The Langlands Transfer Graph: Spectral Edges, Decorrelation Gains, and the Algebraic–Additive Frontier

A Quantitative Reframing of the Langlands Program via the Proof Category

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draft • 2026-04-09

Abstract

We reframe the Langlands program as a systematic edge-construction effort within the proof category **Spec** introduced in [Nagy, 2026a]. Each proved Langlands-type result — from Artin reciprocity to the modularity theorem — corresponds to a morphism between spectral domains carrying a computable decorrelation gain $\delta \in [0, 1]$. We compute δ explicitly for the major known correspondences, finding $\delta \approx 1.0$ uniformly for problems with algebraic correlative structure. We then identify a structural gap: no known or conjectured Langlands transfer provides $\delta > 0$ for *additive* correlative problems (twin primes, binary Goldbach, bounded gaps). This algebraic–additive gap is the quantitative reason why the Langlands program — despite being the most powerful source of new spectral edges in mathematics — has not yet contributed to additive number theory. We characterize what an "additive Langlands transfer" would require, connect the analysis to specific open proof circuits, and propose a research program for identifying the missing morphisms.

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1. Introduction

The Langlands program is the most ambitious systematic effort in modern mathematics to unify number theory and representation theory. Initiated by Langlands's 1967 letter to Weil, it posits a web of correspondences between Galois representations and automorphic forms — correspondences that, when proved, have repeatedly resolved major open problems. The modularity theorem (Wiles, 1995; Breuil–Conrad–Diamond–Taylor, 2001) is the most celebrated example: by establishing that every rational elliptic curve is modular, it immediately implied Fermat's Last Theorem.

Yet the Langlands program, for all its depth, has not touched the classical additive problems of number theory in the way it has reshaped algebraic and multiplicative questions. The twin prime conjecture and the binary Goldbach conjecture remain open. Even after Zhang, Maynard, Tao, and collaborators proved that prime gaps are *bounded* infinitely often, the existence of infinitely many pairs $p, p + 2$ of primes — and the full strength of "bounded gaps" targets such as $H = 2$ — remains out of reach for Langlands-type methods. This is not an accident. We argue that it reflects a structural property of Langlands-type correspondences: they decorrelate *algebraic* constraints with maximal efficiency, but provide zero decorrelation for *additive* constraints.

This claim is made precise using the proof category **Spec** introduced in [Nagy, 2026a]. The key definitions:

- A **spectral domain** $\mathcal{D} = (V, L, \sigma)$ is a vector space V with a spectral operator L and a polarity function σ classifying each spectral component as convolutive (C) or correlative (R).
- A **spectral transfer** $\Phi : \mathcal{D}_1 \to \mathcal{D}_2$ is a morphism that may reclassify components from R to C.
- The **decorrelation gain** $\delta(\Phi) \in [0, 1]$ measures the fraction of correlative spectral energy converted to convolutive.

A problem is solvable when a proof circuit — a loop in **Spec** — achieves total $\delta \geq \delta^*$, the problem's threshold. Harder problems require higher δ^*.

Every proved Langlands correspondence is a morphism in **Spec**. This paper computes δ for each, explains why the gains are uniformly high for algebraic problems, identifies why they vanish for additive problems, and asks what new morphisms would bridge the gap.

1.1 Contributions

1. **Quantitative reframing.** Each Langlands result is assigned a decorrelation gain δ, making the program's power — and its limitations — numerically precise.
2. **The algebraic–additive gap.** Within the **Spec** framework, we formalize (§4.3) why Euler-product–compatible Langlands transfers assign zero decorrelation gain to the additive correlative regimes discussed here, and we explain why this is structurally tied to multiplicative L-function technology.
3. **Requirements for additive transfers.** We characterize what properties a hypothetical "additive Langlands transfer" would need to satisfy.
4. **Proof circuit analysis.** We identify which open problems would become solvable if specific new edges were added to **Spec**.

1.2 Relation to Prior Work

The Langlands program has been surveyed extensively from algebraic, analytic, and geometric perspectives (Gelbart 1984; Bernstein–Gelbart 2003; Frenkel 2007). The novelty here is the *quantitative* perspective: we do not ask "do these objects correspond?" but "how much decorrelation does the correspondence provide, and for which problem types?" The proof category **Spec** was introduced in [Nagy, 2026a] to classify mathematical problems by their spectral structure; this paper is the first application to a specific research program.

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2. The Proof Category Spec

We recall the essential definitions from [Nagy, 2026a] and add the structure needed for Langlands transfers.

2.1 Spectral Domains and Transfers

A spectral domain $\mathcal{D} = (V, L, \sigma)$ consists of a Hilbert space V, a self-adjoint operator L with spectral decomposition $V = \bigoplus_\lambda V_\lambda$, and a polarity function $\sigma : \mathrm{Spec}(L) \to \{C, R\}$ classifying each eigenspace as convolutive or correlative. The convolutive spectral energy is $E_C = \sum_{\sigma(\lambda)=C} \|v_\lambda\|^2$ and the correlative spectral energy $E_R = \sum_{\sigma(\lambda)=R} \|v_\lambda\|^2$.

A spectral transfer $\Phi : \mathcal{D}_1 \to \mathcal{D}_2$ maps (V_1, L_1, σ_1) to (V_2, L_2, σ_2) in a way that may reclassify spectral components. Its decorrelation gain is

$$\delta(\Phi) = \frac{E_R^{(1)} - E_R^{(2)}}{E_R^{(1)}} \in [0, 1],$$

the fraction of correlative energy that becomes convolutive. The composition law is

$$\delta(\Phi_2 \circ \Phi_1) \geq 1 - (1 - \delta(\Phi_1))(1 - \delta(\Phi_2)),$$

ensuring that sequential transfers compound their gains.

2.2 The Arithmetic Domain

For number-theoretic problems, the relevant spectral domain is $\mathcal{D}_{\mathrm{NT}} = (\ell^2(\mathbb{N}), L_{\mathrm{arith}}, \sigma_{\mathrm{arith}})$ where:

- $V = \ell^2(\mathbb{N})$, the space of arithmetic functions.
- L_{arith} has spectral components indexed by characters χ (Dirichlet characters, Hecke characters, etc.).
- The polarity σ depends on the problem: for additive problems ($p + q = n$), the locked components have $\sigma = R$; for multiplicative problems (distribution of primes in arithmetic progressions), the Euler product structure provides $\sigma = C$ for most components.

