The Geometry of Risk: Connecting Spectral Fenton to TQFT and Witten Invariants

Dr. Tamás Nagy

tnagyphd@gmail.com

Speculative

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1. The Analogy with Witten's TQFT

In Topological Quantum Field Theory (TQFT), Ed Witten showed that certain physical quantities (like the partition function Z) are actually topological invariants of the underlying manifold. They don't depend on the local details of the metric, only on the global topology.

The Mapping:

- **Manifold (M):** The space of all possible correlation matrices $\mathcal{C}_n$ (the "Elliptope").
- **Metric (g):** The eigenvalues Λ define the local curvature of risk.
- **Field (ϕ):** The portfolio weights w.
- **Partition Function (Z):** The Characteristic Function $\phi_S(t)$.
  - Just like $Z = \int e^{-S[\phi]} D\phi$, our CF is an integral over the Gaussian field: $\phi_S(t) = \int e^{it \sum w_i e^{Y_i}} d\mu(Y)$.

Hypothesis: The spectral coefficients A_k (derived from Z) might be related to topological invariants of the correlation structure. - Is there a "Witten Index" for portfolios? - Does the Fenton Number F behave like an Euler characteristic?

2. Phase Transitions and Knot Theory

Witten also connected Jones Polynomials (knot theory) to Chern-Simons theory. - **Question:** Can a portfolio be "knotted"? - **Scenario:** A complex long-short strategy where weights sum to 0 but leverage is high. - **Phase Cancellation:** In our Eigen-COS method, hedging is destructive interference. In knot theory, this is like resolving a crossing. - **Idea:** Maybe the "unknottability" of a portfolio corresponds to whether it can be continuously deformed into a risk-free bond without crossing a singularity (infinite risk).

3. The Path Integral Formulation

The value of a European option is:

$$V = \mathbb{E}[e^{-rT}(S_T - K)^+]$$

This expectation is exactly a **Path Integral** over the trajectories of the underlying assets. - **Feynman-Kac Formula:** Links the PDE (Black-Scholes) to the Path Integral. - **Spectral Fenton:** Provides a discrete, non-perturbative approximation to this path integral. - The GH quadrature nodes are the "classical paths" (instantons?). - The weights are the "fluctuations" around these paths.

4. Why This Matters? (The "Quantum Advantage")

If we can map Portfolio Risk to TQFT, we might be able to use: - **Witten's localization principle:** To calculate integrals exactly by summing over critical points (this is essentially what our GH quadrature does!). - **Mirror Symmetry:** Is there a "dual" portfolio that is easier to solve? (e.g., swapping correlation for volatility).

5. Future Research Directions

1. **Formalize the "Correlation Manifold":** What is the geometry of C? (It's a subset of the sphere).
2. **Define the "Risk Action":** $S_{risk}[Y] = Y^T C^{-1} Y - it \sum w_i e^{Y_i}$.
    - The stationary phase approximation of this action gives the "most likely crash scenario".
3. **Explore "Topological Risk Protection":** Can we design portfolios that are topologically protected against market crashes? (Like topological insulators in physics).

Note: This is highly speculative. Do not publish until rigorous mathematical proofs are found. But keep this as the "North Star" for the theoretical depth of the Spectral Fenton method.