Stochastic Calculus

Dr. Tamás Nagy

Dr. Tamás Nagy

tamas@thelatent.space

Skeleton

Abstract

Stochastic Calculus — clean ProofEnv proof.

This paper presents 19 machine-verified theorems building on 6 established facts and 3 hypotheses. All results are formally verified in the Platonic proof kernel (67 verification units, 19 proved statements) and exportable to Lean 4.

---

1. Introduction

2. Further Results

Theorem (qvar_positive). *Qvar Positive.* [Platonic: qvar_positive, domain: stochastic_calculus]

Theorem (ito_isometry). *Ito Isometry.* [Platonic: ito_isometry, domain: stochastic_calculus]

Theorem (qvar_scales_with_time). *Qvar Scales With Time.* [Platonic: qvar_scales_with_time, domain: stochastic_calculus]

Theorem (sigma_square_nonneg). *Sigma Square Nonneg.* [Platonic: sigma_square_nonneg, domain: stochastic_calculus]

Theorem (ito_second_moment_nonneg). *Ito Second Moment Nonneg.* [Platonic: ito_second_moment_nonneg, domain: stochastic_calculus]

Theorem (discount_pos). *Discount Pos.* [Platonic: discount_pos, domain: stochastic_calculus]

Theorem (d1_eq_d2_plus_sigma). *D1 Eq D2 Plus Sigma.* [Platonic: d1_eq_d2_plus_sigma, domain: stochastic_calculus]

Theorem (put_call_parity). *Put Call Parity.* [Platonic: put_call_parity, domain: stochastic_calculus]

Theorem (no_arb_nonneg_price). *No Arb Nonneg Price.* [Platonic: no_arb_nonneg_price, domain: stochastic_calculus]

Theorem (arb_buy_signal). *Arb Buy Signal.* [Platonic: arb_buy_signal, domain: stochastic_calculus]

Theorem (arb_sell_signal). *Arb Sell Signal.* [Platonic: arb_sell_signal, domain: stochastic_calculus]

Theorem (no_arb_both_signs). *No Arb Both Signs.* [Platonic: no_arb_both_signs, domain: stochastic_calculus]

Theorem (discounted_price_nonneg). *Discounted Price Nonneg.* [Platonic: discounted_price_nonneg, domain: stochastic_calculus]

Theorem (martingale_increment_zero_mean). *Martingale Increment Zero Mean.* [Platonic: martingale_increment_zero_mean, domain: stochastic_calculus]

Theorem (d1_d2_spread_eq_sigma_sqrt_T). *D1 D2 Spread Eq Sigma Sqrt T.* [Platonic: d1_d2_spread_eq_sigma_sqrt_T, domain: stochastic_calculus]

Theorem (put_call_parity_nonneg). *Put Call Parity Nonneg.* [Platonic: put_call_parity_nonneg, domain: stochastic_calculus]

Theorem (normal_cdf_complement). *Normal Cdf Complement.* [Platonic: normal_cdf_complement, domain: stochastic_calculus]

Theorem (no_arbitrage_spread_pos). *No Arbitrage Spread Pos.* [Platonic: no_arbitrage_spread_pos, domain: stochastic_calculus]

3. Main Theorems

Theorem (ito_formula_remainder). *Ito Formula Remainder.* [Platonic: ito_formula_remainder, domain: stochastic_calculus]

4. Formal Framework

Hypotheses

- shifted_lower_bound: Shifted Lower Bound
- no_arb_nonneg_is_zero: No Arb Nonneg Is Zero
- itoIsometry_nonneg: Itoisometry Nonneg

Established Facts

- complete_the_square: Complete The Square
- no_arb_nonpos_is_zero: No Arb Nonpos Is Zero
- ftap_hard_zero: Ftap Hard Zero
- itoSimple_zero: Itosimple Zero
- itoSimple_add: Itosimple Add
- multi_reduces_to_single: Multi Reduces To Single

5. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/stochastic_calculus/).

File	Role
stochastic_calculus_proof.py	

6. Discussion

References