2.3 The Automorphic Domain

The automorphic domain $\mathcal{D}_{\mathrm{Aut}} = (L^2(\Gamma \backslash G), L_{\mathrm{Casimir}}, \sigma_{\mathrm{aut}})$ where:

- $V = L^2(\Gamma \backslash G)$ for a reductive group G and arithmetic subgroup Γ.
- L_{Casimir} is the Casimir operator, with spectral decomposition into automorphic representations.
- σ classifies representations: those captured by the Selberg eigenvalue bound are C; exceptional eigenvalues are R.

The Langlands program, in these terms, constructs morphisms $\Phi : \mathcal{D}_{\mathrm{NT}} \to \mathcal{D}_{\mathrm{Aut}}$ and characterizes their decorrelation properties.

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3. Known Langlands Edges and Their Decorrelation Gains

We analyze the four major classes of proved Langlands correspondences, computing δ for each.

3.1 Class Field Theory: Abelian Extensions   Hecke Characters ($\delta = 1.0$)

The correspondence. Artin reciprocity (completed by Tate's thesis, 1950) establishes a bijection between one-dimensional representations of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ — equivalently, characters of abelian Galois extensions — and Hecke characters (Grössencharaktere). The L-function is the invariant: $L(s, \chi_{\mathrm{Gal}}) = L(s, \chi_{\mathrm{Hecke}})$.

The transfer. $\Phi_{\mathrm{CFT}} : \mathcal{D}_{\mathrm{NT}}^{\mathrm{ab}} \to \mathcal{D}_{\mathrm{Aut}}^{\mathrm{GL}(1)}$.

Decorrelation gain. $\delta = 1.0$ for multiplicative problems. The correspondence is an *exact* bijection — every spectral component on the Galois side has a unique partner on the automorphic side, with identical L-function. No information is lost, and the full correlative structure of abelian extensions becomes convolutive in the world of Hecke L-functions, where analytic continuation, functional equations, and the Euler product are available.

What it solves. Dirichlet's theorem on primes in arithmetic progressions, the Chebotarev density theorem, the class number formula — all are consequences of having exact analytic control over abelian L-functions.

What it does not solve. Any problem involving non-abelian Galois groups or additive constraints. The transfer acts only on the abelian fragment $\mathcal{D}_{\mathrm{NT}}^{\mathrm{ab}} \subset \mathcal{D}_{\mathrm{NT}}$.

3.2 Langlands for GL(1): One-Dimensional Representations ($\delta = 1.0$)

This is class field theory viewed through the modern lens. The correspondence $\mathrm{Gal}^{\mathrm{ab}} \leftrightarrow \mathrm{GL}(1, \mathbb{A})$ is the base case of the Langlands program. The decorrelation gain is $\delta = 1.0$ for the same reason: exact bijection, identical L-functions.

The GL(1) case establishes the *template* that all higher Langlands correspondences aim to generalize: for each n-dimensional Galois representation, there should be a unique automorphic representation of $\mathrm{GL}(n)$ with matching L-function.

3.3 The Modularity Theorem: Elliptic Curves Weight-2 Modular Forms ($\delta = 1.0$)

The correspondence. The Shimura–Taniyama–Weil conjecture (proved by Wiles 1995 for semistable curves, completed by Breuil–Conrad–Diamond–Taylor 2001 for all rational elliptic curves) states: for every elliptic curve $E/\mathbb{Q}$ of conductor N, there exists a weight-2 newform $f \in S_2(\Gamma_0(N))$ such that $L(E, s) = L(f, s)$.

The transfer. $\Phi_{\mathrm{mod}} : \mathcal{D}_{\mathrm{EllCurves}/\mathbb{Q}} \to \mathcal{D}_{\mathrm{ModForms}}^{S_2}$.

Decorrelation gain. $\delta = 1.0$ for *algebraic* constraints on elliptic curves. The key property: modular forms inhabit a space with deep analytic structure — q-expansions, Hecke operators, Petersson inner products — that is entirely absent on the elliptic curve side. Algebraic constraints that are opaque in $\mathcal{D}_{\mathrm{EllCurves}}$ become transparently analytic in $\mathcal{D}_{\mathrm{ModForms}}$.

The Fermat circuit. Fermat's Last Theorem follows from a two-step proof circuit:

$$\mathcal{D}_{\mathrm{Diophantine}} \xrightarrow{\text{Frey}} \mathcal{D}_{\mathrm{EllCurves}} \xrightarrow{\Phi_{\mathrm{mod}}} \mathcal{D}_{\mathrm{ModForms}} \xrightarrow{\text{Ribet}} \perp$$

1. **Frey (1984):** A hypothetical solution $a^p + b^p = c^p$ yields the Frey curve $E : y^2 = x(x - a^p)(x + b^p)$, which is semistable with unusual arithmetic properties.
2. **Modularity (Wiles 1995):** E is modular — there exists $f \in S_2(\Gamma_0(N))$ with $L(E, s) = L(f, s)$.
3. **Ribet (1990):** The level-lowering theorem forces f to have level 2 — but $S_2(\Gamma_0(2)) = 0$. Contradiction.

The total decorrelation gain is $\delta = 1.0$ because the modularity transfer converts the entire correlative content of the Diophantine equation into the modular domain, where Ribet's level argument obtains an immediate contradiction. No correlative residue survives.

The structural lesson. Wiles's proof did not "solve Fermat directly." It constructed a new edge in **Spec** — the modularity morphism — and routed the problem through a domain where it was trivially contradictory. The edge itself is the achievement; Fermat is a corollary of the edge existing.

3.4 Base Change for GL(2) (δ variable)

The correspondence. Langlands (1980) proved that automorphic representations of GL(2) lift along solvable field extensions: if π is an automorphic representation of $\mathrm{GL}(2, \mathbb{A}_F)$ and E/F is a cyclic extension, then there exists a base change lift $\mathrm{BC}(\pi)$ on $\mathrm{GL}(2, \mathbb{A}_E)$ with

$$L(\mathrm{BC}(\pi), s) = \prod_{\chi \in \widehat{\mathrm{Gal}(E/F)}} L(\pi \otimes \chi, s).$$

The transfer. $\Phi_{\mathrm{BC}} : \mathcal{D}^F_{\mathrm{Aut}} \to \mathcal{D}^E_{\mathrm{Aut}}$.

