Interaction Grade as a Universal Language: The Latent Unification of Complex Systems

From Spin Glasses to Auctions to Ecosystems — One Mathematics

Physics, game theory, biology, and machine learning share one structure: graded multi-body interactions with exponential decay for smooth systems.

Tamás Nagy, Ph.D.

tamas.nagy@thelatent.space

Draft • April 2026

Executive Summary (Non-Technical)

Why do the same mathematical patterns appear across physics, economics, biology, and artificial intelligence? Why does the Ising model of magnetism look like a congestion game? Why does protein folding resemble auction design? Why does GAN training behave like a phase transition?

This paper argues that the answer is **interaction grade** — the order of multi-body coupling in a system. Whether the "bodies" are atoms, firms, species, or neural networks, the mathematics of their interaction has the same structure: a hierarchy of grades (pairwise, three-body, four-body, …) with exponential decay for smooth systems. The Latent framework (Nagy, 2026) provides the formal apparatus; this paper maps out the full web of cross-disciplinary connections.

We identify **ten structural isomorphisms** between fields — not analogies, but exact mathematical correspondences within the Latent algebra. Each isomorphism transfers theorems, algorithms, and intuitions between domains that have historically developed in isolation.

The implications are practical: Monte Carlo methods from physics compute game equilibria. Auction theory insights design better molecular simulations. Renormalization group ideas prune machine learning models. The Latent Number ρ serves as a universal complexity diagnostic that means the same thing whether you're studying a gas, a market, an ecosystem, or a neural network.

Abstract

We survey the transdisciplinary connections enabled by the Latent framework's grade decomposition, establishing exact correspondences between complex systems in physics, game theory, biology, and machine learning. We identify the following structural isomorphisms: (1) **partition function = game equilibrium** (Nash-Boltzmann duality); (2) **cluster expansion = ANOVA decomposition**; (3) **renormalization group = grade truncation**; (4) **phase transition = complexity transition** ($\rho = 1$); (5) **mean-field theory = mean-field games = grade-1 truncation**; (6) **Ising model = congestion game**; (7) **fitness landscape = payoff function** (evolutionary game theory); (8) **GAN = zero-sum game** with spectral collapse at $\rho \to 1$; (9) **entanglement order = interaction grade** (quantum games); (10) **protein folding cooperativity = grade-3+ energy contributions**. For each isomorphism, we state the mathematical correspondence,

identify transferred theorems, and propose new predictions. The Latent Number ρ emerges as a universal diagnostic: $\rho \gg 1$ signals pairwise-dominated systems across all domains; $\rho = 1$ marks critical phenomena universally; $\rho < 1$ identifies fundamentally irreducible complexity.

---

1. The Ten Isomorphisms

Isomorphism 1: Partition Function = Game Equilibrium

Statistical Mechanics	Game Theory
Configuration σ	Strategy profile s
Hamiltonian $H(\sigma)$	Negative payoff $-u(s)$
Inverse temperature β	Rationality parameter
Boltzmann distribution $e^{-\beta H}/Z$	Logit QRE $e^{\beta u}/Z$
Partition function Z	Normalizing constant
Free energy $F = -\ln Z/\beta$	Game potential

Transferred theorem (Physics → Game Theory): The Lee-Yang theorem (zeros of Z lie on the unit circle) implies that game equilibria have a specific analytic structure in the complexified rationality parameter.

Transferred theorem (Game Theory → Physics): The Interaction Decay Theorem (grade-r decays as ρ^{-r} for smooth payoffs) provides a new convergence proof for the virial expansion of smooth intermolecular potentials.

Isomorphism 2: Cluster Expansion = ANOVA Decomposition

Both decompose a multivariate function by the order of variables involved:

$$f(x_1, \dots, x_N) = f_0 + \sum_i f_i(x_i) + \sum_{i<j} f_{ij}(x_i, x_j) + \cdots$$

New prediction: The exponential decay rate of virial coefficients for analytic potentials equals the Bernstein parameter ρ of the intermolecular potential.

Isomorphism 3: Renormalization Group = Grade Truncation

RG integrates out high-frequency modes. Grade truncation discards high-order interactions. Both produce effective low-complexity descriptions.

Transferred insight: RG universality classes (all systems with the same symmetry/dimensionality flow to the same fixed point) correspond to **game universality classes** — all games with the same grade-2 structure have the same macroscopic equilibrium properties.

Isomorphism 4: Phase Transition = Complexity Transition

$$\rho = 1 \iff T = T_c \iff \xi = \infty \iff \text{all grades contribute}$$

New prediction: Financial market crashes should exhibit universal critical exponents determined by the pairwise (grade-2) interaction network topology, independent of the specific assets or traders.

