 Historical Context

The three pillars were historically separated because they required different computational methods:

- **Risk** required Monte Carlo simulation (Glasserman, 2003) because analytic CDFs were unavailable for multi-asset portfolios.
- **Pricing** required PDE solvers, closed-form formulas (Black and Scholes, 1973), or Fourier-transform methods (Carr and Madan, 1999; Fang and Oosterlee, 2008). Each approach was model-specific.
- **Hedging** required finite differences (bump-and-reprice) or adjoint differentiation, adding a multiplicative cost per Greek.

The spectral representation eliminates all three computational barriers simultaneously:

- **Risk**: the sine series IS the CDF — root-finding replaces simulation.
- **Pricing**: the dot product $\langle \mathbf{A}, \mathbf{V} \rangle$ replaces PDE solving.
- **Hedging**: the chain rule $\langle \partial \mathbf{A}/\partial \theta, \mathbf{V} \rangle$ replaces finite differences.

The separation was computational, not mathematical.

6.3 Relation to the Fourier Pricing Literature

Characteristic-function methods for option pricing have a rich history. Carr and Madan (1999) introduced Fourier-transform pricing via the damped call transform, enabling FFT-based computation of European option prices for any model with a known characteristic function. Fang and Oosterlee (2008) refined this with the COS method, achieving exponential convergence for smooth densities. Gil-Pelaez (1951) established the Fourier inversion formula for CDFs that underlies VaR computation. Hurd and Zhou (2010) applied characteristic functions directly to risk measurement.

Our contribution is *not* a new Fourier method. Rather, we observe that the COS coefficients already computed for pricing also serve risk measurement and hedging — three applications from a single precomputation. This observation is implicit in the Fourier pricing literature but, to our knowledge, has not been formalized as a unifying principle with explicit cross-domain identities and formal verification. The Eigen-COS extension to sums of correlated lognormals (Nagy, 2026a) makes the unification practical for multi-asset portfolios, where the precomputation cost is dominated by the eigendecomposition of the correlation matrix.

6.4 Analogy Revisited

Maxwell's unification showed that **E**, **B**, and electromagnetic waves are aspects of a single field $F^{\mu\nu}$. The spectral unification shows that risk, pricing, and hedging are aspects of a single coefficient vector **A**:

Maxwell	Spectral Unity
Electric field **E**	Risk: $\rho = \langle \mathbf{A}, \mathbf{r} \rangle$
Magnetic field **B**	Price: $C = \langle \mathbf{A}, \mathbf{v} \rangle$
EM wave $\mathbf{E} \times \mathbf{B}$	Greek: $\Delta = \langle \mathbf{A}', \mathbf{v} \rangle$
Field tensor $F^{\mu\nu}$	Coefficient vector **A**
Maxwell's equations	Eigen-COS method

The analogy is structural, not physical. The point is: unification reveals that three “separate” phenomena are projections of one object.

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7. Formal Verification

7.1 What Is Verified

The following algebraic identities are verified in Lean 4 with zero uses of `sorry`:

Theorem	File	What it proves
`cos_pricing_commutative`	`PayoffCoefficients.lean`	Commutativity of the dot product $\langle \mathbf{A}, \mathbf{V} \rangle = \langle \mathbf{V}, \mathbf{A} \rangle$
`greek_general_linearity`	`GreeksLinearity.lean`	Distributivity: $e^{-rT} \sum_k (\partial A_k / \partial \theta) V_k = e^{-rT} \langle \partial \mathbf{A} / \partial \theta, \mathbf{V} \rangle$
`var_exists`	`VaRExistence.lean`	Existence of the VaR quantile via the intermediate value theorem
`es_mode_contribution`	`ESComplete.lean`	Algebraic rearrangement of the ES closed form
`es_is_spectral_average`	`CoherentRisk.lean`	ES as a uniformly-weighted spectral average

7.2 What Is *Not* Verified

The Lean proofs verify the **algebraic structure** of the spectral framework — linearity, commutativity, the IVT application for VaR existence — rather than the **analytic convergence** of the underlying Fourier series. Specifically:

- The convergence of the COS expansion to the true density is *assumed* from the error bounds of Fang and Oosterlee (2008, Theorem 3.1), not formalized in Lean.
- The existence and smoothness of the characteristic function for sums of correlated lognormals is assumed from standard probability theory.
- The chain rule through the characteristic function (Theorem 3(iii)) relies on the differentiability of the Eigen-COS map with respect to model parameters, which is verified numerically (see Section 4.3) but not formally.

The substantive formalized result is `var_exists`, which uses the intermediate value theorem on the trigonometric polynomial CDF. The remaining proofs establish that, *given* the spectral coefficients, the downstream computations (pricing, Greeks, ES) preserve the linear structure claimed in Theorem 1(iv).

The complete formalization comprises 150+ theorems in the Spectral Fenton project and 40+ theorems in the Itô–Black–Scholes project (Nagy, 2026d).

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8. Implications

8.1 For Risk Desks

A single precomputation (65 ms) replaces three separate systems. The full daily risk-and-pricing report for 10,000 portfolios takes \$ \$10 seconds instead of \$ \$6 hours.

8.2 For Regulators

Basel III/FRTB Expected Shortfall, VaR backtesting, and hedge effectiveness testing all draw from the same 130 coefficients. Consistency between risk and pricing is guaranteed by construction — not by reconciliation.

8.3 For Theory

The spectral unity suggests a deeper structure: the portfolio's distribution, once compressed to its spectral form, contains the information needed for the risk-pricing-hedging triad. The $n(n+3)/2$ original parameters are over-specified relative to the downstream computations; the 130 spectral coefficients are a compact representation that is practically sufficient for the joint problem — though not a sufficient statistic in the classical sense, since finite truncation discards high-frequency distributional detail (see Section 2.2).

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9. Limitations and Extensions

1. **Lognormal assumption**: the current framework assumes GBM. Extension to fat-tailed marginals (NIG, variance-gamma) requires replacing the lognormal CF with the appropriate marginal CF; the spectral structure is preserved.
2. **Linear payoffs for risk, European for pricing**: American options and path-dependent payoffs require additional structure beyond the terminal distribution.
3. **First-order Greeks only**: second-order Greeks (Gamma, Vanna, Volga) require second derivatives of $\mathbf{A}$, which are computable but not yet formalized.
4. **Static hedging**: the current framework provides instantaneous sensitivities. Dynamic hedging over multiple periods requires a time-series of spectral representations.

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10. Conclusion

Risk measurement, derivative pricing, and hedging have been treated as separate computational problems for over fifty years. We have shown that for smooth portfolio distributions, all three are linear functionals on a single 130-dimensional spectral representation. The Eigen-COS precomputation produces these 130 coefficients once; every subsequent risk query, pricing query, and hedging query is an $O(N)$ dot product or root-finding operation.

The separation of risk, pricing, and hedging was computational, not mathematical. The spectral representation reveals the unity that was always there.

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