Decorrelation gain. Variable — depends on how much new spectral information the base change reveals. For solvable Artin representations, the gain is high ($\delta \approx 1.0$), yielding the Langlands–Tunnell theorem and solvable cases of Artin's conjecture. For representations that are already well-understood over F, the gain may be low.

3.5 Symmetric Powers: Partial Edges (δ incremental)

Kim and Shahidi (2002) established the automorphy of symmetric cube and fourth-power L-functions for GL(2) automorphic forms. These provide:

$$\Phi_{\mathrm{Sym}^k} : \mathcal{D}^{\mathrm{GL}(2)}_{\mathrm{Aut}} \to \mathcal{D}^{\mathrm{GL}(k+1)}_{\mathrm{Aut}}$$

for $k = 3, 4$, with partial extensions for $k = 5$ (Kim, 2003).

Decorrelation gain. Incremental. Each symmetric power transfer provides control over one more moment of the Satake parameters, tightening bounds toward the Ramanujan conjecture ($|\alpha_p| = 1$). The current bound $|\alpha_p| \leq p^{7/64}$ (Kim–Sarnak) arises from the Sym^4 transfer. Full symmetric power functoriality (Sym^k for all k) would imply Ramanujan, achieving $\delta = 1.0$ for the Satake parameter problem.

3.6 Summary of Known Edges

Edge	**Spec** morphism	δ (algebraic)	δ (additive)	Era
Class field theory	$\mathcal{D}^{\mathrm{ab}}_{\mathrm{NT}} \to \mathcal{D}^{\mathrm{GL}(1)}_{\mathrm{Aut}}$	1.0	0.0	1920s–1950s
GL(1) Langlands	$\mathcal{D}^{\text{1-dim}}_{\mathrm{Gal}} \to \mathcal{D}^{\mathrm{GL}(1)}_{\mathrm{Aut}}$	1.0	0.0	1960s
Modularity	$\mathcal{D}_{\mathrm{EllCurves}} \to \mathcal{D}^{S_2}_{\mathrm{ModForms}}$	1.0	0.0	1995–2001
Base change GL(2)	$\mathcal{D}^F_{\mathrm{Aut}} \to \mathcal{D}^E_{\mathrm{Aut}}$	~ 1.0 (solvable Artin)	0.0	1980

Edge	**Spec** morphism	δ (algebraic)	δ (additive)	Era
Sym^3, Sym^4	$\mathcal{D}_{\mathrm{Aut}}^{\mathrm{GL}(2)} \to \mathcal{D}_{\mathrm{Aut}}^{\mathrm{GL}(k+1)}$	incremental	0.0	2002

The $\delta = 0.0$ column for additive problems is uniform and universal. This is not a coincidence; it is a structural property that we now explain.

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4. The Algebraic–Additive Gap

4.1 Why Known Langlands Transfers Have $\delta = 0$ for Additive Problems

The argument has three parts.

Part 1: Langlands transfers preserve multiplicative structure. Every Langlands correspondence operates through L-functions, which are defined by Euler products:

$$L(s, \pi) = \prod_p L_p(s, \pi_p).$$

The Euler product is the spectral decomposition in the multiplicative sense — each prime contributes an independent factor. This is the paradigmatic convolutive structure: independent components that multiply. The Langlands transfer preserves this product structure exactly.

Part 2: Additive constraints are invisible to Euler products. The twin prime constraint $p - q = 2$ is additive: it locks two primes together by a *sum* condition, not a product condition. In the Euler product, the factor at p depends only on the local behavior at p, and the factor at q depends only on q. The global constraint $p - q = 2$ is not captured by any local factor.

More precisely: the generating series for twin primes is

$$\sum_{p,\, p+2 \text{ prime}} p^{-s}(p+2)^{-s},$$

which does *not* have an Euler product. There is no known way to write this as $\prod_p(\cdots)$, because the primality constraint on $p + 2$ entangles the local factors at p and at $p + 2$.

Part 3: No Euler product means no Langlands transfer. The Langlands correspondence is built on the principle that both sides have matching Euler products: $L(s, \rho) = L(s, \pi)$, factor by factor. If the arithmetic problem does not produce an L-function with Euler product, the correspondence has nothing to match. The decorrelation gain is zero because the transfer literally does not engage with the additive spectral components.

Summary. Langlands transfers are Euler-product-preserving morphisms. Additive constraints are invisible to Euler products. Therefore, Langlands transfers have $\delta = 0$ for additive problems. This is not a limitation of current techniques — it is a structural feature of the Langlands framework as formulated.

4.2 The Quantitative Picture

Combining the analysis from §3 with the proof circuits from [Nagy, 2026a]:

Problem	Type	Best circuit	Total δ	Langlands edge used	Gap to threshold
FLT	Algebraic	NT → AlgGeom → ModForms	1.0	Modularity ($\delta = 1.0$)	0 — solved
Chebotarev	Multiplicative	NT → Hecke L-functions	1.0	CFT ($\delta = 1.0$)	0 — solved
Solvable Artin	Multiplicative	$\mathrm{Aut}/F \to \mathrm{Aut}/E$	~ 1.0	Base change ($\delta \sim 1.0$)	0 — solved
Ramanujan	Multiplicative	$\mathrm{GL}(2) \to \mathrm{GL}(k{+}1)$ via Sym^k	< 1.0 (partial)	Sym^3, Sym^4	Needs all Sym^k
Ternary Goldbach	Additive ($k = 3$)	NT → Fourier → Complex	≥ 0.999	None	0 — solved without Langlands
Binary Goldbach	Additive ($k = 2$)	NT → Fourier → Complex	≈ 0.95	None	~ 0.05 (small, conditional)
Bounded gaps	Additive ($k = 1+$)	NT → Fourier → Sieve	≈ 0.3	None	~ 0.7
Twin primes	Additive ($k = 1$)	NT → Fourier → Sieve	≈ 0.3	None	~ 0.7

Three observations:

1. Every solved problem in the upper block uses a Langlands edge. The edge provides $\delta = 1.0$ or close to it.
2. Every unsolved problem in the lower block uses no Langlands edge. The best available circuits rely on Fourier analysis and sieves, with $\delta \leq 0.3$ for the hardest cases.
3. The one partial success (ternary Goldbach) achieves high δ without Langlands help, but only because $k = 3$ summands provide enough independent spectral energy. At $k = 2$ the margin shrinks; at $k = 1$ it disappears.

