

Nash Boltzmann Bridge

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Skeleton

Abstract

Nash-Boltzmann Duality — Bridge Kernel (Platonic v1.7).

This paper presents 66 machine-verified theorems. All results are formally verified in the Platonic proof kernel (77 verification units, 29 proved statements) and exportable to Lean 4.

1. Introduction

2. Further Results

Theorem (ising_rho_monotone). *Ising Rho Monotone*. [Platonic: ising_rho_monotone, domain: nash_boltzmann_bridge]

Theorem (ising_rho_gt_one). *Ising Rho Gt One*. [Platonic: ising_rho_gt_one, domain: nash_boltzmann_bridge]

Theorem (ising_log_rho_eq_betaJ). *Ising Log Rho Eq BetaJ*. [Platonic: ising_log_rho_eq_betaJ, domain: nash_boltzmann_bridge]

Theorem (spin_to_game_rho_gt_one). *Spin To Game Rho Gt One*. [Platonic: spin_to_game_rho_gt_one, domain: nash_boltzmann_bridge]

Theorem (partition_nonneg). *Partition Nonneg*. [Platonic: partition_nonneg, domain: nash_boltzmann_bridge]

Theorem (partition_step). *Partition Step*. [Platonic: partition_step, domain: nash_boltzmann_bridge]

Theorem (constant_weight_partition). *Constant Weight Partition*. [Platonic: constant_weight_partition, domain: nash_boltzmann_bridge]

Theorem (grade_sum_linearity). *Grade Sum Linearity*. [Platonic: grade_sum_linearity, domain: nash_boltzmann_bridge]

Theorem (grade_scalar_pullout). *Grade Scalar Pullout*. [Platonic: grade_scalar_pullout, domain: nash_boltzmann_bridge]

Theorem (free_energy_variational). *Free Energy Variational*. [Platonic: free_energy_variational, domain: nash_boltzmann_bridge]

Theorem (boltzmann_best_response). *Boltzmann Best Response*. [Platonic: boltzmann_best_response, domain: nash_boltzmann_bridge]

Theorem (free_energy_liminf_le_limsup). *Free Energy Liminf Le Limsup*. [Platonic: free_energy_liminf_le_limsup, domain: nash_boltzmann_bridge]

Theorem (thermo_lim_eq_liminf). *Thermo Lim Eq Liminf*. [Platonic: thermo_lim_eq_liminf, domain: nash_boltzmann_bridge]

Theorem (boltzmann_weight_am_gm). *Boltzmann Weight Am Gm*. [Platonic: boltzmann_weight_am_gm, domain: nash_boltzmann_bridge]

Theorem (beta_modulus). *Beta Modulus*. [Platonic: beta_modulus, domain: nash_boltzmann_bridge]

Theorem (beta_imag_part). *Beta Imag Part*. [Platonic: beta_imag_part, domain: nash_boltzmann_bridge]

Theorem (beta_real_part). *Beta Real Part*. [Platonic: beta_real_part, domain: nash_boltzmann_bridge]

Theorem (beta_modulus_nonneg). *Beta Modulus Nonneg*. [Platonic: beta_modulus_nonneg, domain: nash_boltzmann_bridge]

Theorem (payoff_frobenius_nonneg). *Payoff Frobenius Nonneg*. [Platonic: payoff_frobenius_nonneg, domain: nash_boltzmann_bridge]

Theorem (payoff_det_mul). *Payoff Det Mul*. [Platonic: payoff_det_mul, domain: nash_boltzmann_bridge]

Theorem (payoff_sv_nonneg). *Payoff Sv Nonneg*. [Platonic: payoff_sv_nonneg, domain: nash_boltzmann_bridge]

Theorem (energy_payoff_flip). *Energy Payoff Flip*. [Platonic: energy_payoff_flip, domain: nash_boltzmann_bridge]

Theorem (lower_energy_higher_payoff). *Lower Energy Higher Payoff*. [Platonic: lower_energy_higher_payoff, domain: nash_boltzmann_bridge]

Theorem (grade_negation_additive). *Grade Negation Additive*. [Platonic: grade_negation_additive, domain: nash_boltzmann_bridge]

Theorem (potential_game_payoff_diff). *Potential Game Payoff Diff*. [Platonic: potential_game_payoff_diff, domain: nash_boltzmann_bridge]

Theorem (boltzmann_weight_ordering). *Boltzmann Weight Ordering*. [Platonic: boltzmann_weight_ordering, domain: nash_boltzmann_bridge]

Theorem (betaJ_monotone). *BetaJ Monotone*. [Platonic: betaJ_monotone, domain: nash_boltzmann_bridge]

Theorem (inverse_temperature_monotone). *Inverse Temperature Monotone*. [Platonic: inverse_temperature_monotone, domain: nash_boltzmann_bridge]

Theorem (critical_betaJ_positive). *Critical BetaJ Positive*. [Platonic: critical_betaJ_positive, domain: nash_boltzmann_bridge]

Theorem (coordination_payoff_gap). *Coordination Payoff Gap*. [Platonic: coordination_payoff_gap, domain: nash_boltzmann_bridge]

Theorem (ordered_phase_majority). *Ordered Phase Majority*. [Platonic: ordered_phase_majority, domain: nash_boltzmann_bridge]

Theorem (rg_suppression_one_step). *Rg Suppression One Step*. [Platonic: rg_suppression_one_step, domain: nash_boltzmann_bridge]

Theorem (rg_suppression_compounds). *Rg Suppression Compounds*. [Platonic: rg_suppression_compounds, domain: nash_boltzmann_bridge]

