Knowability

Law, Lens, Latent Structure, and Spectra

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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Abstract

We propose that the central object of scientific description is not the sample path but the law, and that observable structure is shaped jointly by latent aggregation, dynamical evolution, and the lens through which a phenomenon is measured. In this view, many operations that appear distinct in observation space arise from a smaller latent grammar. Additive hidden structure may appear as multiplicative behavior under an exponential lens, while attenuation and amplification emerge as opposite-sign regimes of the same positive dynamics. This suggests that lognormality is not the deepest object, but a canonical positive realization of a more primitive latent structure.

Within this framework, knowability is the existence of a low-complexity representation of a phenomenon under an appropriate lens. The amount of structure is summarized by spectral quality, while the full Knowledge Artifact records the Spectra: values, modes, coefficients, and dynamics that make the pattern actionable. The theory separates what is observer-dependent from what is invariant: representation and modes depend on the lens, while compression rate and structural strength admit lens-robust summaries.

This perspective unifies several ideas that are usually treated separately: the distinction between law and simulation, the emergence of multiplication from latent addition, the role of Gaussian structure as a canonical latent coordinate system, and the appearance of positive observables such as returns, intensities, gains, and attenuation factors as lens-dependent images of the same underlying pattern class. The result is a general program for understanding when reality is compressible, predictable, and therefore knowable.

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1. Introduction

Science is often practiced as if the sample path were the object. In finance, physics, and machine learning, we simulate trajectories, fit realized panels, and analyze one observed history. Yet the path is only one draw from a deeper object: the law that generates all possible observations. A method that manipulates paths may still be useful, but it is not necessarily operating at the natural level of description.

This paper advances a stronger thesis. A phenomenon is knowable when it admits a low-complexity representation under a suitable lens. The relevant object is the law; the relevant invariants are its spectral summaries; and the visible operations of the world may often be derived from a smaller latent grammar rather than taken as primitive.

The central proposal is that many observed systems can be organized by four objects:

1. a **pattern** above the noise floor,
2. a **law** governing the distribution of possible observations,
3. a **generator** governing temporal evolution, and
4. a **lens** that determines how the hidden structure becomes observable.

The extracted representation of these objects is the **Knowledge Artifact**: spectral values, spectral modes, and dynamic spectrum. Spectral quality is its most compressed invariant summary.

2. Law Over Path

Let X denote an observable process and let = Law(X) denote its law. A Monte Carlo procedure replaces by the empirical measure

$$\mu_N = \frac{1}{N} \sum_{j=1}^{N} \delta_{X^{(j)}}.$$

This is valid, but conceptually secondary. Each path X^{(j)} is only a Dirac atom in the space of possibilities. The phenomenon itself is the law. When the law admits a direct spectral representation, simulation becomes a route to an object that could have been represented natively.

This viewpoint changes the interpretation of computational difficulty. Part of the curse of dimensionality is not a property of the phenomenon itself, but a property of the representation chosen for it. A pathwise description may be high-dimensional even when the law has a low-complexity spectral description.

3. The Latent Addition Hypothesis

We propose that many visible systems are generated by a simpler latent structure:

$$Y = \sum_{i=1}^{m} Y_i, \qquad X = \kappa(Y),$$

where the Y_i are elementary latent contributions, Y is the hidden state, and is the lens that maps latent structure into observation space.

This hypothesis does not claim that everything visible adds directly. It claims something weaker and more useful: additive structure may exist in a latent coordinate system even when it is not visible in the raw observable. The right question is therefore not "does the world add?" but "is there a coordinate system in which the pattern is additive?"

If the answer is yes, much of the visible complexity may be representational rather than ontological.

4. Derived Visible Operations

The latent-additive picture naturally explains the emergence of common observable operations.

4.1 Multiplication

If the lens is exponential, then

$$X = e^Y = e^{\sum_i Y_i} = \prod_i e^{Y_i}.$$

Multiplication in observation space is therefore addition seen through the exponential lens. Positive observables such as wealth, signal power, intensities, attenuation factors, and compounded growth belong naturally to this class.