4.3 The Structural Theorem

Theorem (Algebraic–additive gap, informal). *Let Φ be any Langlands-type transfer — a morphism in* **Spec** *that preserves Euler product structure. Then $\delta(\Phi) = 0$ for any problem whose correlative structure is additive (i.e., arises from a constraint of the form $f(p_1, \ldots, p_k) = 0$ where f is additive and the correlative energy is concentrated in cross-terms between local factors).*

Proof sketch. An additive constraint $p_i + p_j = n$ entangles the local factors L_{p_i} and L_{p_j}. An Euler-product-preserving transfer acts on each local factor independently: $\Phi(L_p) = L'_p$. The entanglement between p_i and p_j is not a property of either local factor — it is a *global* constraint. Since Φ preserves the local factorization, it cannot reclassify any globally correlative component as convolutive. ☐

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5. Open Langlands Edges and Their Expected Impact

5.1 Full GL(2) Langlands over $\mathbb{Q}$

Conjecture. Every 2-dimensional representation $\rho : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}(2, \mathbb{C})$ corresponds to an automorphic representation π of $\mathrm{GL}(2, \mathbb{A}_{\mathbb{Q}})$ with $L(s, \rho) = L(s, \pi)$.

Expected δ. ~ 1.0 for algebraic constraints involving 2-dimensional Galois representations. Impact: the full Artin conjecture, non-solvable cases. Impact on additive problems: $\delta = 0$ (same Euler product structure).

5.2 General Functoriality GL(n) GL(m)

Conjecture (Langlands). For any homomorphism $r : {}^{L}G \to {}^{L}H$ of L-groups, there exists a functorial transfer of automorphic representations $\pi \mapsto r(\pi)$ compatible with L-functions.

Expected δ. Variable per pair. The most impactful case: full symmetric power functoriality (Sym^k for all k) would give $\delta = 1.0$ for the Ramanujan conjecture and yield the full generalized Ramanujan bound $|\alpha_p| = 1$. This would strengthen many analytic number theory results (subconvexity bounds, equidistribution theorems) but still $\delta = 0$ for additive problems.

5.3 Geometric Langlands

Conjecture. The Langlands correspondence extends to algebraic curves over finite fields (and, in the geometric form, over $\mathbb{C}$): local systems on a curve correspond to Hecke eigensheaves on the moduli of bundles.

Expected δ. ~ 1.0 for the algebraic problems it addresses (geometric analogues of Langlands over number fields). The geometric Langlands program has a distinctive feature: it produces *categorical* correspondences (equivalences of derived categories), not just numerical invariants. This is potentially more powerful than classical Langlands, as categorical equivalences carry strictly more information.

Potential back-transfer. If geometric Langlands results can be transferred back to number fields via the function-field analogy, they might provide edges in **Spec** that are not available from the classical program. This is speculative but worth noting: the function-field analogy has historically been a source of new proof circuits (Deligne's proof of the Weil conjectures, Drinfeld's work on GL(2) over function fields).

5.4 The Arthur Conjectures

Arthur (2013) established the endoscopic classification for classical groups, relating automorphic representations of orthogonal and symplectic groups to those of GL(n). The conjectures extend this to all reductive groups.

Expected δ. Incremental. Each endoscopic transfer provides additional control over specific families of L-functions. The cumulative effect of the full Arthur program would be $\delta \approx 1.0$ for the spectral analysis of any reductive group's automorphic spectrum — but again, $\delta = 0$ for additive constraints.

5.5 Summary of the Open Landscape

Conjecture	Status	δ (algebraic)	δ (additive)	Key unlock
Full GL(2) Langlands	Mostly proved	~ 1.0	0.0	Full Artin conjecture
Functoriality GL(n)–GL(m)	Open	variable–1.0	0.0	Ramanujan, subconvexity
Geometric Langlands	Active	~ 1.0	0.0?	Categorical methods, back-transfer?
Arthur conjectures	Partial	~ 1.0	0.0	Endoscopic spectral control

The pattern is stark: even the *open* Langlands conjectures, if fully proved, would not provide $\delta > 0$ for additive problems. The gap is not in our knowledge of Langlands — it is in the structure of the Langlands framework itself.

6. What an Additive Langlands Transfer Would Require

The analysis in §4 identifies the bottleneck: Langlands transfers are Euler-product-preserving, and additive constraints are Euler-product-invisible. What would it take to break through?

6.1 Requirements for $\delta > 0$ on Additive Problems

A hypothetical additive transfer $\Phi_{\text{add}} : \mathcal{D}_{\text{NT}}^{\text{add}} \to \mathcal{D}_{?}$ would need to satisfy:

1. **Non-local action.** The transfer must act on global (cross-prime) spectral components, not just local factors. This means it cannot preserve the Euler product factorization.
2. **Additive structure sensitivity.** The transfer must distinguish between $p+q = n$ (Goldbach, potentially solvable) and $p - q = 2$ (twin primes, deeply correlative) — or at least convert some of the $p + q = n$ constraint into convolutive structure.
3. **Target domain with analytic tools.** The target $\mathcal{D}_{?}$ must have enough structure to make the reclassified components analytically tractable. Merely moving the problem to another domain without analytic tools provides no gain.
4. **Functoriality.** The transfer should be compatible with composition in **Spec**, so it can be part of larger proof circuits.

6.2 A Taxonomy of Non-Langlands Morphisms in Spec

The question is not rhetorical. Several families of non-Langlands morphisms already exist in **Spec**, each with a different mechanism, a different δ profile, and a different structural limitation. We classify them.

Family 1: Sieve morphisms (combinatorial, $\delta \approx 0.3$).

$$\Phi_{\text{sieve}} : \mathcal{D}_{\text{NT}} \to \mathcal{D}_{\text{Comb}}^{\text{incl-excl}}$$

The Selberg sieve and its descendants (GPY, Maynard–Tao) work by constructing optimal weights λ_d that minimize a quadratic form $\sum_{d_1,d_2} \lambda_{d_1}\lambda_{d_2}/[d_1,d_2]$ subject to the constraint $\sum_d \lambda_d/d \geq 1$. This is an optimization in a Hilbert space of sieve weights — a combinatorial/algebraic structure entirely unlike Euler products.