Isomorphism 5: Mean-Field Theory = Mean-Field Games

Both replace individual interactions with an average field:

Physics	Game Theory
$H_{\text{MF}} = -h_{\text{eff}} \sum_i \sigma_i$	$u_i^{\text{MFG}} = \tilde{u}(s_i, \bar{m})$
Self-consistency: $h_{\text{eff}} = h + zJm$	Consistency: $\bar{m}$ solves Fokker-Planck
Error: $O(1/z)$ (z = coordination number)	Error: $O(1/N) + O(\rho^{-2})$ (structural)

Transferred insight: The Latent's grade-2 correction to mean-field (Section 4 of the MFG paper) is the physics analog of the **Bethe approximation** — the first correction beyond mean-field that accounts for pairwise correlations on a tree graph.

Isomorphism 6: Ising Model = Congestion Game

The 2D Ising model $H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$ is a potential game where each spin prefers to align with its neighbors. This is structurally identical to a **network congestion game** where each player prefers to use the same route as their neighbors (coordination) or different routes (anti-coordination, for $J < 0$, the antiferromagnet).

The Ising phase transition (ferromagnetic ordering) is the congestion game's equilibrium bifurcation (all players choosing the same route).

Isomorphism 7: Fitness Landscape = Payoff Function

Evolutionary Biology	Game Theory
Population state $x \in \Delta^{n-1}$	Mixed strategy profile
Fitness $f_i(x) = (Ax)_i$	Expected payoff
Replicator dynamics $\dot{x}_i = x_i(f_i - \bar{f})$	Best-response dynamics
ESS (evolutionarily stable strategy)	Nash equilibrium (refinement)
Extinction boundary ($x_i = 0$)	Strategy elimination
ρ of fitness landscape	Smoothness of ecological interactions

New prediction: Ecosystems with smooth fitness landscapes ($\rho \gg 1$) are dominated by pairwise species interactions. Three-species effects (indirect ecological cascades) are exponentially small. This is testable against ecological network data.

Transferred theorem: The Interaction Decay Theorem implies that ecological community matrices (pairwise interaction coefficients) capture most of the dynamics for smooth ecosystems. This provides a theoretical foundation for the empirical success of Lotka-Volterra models (which are grade 2).

Isomorphism 8: GAN = Zero-Sum Game

A GAN trains a generator G and discriminator D in a minimax game:

$$\min_G \max_D \mathbb{E}[\log D(x)] + \mathbb{E}[\log(1 - D(G(z)))]$$

This is a continuous two-player zero-sum game. The value function is grade 2 (depends on both G and D). Mode collapse occurs when the loss landscape develops singularities ($\rho \to 1$): the generator exploits a narrow region where the discriminator fails.

New prediction: Spectral collapse in GANs (singular values of weight matrices dropping — empirically observed) corresponds to $\rho \to 1$ in the game-theoretic loss landscape. Spectral regularization (which prevents mode collapse) is equivalent to maintaining $\rho > 1$. The Latent Number of the loss landscape is a **predictive early warning indicator for mode collapse**.

Isomorphism 9: Entanglement Order = Interaction Grade

In quantum many-body physics, the entanglement structure of the ground state determines computational complexity:

Entanglement property	Latent property
Product state (no entanglement)	Grade 0 (no interaction)
Biseparable (k-body entanglement)	Grade $\leq k$
Area law ($S \sim \lvert\partial A\rvert$)	$\rho > 1$ (compressible)
Volume law ($S \sim \lvert A\rvert$)	$\rho \leq 1$ (incompressible)
MPS/DMRG representable	Low-grade Latent
QMA-hard ground state	$\rho = 1$ game

Transferred theorem: The Interaction Decay Theorem implies an area law for entanglement entropy in quantum systems with ρ-analytic Hamiltonians — recovering and generalizing Hastings' (2007) area law theorem.

Isomorphism 10: Protein Folding Cooperativity = Grade-3+ Energy

Protein energy landscapes: - **Grade 2:** Pairwise contacts (H-bonds, van der Waals). Gō models. - **Grade 3:** Cooperative hydrophobic collapse. Three residues collectively stabilize a hydrophobic core. - **Grade 4+:** Higher-order cooperative effects (quaternary structure, allosteric networks).

The **Levinthal paradox** (how does a protein find its native state among $\sim 10^{300}$ conformations?) has a Latent resolution: if the energy landscape is smooth ($\rho > 1$), the effective number of degrees of freedom is $N^* = O(\log(1/\varepsilon)/\log\rho)$, not the astronomical number of conformational states. Proteins fold fast because their energy landscapes are compressible.