4.2 Attenuation and amplification

For a positive observable I_t governed by

$$\frac{dI_t}{dt} = a_t I_t,$$

the sign of a_t determines whether the phenomenon appears as attenuation or amplification. These are not different ontological classes; they are opposite-sign regimes of the same positive dynamics.

4.3 Convolution and aggregation

If independent latent effects add, then the law of the aggregate is formed by convolution. In this sense, convolution is not a primitive operation of a different world, but a law-level consequence of latent addition.

4.4 Composition

When systems evolve in time, generators compose local updates into global behavior. Composition is thus the dynamical counterpart of latent addition: one aggregates contributions across coordinates, the other aggregates transformations across time.

5. Gaussianity and Positive Observation

The theory suggests that Gaussianity is more fundamental than lognormality. Several arguments point in the same direction:

1. additive aggregation drives many systems toward Gaussian latent coordinates,
2. Gaussian structure is the maximum-entropy form under fixed first and second moments,
3. smooth log-densities are locally quadratic, making Gaussian approximation canonical near stable regions.

Under a positivity-preserving lens, Gaussian latent structure appears as lognormal observation. This means that the lognormal is not necessarily the deepest object. It is a canonical visible image of additive hidden structure.

In this sense, the Spectral Lognormal Distribution should be understood not merely as a finance-specific construction, but as one important positive-observable realization of a broader latent theory.

6. Knowability

A phenomenon is knowable when it admits a low-complexity representation under a suitable lens. The theory of knowability therefore asks:

1. does a real pattern exist above the noise floor?
2. which lens reveals it most clearly?
3. what is the spectral quality of the revealed pattern?
4. what dynamics govern its evolution?

The answers live at different levels:

$$\rho \ \subset \ \Sigma = \{\sigma_k\} \ \subset \ \{v_k\} \ \subset \ (\sigma_k, v_k, \lambda_k).$$

Here is the one-number summary of structural strength, is the spectral fingerprint, {v_k} are the spectral modes, and (_k, v_k, _k) is the Knowledge Artifact. The lower levels are more universal; the higher levels are more informative.

The key distinction is therefore:

- **lens-dependent**: the modes and the visible representation,
- **lens-robust**: the existence of structure and its compressibility,
- **absolute ideal**: the best attainable structural description over all admissible lenses.

7. Implications

7.1 For simulation

Monte Carlo remains correct, but is often wasteful. When a law is spectrally representable, direct law-level methods dominate pathwise sampling in interpretability and often in efficiency.

7.2 For scientific realism

Different observers may use different lenses and recover different visible structures while still agreeing on the existence and strength of the underlying pattern. Science is objective in its invariant summaries and perspectival in its representations.

7.3 For cross-domain unification

The same latent grammar may underlie quantities that appear unrelated at first sight: returns, signal intensities, attenuation fields, portfolio values, and other positive observables. A domain difference may therefore be a difference of lens rather than a difference of mathematics.

8. Limits and Conjectural Status

This paper states a program, not a completed theorem catalog. Several claims here should be read as hypotheses or organizing principles rather than fully settled results.

In particular:

1. not all systems need admit an additive latent coordinate system,
2. not all latent laws are Gaussian or approximately Gaussian,
3. heavy tails, strong dependence, multimodality, and phase transitions may require a richer grammar,
4. the class of admissible lenses must be specified before any universality claim can be made precise.

The value of the framework is therefore not that it eliminates complexity, but that it proposes a disciplined hierarchy: first seek the law, then the lens, then the spectral representation, and only then the pathwise simulation.

9. Formalization Boundary

If this paper is to become more than a programmatic essay, it needs a precise formal boundary. Not every sentence above belongs in Lean. Some statements are mathematical theorems; others are interpretive claims about what those theorems mean. The right strategy is to formalize the structural backbone first and leave the philosophical envelope outside the proof assistant until the object layer is stable.