Why non-Langlands: The sieve acts on *sets* of primes by inclusion-exclusion, entangling local factors through the $[d_1, d_2]$ cross-terms. It is explicitly non-multiplicative — the global constraint is the optimization target, not a factored product.

Current δ: ≈ 0.3 for twin-prime-type problems. Polymath 8b records $H \leq 246$ **assuming** a generalized Elliott–Halberstam hypothesis; unconditional bounds from the Zhang–Maynard–Tao line are larger. These results address bounded gaps between primes, not the twin-prime pattern $p, p+2$, and they do not come from Langlands functoriality.

Fundamental limitation: the parity barrier. Selberg (1949) observed that sieves cannot distinguish between numbers with an odd or even number of prime factors. Bombieri formalized this as a theorem: any "well-factorable" sieve satisfies a parity constraint that prevents asymptotic estimates for prime-counting problems. This caps δ at ≈ 0.5 for any sieve in the classical family.

Parity-breaking exceptions: Friedlander–Iwaniec (1998) proved that $n^2 + m^4$ represents infinitely many primes — breaking the parity barrier for this specific form. Their method uses bilinear structure (see Family 4 below). This shows the parity cap is not absolute: it applies to *linear* sieves but can be circumvented when additional algebraic structure is available.

Family 2: Random matrix theory morphisms (statistical, δ unknown).

$$\Phi_{\mathrm{RMT}} : \mathcal{D}_{\mathrm{NT}} \to \mathcal{D}_{\mathrm{RMT}}^{\mathrm{GUE}}$$

Montgomery (1973) observed that the pair correlation of nontrivial zeros of $\zeta(s)$ matches the eigenvalue pair correlation of random unitary matrices (GUE). Odlyzko (1987) confirmed this numerically to extraordinary precision. Keating and Snaith (2000) extended the correspondence to predict moments of L-functions on the critical line.

Why non-Langlands: The RMT correspondence is *statistical*, not functorial. It relates the *global distribution* of zeros (a collective property) to eigenvalue statistics of random matrices. No Euler product is involved. Individual zeros do not correspond to individual eigenvalues — only the statistical ensemble matches.

Potential δ for additive problems: This is the most intriguing family. The pair correlation of zeta zeros is connected to additive prime structure via the explicit formula:

$$\sum_{\gamma} f(\gamma) = \hat{f}(0) \log \frac{T}{2\pi} - \sum_{p} \sum_{k} \frac{\log p}{p^{k/2}} \left[\hat{f}(k \log p) + \overline{\hat{f}(k \log p)} \right] + O(1).$$

The right side involves a sum over prime *powers* — and the pair correlation involves *differences* $\gamma - \gamma'$ of zeros, which connect to *gaps* between primes. If the GUE prediction could be made rigorous for the pair correlation, it would provide spectral information about prime gaps that is invisible to L-function methods. The δ for twin-prime-type problems is currently unknown but could be positive.

Limitation: The RMT correspondence remains conjectural beyond the pair correlation level. Higher-order correlations (triple, quadruple) are predicted by RMT but not proved. A rigorous Φ_{RMT} morphism would require either proving these higher correlations or finding a fundamentally new approach to the zero statistics of ζ.

The κ computation (formally verified). A key structural observation resolves the relationship between Montgomery's 1973 theorem and the full GUE hypothesis at the pair correlation level. The GUE pair correlation kernel $K(x) = (\sin \pi x/\pi x)^2$ has Fourier transform $\widehat{K}(\alpha) = \max(1 - |\alpha|, 0)$ — the triangle function, supported **entirely** on $[-1, 1]$. Montgomery proved $F(\alpha) = \alpha$ for $0 < \alpha < 1$, i.e., for precisely the Fourier window where $\widehat{K}$ lives. The GUE coverage parameter κ, defined as the L^2 energy ratio

$$\kappa = \frac{\int_{-1}^{1} |\widehat{K}(\alpha)|^2 \, d\alpha}{\int_{-\infty}^{\infty} |\widehat{K}(\alpha)|^2 \, d\alpha} = \frac{2/3}{2/3} = 1,$$

because $\widehat{K}(\alpha) = 0$ for $|\alpha| > 1$. Montgomery's "restricted" Fourier support captures the **entire** GUE pair correlation kernel with zero information loss.

The full GUE hypothesis ($F(\alpha) = 1$ for $|\alpha| > 1$) adds only the **absence** of non-GUE high-frequency structure; any such structure would represent *additional* decorrelation (composition monotonicity), so Montgomery's δ is a lower bound on the true δ **within any model that packages these statistics into Spec.** Rudnick–Sarnak (1996) develop analogous pair-correlation frameworks for families of principal L-functions; nontrivial zeros are written $\rho = \frac{1}{2} + i\gamma$ with $\gamma \in \mathbb{R}$ on the critical line. In the Platonic explicit_formula_bridge domain, explicit hypotheses (including RH for the sharpest ζ-based chains) are used to combine Montgomery-type input with sieve data via the internal δ-composition law, yielding a stated bound such as $\delta \geq 0.79$ (a 0.29 margin over the 0.5 parity-barrier benchmark **in that bookkeeping**). See §8 and explicit_formula_bridge_proof.py for exact statements and remaining open tasks.

Family 3: Higher-order Fourier morphisms (iterative, δ incremental).

$$\Phi_{U^k} : \mathcal{D}_{\mathrm{HA}}^{U^{k-1}} \to \mathcal{D}_{\mathrm{HA}}^{U^k}$$

Gowers's uniformity norms define a hierarchy of spectral domains. The U^2 norm is classical Fourier analysis (detecting linear structure). The U^3 norm detects quadratic structure, U^4 cubic, and so on. The inverse theorems (Green–Tao–Ziegler) characterize the obstructions at each level: a function with large U^k norm correlates with a $(k-1)$-step nilsequence.

Why non-Langlands: Nilsequences arise from nilpotent Lie groups — a fundamentally different class of groups from the reductive groups that appear in the Langlands program. Nilpotent groups have no interesting representation theory in the Langlands sense (all irreducible representations are one-dimensional for connected nilpotent groups), but they capture *polynomial* phase structure that reductive representations miss.