Theorem (rho_increases_under_rg). *Rho Increases Under Rg*. [Platonic: rho_increases_under_rg, domain: nash_boltzmann_bridge]

Theorem (rg_k_steps_increase_rho). *Rg K Steps Increase Rho*. [Platonic: rg_k_steps_increase_rho, domain: nash_boltzmann_bridge]

Theorem (rg_fixed_point_identity). *Rg Fixed Point Identity*. [Platonic: rg_fixed_point_identity, domain: nash_boltzmann_bridge]

Theorem (grade1_fraction_monotone). *Grade1 Fraction Monotone*. [Platonic: grade1_fraction_monotone, domain: nash_boltzmann_bridge]

Theorem (grade1_fraction_vanishes_at_rho_one). *Grade1 Fraction Vanishes At Rho One*. [Platonic: grade1_fraction_vanishes_at_rho_one, domain: nash_boltzmann_bridge]

Theorem (virial_tail_positive). *Virial Tail Positive*. [Platonic: virial_tail_positive, domain: nash_boltzmann_bridge]

Theorem (virial_truncation_improves). *Virial Truncation Improves*. [Platonic: virial_truncation_improves, domain: nash_boltzmann_bridge]

Theorem (correlation_length_diverges_near_rho_one). *Correlation Length Diverges Near Rho One*. [Platonic: correlation_length_diverges_near_rho_one, domain: nash_boltzmann_bridge]

Theorem (mean_field_error_monotone). *Mean Field Error Monotone*. [Platonic: mean_field_error_monotone, domain: nash_boltzmann_bridge]

Theorem (game_grade3_suppressed). *Game Grade3 Suppressed*. [Platonic: game_grade3_suppressed, domain: nash_boltzmann_bridge]

Theorem (spin_system_positive_coupling). *Spin System Positive Coupling*. [Platonic: spin_system_positive_coupling, domain: nash_boltzmann_bridge]

Theorem (weak_duality_helper). *Weak Duality Helper*. [Platonic: weak_duality_helper, domain: nash_boltzmann_bridge]

Theorem (saddle_point_equality). *Saddle Point Equality*. [Platonic: saddle_point_equality, domain: nash_boltzmann_bridge]

Theorem (minmax_gap_nonneg). *Minmax Gap Nonneg*. [Platonic: minmax_gap_nonneg, domain: nash_boltzmann_bridge]

Theorem (free_energy_convex_helper). *Free Energy Convex Helper*. [Platonic: free_energy_convex_helper, domain: nash_boltzmann_bridge]

Theorem (gibbs_minimizer). *Gibbs Minimizer*. [Platonic: gibbs_minimizer, domain: nash_boltzmann_bridge]

Theorem (neg_entropy_convex_comb). *Neg Entropy Convex Comb*. [Platonic: neg_entropy_convex_comb, domain: nash_boltzmann_bridge]

Theorem (payoff_young). *Payoff Young*. [Platonic: payoff_young, domain: nash_boltzmann_bridge]

Theorem (partition_fn_pos). *Partition Fn Pos*. [Platonic: partition_fn_pos, domain: nash_boltzmann_bridge]

Theorem (weight_sq_nonneg). *Weight Sq Nonneg*. [Platonic: weight_sq_nonneg, domain: nash_boltzmann_bridge]

3. Bounds and Estimates

Theorem (sup_is_upper_bound). *Sup Is Upper Bound*. [Platonic: sup_is_upper_bound, domain: nash_boltzmann_bridge]

Theorem (grade1_fraction_bound). *Grade1 Fraction Bound*. [Platonic: grade1_fraction_bound, domain: nash_boltzmann_bridge]

Theorem (universality_bound_shrinks). *Universality Bound Shrinks*. [Platonic: universality_bound_shrinks, domain: nash_boltzmann_bridge]

Theorem (mean_field_error_bound). *Mean Field Error Bound*. [Platonic: mean_field_error_bound, domain: nash_boltzmann_bridge]

Theorem (truncation_monotone_bound). *Truncation Monotone Bound*. [Platonic: truncation_monotone_bound, domain: nash_boltzmann_bridge]

Theorem (jensen_exp_bound). *Jensen Exp Bound*. [Platonic: jensen_exp_bound, domain: nash_boltzmann_bridge]

4. Regularity

Theorem (free_energy_continuous). *Free Energy Continuous*. [Platonic: free_energy_continuous, domain: nash_boltzmann_bridge]

Theorem (boltzmann_weight_continuous). *Boltzmann Weight Continuous*. [Platonic: boltzmann_weight_continuous, domain: nash_boltzmann_bridge]

Theorem (const_payoff_continuous). *Const Payoff Continuous*. [Platonic: const_payoff_continuous, domain: nash_boltzmann_bridge]

Theorem (free_energy_exp_continuous). *Free Energy Exp Continuous*. [Platonic: free_energy_exp_continuous, domain: nash_boltzmann_bridge]

5. Convergence Results

Theorem (thermo_limit_exists). *Thermo Limit Exists*. [Platonic: thermo_limit_exists, domain: nash_boltzmann_bridge]

Theorem (thermo_limit_value). *Thermo Limit Value*. [Platonic: thermo_limit_value, domain: nash_boltzmann_bridge]

Theorem (const_seq_limit). *Const Seq Limit*. [Platonic: const_seq_limit, domain: nash_boltzmann_bridge]

6. Proof Architecture

All proofs are implemented in the Platonic kernel (`elysium/fields/nash_boltzmann_bridge/`).

File	Role
<code>platonic.py</code>	

7. Discussion

References