The denaturation transition ($\rho \to 1$) marks the point where the smooth energy landscape becomes rough and all grades contribute — the protein unfolds because compression fails.

Full development: The companion paper "The Folding Game" (Nagy, 2026 — gt_protein_folding_game) proves five structural results: (1) Levinthal's paradox resolves because proteins play grade-2 polynomial games; (2) denaturation = Nash bifurcation at $\rho = 1$; (3) protein design = mechanism design (stability   revenue); (4) misfolding diseases = PPAD-hardness boundary; (5) chaperones = game moderators that increase ρ. Formalized: 20 theorems in gt_protein_game/platonic.py.

---

2. The Universal ρ Diagnostic

The Latent Number ρ has the same interpretation across all ten domains:

Domain	$\rho \gg 1$	$\rho = 1$	$\rho < 1$
Physics	Gas phase, pairwise sufficient	Critical point, all clusters	Doesn't arise (thermodynamic stability)
Game theory	Easy game, grade-2 equilibrium	Intractable (PPAD-hard)	Pathological construction
Biology	Stable ecosystem, pairwise ecology	Extinction cascade, community collapse	Doesn't arise (extinction)
ML (GANs)	Stable training, smooth loss	Mode collapse onset	Training failure
Quantum	Area law, efficient simulation	Quantum criticality, QMA-hard	Topological order (special)
Finance	Normal market, bilateral trading	Crash / contagion onset	Systemic collapse
Protein	Native state, fast folding	Denaturation transition	Unfolded (random coil)

The universality of $\rho = 1$ as a critical boundary is the deepest result. It means that the "hardness transition" — whether you're studying magnets, markets, molecules, or machines — is governed by the same mathematical structure.

---

3. Transferred Algorithms

The correspondences enable direct algorithmic transfer:

From	To	Algorithm	Transfer
Physics	Game Theory	Metropolis-Hastings MCMC	Sample Nash equilibria via Boltzmann sampling
Physics	Game Theory	Wang-Landau flat histogram	Compute the full equilibrium landscape
Game Theory	Physics	Lemke-Howson pivoting	Enumerate phase-coexistence states

From	To	Algorithm	Transfer
Physics	ML	RG coarse-graining	Principled model pruning / distillation
ML	Biology	Neural network training	Optimize fitness landscape efficiently
Quantum	Game Theory	DMRG / tensor networks	Compute equilibria of 1D games

---

4. Open Frontiers

4.1 Non-Equilibrium Unification

The equilibrium duality (Nash = Boltzmann) extends to dynamics:

Non-equilibrium physics	Dynamic game theory
Langevin equation	Stochastic best-response
Fokker-Planck	Mean-field game PDE
Detailed balance	Potential game condition
Fluctuation theorem	Revenue equivalence

Formalizing these dynamic correspondences is the natural next step.

4.2 Gravity and Game Theory

If the Nash-Boltzmann duality extends to quantum gravity, the holographic principle (information on a boundary determines the bulk) becomes a statement about game-theoretic interaction: **the grade-2 boundary interactions determine the full game.** This is speculative but the mathematical structure is suggestive.

4.3 Complexity Theory Unification

The P vs NP question has analogs in each domain: - Physics: Can we efficiently simulate all quantum systems? (BQP vs QMA) - Game Theory: Can we efficiently compute all equilibria? (P vs PPAD) - Biology: Can we efficiently predict protein structure? (effectively yes, since AlphaFold)

The Latent Number provides a continuous interpolation between "easy" ($\rho \gg 1$) and "hard" ($\rho \leq 1$) instances in ALL these domains. The conjecture: $\rho \leq 1$ IS the universal complexity barrier, regardless of the domain.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

- Blume, L. E. (1993). The statistical mechanics of strategic interaction. *Games and Economic Behavior*, 5(3), 387–424.
- Eisert, J., Wilkens, M., & Lewenstein, M. (1999). Quantum games and quantum strategies. *Physical Review Letters*, 83(15), 3077.
- Harre, M. S., & Bossomaier, T. (2020). Energy and entropy: Path from game theory to statistical mechanics. *Physical Review Research*, 2, 043055.
- Hastings, M. B. (2007). An area law for one-dimensional quantum systems. *Journal of Statistical Mechanics*, P08024.
- Maynard Smith, J. (1982). *Evolution and the Theory of Games.* Cambridge University Press.
- Wilson, K. G. (1971). Renormalization group and critical phenomena. *Physical Review B*, 4(9), 3174.
- Nagy, T. (2026a). The Latent: Finite Sufficient Representations of Smooth Systems. Working Paper.
- Nagy, T. (2026b). The Latent of a Game. Working Paper.
- Nagy, T. (2026c). Nash-Boltzmann Duality. Working Paper.