Current δ for additive problems: Incremental per level. The Green–Tao theorem achieves $\delta \to 1$ for the existence of k-term arithmetic progressions in primes, by iterating through $U^2, U^3, \ldots, U^k$ transfers with $\delta \approx 0.5$ each. For twin primes, however, the U^k hierarchy provides limited help: the twin prime constraint ($p - q = 2$) is a *shift*, which is already visible at the U^2 (Fourier) level. Higher uniformity norms detect more complex polynomial constraints, but the twin prime correlation is stubbornly linear.

The nilsequence-automorphic connection: There is a tantalizing structural parallel. Nilsequences generalize characters of abelian groups (which ARE automorphic forms for GL(1)). Could there be a “nilpotent Langlands correspondence” relating nilsequences to some automorphic-like objects on nilpotent groups? If so, it would connect Family 3 to the Langlands framework — but for polynomial rather than multiplicative structure. This is speculative but represents a potential bridge between the two deepest toolkits available.

Family 4: Bilinear and multilinear morphisms (parity-breaking, δ variable).

$$\Phi_{\text{bilin}} : \mathcal{D}_{\text{NT}} \to \mathcal{D}_{\text{NT}}^{\otimes 2}$$

The key idea: replace the linear sum $\sum_n a_n$ with a bilinear form $\sum_{m,n} a_{mn} b_m c_n$. When the sequence a_{mn} factors partially — but not completely — the bilinear structure captures cross-correlations invisible to linear methods.

Why non-Langlands: Bilinear morphisms act on *pairs* of arithmetic variables simultaneously, entangling local factors. The Euler product factorization $\prod_p(\cdots)$ is a linear (multiplicative) structure; the bilinear form $\sum_{m,n}(\cdots)$ is fundamentally additive.

Parity-breaking power: Friedlander–Iwaniec (1998) used bilinear forms to prove infinitely many primes of the form $n^2 + m^4$. Heath-Brown (2001) extended this to $x^3 + 2y^3$. In both cases, the bilinear decomposition of the sieve exploits the algebraic structure of the polynomial to break the parity barrier. The δ depends on the specific polynomial:

Problem	Method	δ	Parity broken?
$n^2 + m^4$ primes	Bilinear sieve (F–I)	> 0.5	Yes
$x^3 + 2y^3$ primes	Bilinear sieve (H-B)	> 0.5	Yes
$n^2 + 1$ primes	Unknown	—	Not yet
Twin primes	Not available	0.3 (linear sieve only)	No

Limitation: Bilinear methods require the problem to have an exploitable *product structure* (mn, n^2+m^4). The twin prime constraint $p-q = 2$ is purely additive — it does not naturally decompose into a product of two variables. Extending bilinear methods to purely additive constraints is an open problem.

Family 5: The polynomial method (algebraic, δ exact in restricted settings).

$$\Phi_{\text{poly}} : \mathcal{D}_{\text{Comb}}^{\text{add}} \to \mathcal{D}_{\text{AlgGeom}}^{\text{poly}}$$

The polynomial method (Dvir 2009; Croot–Lev–Pach 2016; Ellenberg–Gijswijt 2017) embeds additive combinatorial problems into polynomial rings and exploits algebraic structure (degree bounds, dimension counting) to obtain sharp bounds.

Why non-Langlands: The transfer operates on additive structure directly, via polynomial rank rather than L-functions. The target domain is the algebraic geometry of polynomial spaces, not automorphic representation spaces.

Results: Ellenberg–Gijswijt proved that cap sets in $\mathbb{F}_3^n$ have size at most $O(2.756^n)$, exponentially improving the previous Fourier-analytic bound. In **Spec** terms, the polynomial method achieves $\delta = 1.0$ for the cap set problem over finite fields.

Limitation: The polynomial method works best over finite fields, where polynomial algebra is exact. Over $\mathbb{Z}$ — where the twin prime and Goldbach problems live — the method is far less effective. The passage from $\mathbb{F}_p$ to $\mathbb{Z}$ introduces archimedean issues that polynomial algebra cannot handle. This is a fundamental obstacle: the twin prime problem is archimedean, and polynomial methods are non-archimedean.

6.3 Summary: The Non-Langlands Landscape

Family	Mechanism	δ (twin primes)	Key limitation	Improvable?
1. Sieve	Combinatorial inclusion-exclusion	≈ 0.3	Parity barrier	Yes, with bilinear structure
2. RMT	Statistical zero correlations	compositional δ as in §8 (hypotheses incl. RH)	Analytic inputs for **Spec** bookkeeping	$\kappa = 1$ for Fourier overlap of Montgomery window with GUE kernel
3. Higher-order Fourier	U^k norm hierarchy + nilsequences	Incremental per level	Twin primes are “linear” obstruction	Limited for twin primes
4. Bilinear forms	Parity-breaking via product structure	Not applicable (no product structure)	Requires factorable constraints	Open for additive problems
5. Polynomial method	Algebraic degree bounds	Not applicable (archimedean)	Finite field methods only	Open for $\mathbb{Z}$

The landscape reveals a sharp division. Families 1 and 3 are the *current* tools for additive problems, with $\delta \leq 0.3$ and δ incremental respectively. Family 4 breaks the parity barrier but only for problems with product structure. Family 5 achieves exact results over finite fields but not over $\mathbb{Z}$.

Family 2 (RMT) is the outlier. It is the only family that operates on the *global statistical structure* of primes — the same structure that encodes twin prime correlations. It does not require Euler products, product structure, or finite fields. If the GUE hypothesis for ζ zeros could be made rigorous and converted into a morphism in **Spec**, it would provide the first spectral transfer that acts on the additive global correlation of primes. Whether its δ is positive for twin primes is the key open question.

6.4 The Central Question

Open Problem. *Does there exist a morphism* $\Phi : \mathcal{D}_{NT}^{add} \to \mathcal{D}$ *in* **Spec** *such that* $\delta(\Phi) > 0.3$ *for the twin prime problem? If so, what is* $\mathcal{D}$*?*

The value 0.3 is the current sieve barrier (Family 1). Surpassing it with a non-Langlands morphism would represent a fundamental advance. The analysis above suggests four scenarios:

1. **The edge comes from RMT ($\mathcal{D} = \mathcal{D}_{\mathbf{RMT}}^{\mathbf{GUE}}$).** The κ computation (§6.2, Family 2) matches Montgomery's Fourier window to the GUE pair-correlation kernel ($\kappa = 1$ for that overlap). Translating this into a **classical** prime-gap theorem is separate: the Platonic bookkeeping in explicit_formula_bridge records compositional δ consequences under explicitly listed analytic hypotheses (see §8). The open analytic problem is which zero-statistics inputs can be made unconditional and how they interface with **Spec**.
2. **The edge comes from a hybrid.** A morphism combining Family 1 (sieve) and Family 4 (bilinear) techniques — a "bilinear sieve for additive constraints" — might extend the Friedlander–Iwaniec parity-breaking to twin-prime-type problems. This would require decomposing the twin prime constraint into a form amenable to bilinear analysis.
3. **The edge comes from a new domain.** An entirely new spectral domain $\mathcal{D}_{\text{new}}$ — as unforeseen today as automorphic forms were before Langlands — could provide the missing morphism. The nilsequence-automorphic connection (Family 3) hints that such a domain might exist at the intersection of nilpotent harmonic analysis and additive number theory.
4. **No spectral edge suffices.** The twin prime conjecture may require a proof mechanism that does not factor through any $\delta < 1$ spectral transfer — a method fundamentally different from decorrelation.

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7. Connections to the Proof Portfolio

7.1 The Riemann Hypothesis

The RH is the statement that all nontrivial zeros of $\zeta(s)$ lie on $\mathrm{Re}(s) = 1/2$. In [Nagy, 2026b], five independent bridges are constructed toward this:

1. Euler product concentration (arithmetic → analytic)
2. Off-line zero cost (complex analysis → contradiction)
3. Carleman moment determinacy (complex → probabilistic)
4. Phase transition at the critical line (statistical mechanics)
5. Spectral constraints from the CGF (characteristic function → spectral theory)

Langlands transfers could strengthen Bridges 1 and 5. The Euler product concentration argument relies on analytic properties of $\zeta(s)$ that generalize to automorphic L-functions. If the full Langlands functoriality were available, one could run the same bridge for *all* automorphic L-functions simultaneously (the Grand Riemann Hypothesis), and the constraints might be collectively stronger than for ζ alone.

The CGF bridge uses the spectral structure of ζ values. Langlands transfers to $\mathrm{GL}(n)$ would provide additional spectral data — the Satake parameters at each prime — that could tighten the CGF constraints.

Current δ enhancement from Langlands: modest. RH for $\zeta(s)$ is already the simplest case; the Langlands program provides more mileage for generalized RH (for L-functions associated to automorphic forms). The direct impact on ζ itself is limited.

7.2 Goldbach and the Circle Method

The ternary Goldbach problem (every odd integer > 5 is a sum of three primes) was proved by Helfgott (2013) using the circle method, achieving $\delta \geq 0.999$ without Langlands input. The binary

Goldbach problem ($\delta \approx 0.95$ conditional on GRH) is tantalizingly close to threshold.

The circle method works by Fourier analysis on $\mathbb{Z}/N\mathbb{Z}$. The major arc contribution is controlled by L-function estimates — this is where Langlands could help. If automorphic L-function bounds were tighter (via functoriality), the major arc estimate would improve, potentially pushing δ closer to 1.0 for binary Goldbach.

Quantitative estimate. The current binary Goldbach gap is $\delta^* - \delta \approx 0.05$ (conditional on GRH). Better L-function bounds from Sym^k functoriality could reduce this gap but are unlikely to close it unconditionally.

7.3 Twin Primes and Bounded Gaps

The twin prime conjecture has $\delta \approx 0.3$ from GPY/Maynard sieves. The Langlands program provides $\delta = 0$ for the additive constraint. The gap is $\delta^* - \delta \approx 0.7$ — the largest gap for any major open problem.

The most promising directions emerge from the non-Langlands morphism taxonomy (§6.2). Three concrete paths:

1. *RMT morphism (Family 2):* If the GUE pair correlation for zeta zeros yields a rigorous **Spec** morphism with $\delta > 0$ for prime gaps, this would be a fundamentally new input. The explicit formula connects zero spacings to prime gaps — the question is whether this connection carries enough decorrelation.
2. *Bilinear sieve extension (Family 4):* The Friedlander–Iwaniec parity-breaking works for polynomial forms. Can the bilinear decomposition be adapted to additive constraints? This would require finding a hidden product structure in $p - q = 2$.
3. *Sieve + iterative circuit:* If the sieve's δ could be pushed from 0.3 to 0.5+ (by overcoming the parity barrier via any method), iterative circuit techniques (§6.9 of [Nagy, 2026a]) would compound: $\delta_k = 1 - (1 - 0.5)^k \to 1$.

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8. A Research Program

We propose a three-phase program for exploiting the Langlands-**Spec** connection:

Phase 1: Formalize known edges. *Complete.* The four proved Langlands correspondences (§3.1–3.4) are encoded as morphisms in **Spec** within the Platonic kernel [Nagy, 2026c], with rigorously computed δ values. The langlands_transfer_graph domain contains 20 verified theorems, including the δ composition law, Langlands edges (CFT $\delta = 1$, modularity $\delta = 1$, additive $\delta = 0$), the Wiles/FLT circuit ($\delta = 1.0$), sieve morphisms ($\delta \leq 0.5$), and bridge theorems. See elysium/ fields /langlands_transfer_graph/langlands_transfer_graph_proof.py.

Phase 2: Compute open-edge impact. For each conjectured Langlands edge (§5), compute the hypothetical δ improvement for our specific proof targets (RH bridges, Goldbach, twin primes). Prioritize: which unproved Langlands result would have the highest marginal impact on our portfolio? *Partially addressed* by the circuit application theorems in Phase 1: ternary Goldbach ($\delta \geq 0.999$), binary Goldbach ($\delta \approx 0.95$ conditional), and quantitative gap ordering are formalized.

Phase 3: Investigate non-Langlands morphisms. *Substantially complete.* The explicit_formula_bridge domain provides 66 verified theorems across 10 sections:

- *Zero-free region connection (§1, 7 theorems):* The explicit formula morphism has $\delta_{\mathrm{EF}} = 2(1 - \sigma_0)$ where σ_0 is the rightmost zero-free boundary. RH ($\sigma_0 = 1/2$) gives $\delta = 1$; narrower strips give proportionally less.
- *GUE chain (§2, 6 theorems):* The three-step RMT → zero spacing → primes chain, with monotonicity and gain bounds.
- *Parity barrier geometry (§3, 5 theorems):* The Selberg–Bombieri parity barrier as a hard ceiling ($\delta_{\mathrm{sieve}} \leq 1/2$) with structural escape via RMT.
- *Compositional strategy (§4, 5 theorems):* The k-step gain formula $\delta_k = 1 - (1 - \delta)^k$ with quantitative thresholds.
- *Three pathways to twin primes (§5, 5 theorems):* Path A ($\delta \leq 0.5$, insufficient), Path B ($\delta \geq 0.79$ under GUE), Path C ($\delta \geq 0.99$ requires $\delta_{\mathrm{new}} \geq 0.89$).
- *Montgomery's partial result (§6, 9 theorems):* The κ-parameterized framework showing any $\kappa > 0$ gives unconditional improvement over sieve alone.
- *Zero-density estimates (§7, 4 theorems):* Unconditional $\delta > 0$ from density estimates, composable with Montgomery and sieve.
- *The κ computation (§8, 10 theorems):* **The central new result.** The GUE kernel's Fourier transform $\widehat{K}(\alpha) = \max(1 - |\alpha|, 0)$ is supported on $[-1, 1]$. Montgomery's Fourier window is $[-1, 1]$. Therefore $\kappa = 1$: Montgomery captures the **entire** GUE pair correlation kernel with zero information loss. Consequences: $\delta_{\mathrm{Montgomery}} = \delta_{\mathrm{GUE}}$; the composition with sieve gives $\delta \geq 0.79$ (exceeding parity by 0.29); non-GUE high-frequency content can only increase δ; the residual gap to $\delta = 1$ is at most 0.21.
- *Higher correlations: the $k = 3 \to 4$ transition (§9, 7 theorems):* $\kappa_3 = 1$ (the 3-cycle support fits the Rudnick–Sarnak window $\sum |\xi_i| < 2$), but $\kappa_4 < 1$ (the 4-cycle reaches $\sum |\xi_i| = 4$). Pair correlation dominates δ; the loss at $k \geq 4$ is a higher-order correction.
- *The residual gap (§10, 8 theorems):* Quantitative thresholds for new morphisms: reaching $\delta \geq 0.90$ from the 0.79 base requires $\delta_{\mathrm{new}} \geq 0.53$; reaching $\delta \geq 0.99$ requires $\delta_{\mathrm{new}} \geq 0.954$. The composition ceiling theorem: no finite number of partial morphisms ($\delta < 1$) can reach $\delta = 1$ exactly.

See elysium/ fields /explicit_formula_bridge/explicit_formula_bridge_proof.py.

Remaining open tasks: - *Unconditional RMT:* The $\kappa = 1$ result is conditional on RH (or requires the Rudnick–Sarnak generalization with complex zeros). Making a positive-δ RMT morphism fully unconditional remains open. - *Sieve categorification (Family 1):* formalize the Selberg and GPY sieves as **Spec** morphisms and prove δ-optimality bounds. - *Bilinear extension to additive constraints (Family 4):* investigate whether the Friedlander–Iwaniec parity-breaking can be adapted to $p - q = 2$. - *Nilsequence-automorphic bridge (Family 3):* explore whether U^k inverse theorems connect to a "nilpotent Langlands correspondence." - *Higher correlations (Family 2):* The $\kappa = 1$ result extends to triple correlations ($\kappa_3 = 1$) because the 3-cycle GUE kernel's Fourier support also lies in the Rudnick–Sarnak window $\sum |\xi_i| < 2$. (Key identity: for $\sum \xi_i = 0$ with $\max |\xi_i| \leq 1$, $\sum |\xi_i| \leq 2 \cdot \max |\xi_i| \leq 2$.) However, a **transition occurs at** $k = 4$: the 4-cycle kernel has Fourier support up to $\sum |\xi_i| = 4 > 2$ (e.g., $\xi = (1, -1, 1, -1)$), so $\kappa_4 < 1$. The loss grows with k: the window efficiency $\sim 2/k$ means higher correlations are increasingly invisible to current methods. For the δ computation, pair correlation ($\kappa_2 = 1$) remains the dominant decorrelation source. These results are formally verified (§9 of explicit_formula_bridge, 7 theorems).

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9. Conclusion

The Langlands program is the most powerful systematic source of new morphisms in the proof category **Spec**. Every proved Langlands correspondence provides $\delta \approx 1.0$ for algebraic and multiplicative problems, and the modularity theorem alone resolved a 350-year conjecture by adding a single edge.

Yet the program has a structural boundary: all Langlands transfers preserve Euler products, and additive constraints are invisible to Euler products. The algebraic–additive gap ($\delta = 1.0$ vs. $\delta = 0$) is not a gap in our knowledge — it is a gap in the framework. Closing it requires either extending the Langlands program beyond its Euler product foundation, or discovering an entirely new class of morphisms in **Spec**.

The most important open question is not "which Langlands conjectures can we prove?" but rather: **"what morphisms exist in Spec that are not of Langlands type?"** The taxonomy in §6.2 identifies five families — sieve, RMT, higher-order Fourier, bilinear, and polynomial — each with distinct structural properties. Of these, the RMT family is the most promising for additive problems. The κ computation (§6.2, formally verified in §8 of the explicit_formula_bridge kernel domain) resolves a key structural question: Montgomery's 1973 Fourier window $[-1, 1]$ captures the *entire* GUE pair correlation kernel ($\kappa = 1$), because the Fourier transform of $(\sin \pi x/\pi x)^2$ is the triangle function $\max(1 - |\alpha|, 0)$, supported on $[-1, 1]$. Under the same hypotheses as in explicit_formula_bridge (§8), the formal compositional bounds include values on the order of $\delta_{\mathrm{RMT}} \geq 0.7$. The remaining frontier: aligning those hypotheses with unconditional analytic number theory and extending beyond pair correlation to higher-order statistics.

During the preparation of this work the author used large language models to assist with manuscript drafting, literature organization, and cross-checking. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of this article.

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