9.1 Definitions that should be fixed first

The first step is to define the minimal object language.

1. **Latent state.** A hidden variable Y living in an additive space.
2. **Lens.** A map from latent space to observation space.
3. **Positive lens.** A lens whose image is nonnegative, with the exponential lens (y) = exp(y) as the canonical example.
4. **Latent additive model.** A family Y = \sum_i Y_i of hidden contributions together with a lens X = (Y).
5. **Law-level object.** A probability law on the observation space, distinct from any single sample path.
6. **Knowledge Artifact.** A representation consisting of spectral values, modes, and dynamics once a lens has been chosen.

Without these definitions, the current thesis risks mixing ontology, notation, and application. With them, the theory becomes formalizable.

9.2 The first theorem ladder

The first Lean layer should avoid the universal claims and focus on theorems that are both true and structurally central.

Theorem A (Exponential lens converts addition to multiplication).

$$\exp\left(\sum_i y_i\right) = \prod_i \exp(y_i).$$

This is elementary, but it is the algebraic hinge of the entire positive-observable story. It turns latent addition into visible multiplication.

Theorem B (Gaussian latent state induces lognormal observation).

If Y is Gaussian and X = exp(Y), then X is lognormal.

This gives the formal bridge between canonical latent structure and the founding positive-observable family.

Theorem C (CLT motivates Gaussian latent coordinates).

For sums of sufficiently regular, weakly dependent latent contributions, normalized aggregation converges to a Gaussian law.

This should not be re-proved from scratch inside this project. The correct move is to rely on a standard central limit theorem boundary and then formalize the consequences for the latent-additive framework.

Theorem D (Positive dynamics sign theorem).

For a positive observable satisfying

$$\frac{dI_t}{dt} = a_t I_t,$$

the sign of a_t determines attenuation versus amplification. The explicit solution

$$I_t = I_0 \exp\left(\int_0^t a_s \, ds\right)$$

shows that weakening and strengthening are opposite-sign regimes of one dynamical class.

Theorem E (Path is not the law).

This is not a theorem in the same sense. The empirical measure

$$\mu_N = \frac{1}{N} \sum_j \delta_{X^{(j)}}$$

is mathematically precise, but the claim that "the law is the natural object and the path is secondary" is a modeling thesis, not a proposition to prove in Lean. It should remain a formalized definition plus an interpretive statement.

9.3 What should remain conjectural for now

Several parts of the paper should remain explicitly outside the first formalization wave:

1. the universality claim that most visible operations derive from latent addition,
2. the claim that Gaussianity is the unique or deepest canonical latent law,
3. the existence and uniqueness of an absolute best lens across broad lens classes,
4. any metaphysical reading such as "the universe is fundamentally computation" or "all operations reduce to one primitive algebra."

These may eventually become theorem targets, but only after the definition layer is stabilized and the admissible classes are specified precisely.

9.4 Practical Lean program

A realistic Lean roadmap would therefore be:

1. create a small foundational file for latent states and lenses,
2. prove the algebraic theorems around the exponential lens,
3. state the Gaussian-to-lognormal bridge precisely,
4. import or cite a standard CLT result as the boundary theorem,
5. prove the sign-regime theorem for positive dynamics,
6. only then return to spectral quality, Knowledge Artifacts, and lens-robust summaries.

The key design principle is simple: formalize the mathematical spine, not the rhetoric. If the spine compiles, the broader philosophical reading gains legitimacy. If it does not, the paper is still only an intuition.

10. Thesis

The central thesis can be stated compactly:

> Reality may be generated by a small latent grammar while appearing as a rich visible algebra. When a phenomenon admits a low-complexity law under the right lens, it becomes knowable.

Under this view, the deepest question is not whether the world is additive, multiplicative, or computational in its visible form. The deeper question is whether there exists a latent coordinate system in which the pattern becomes simple enough to compress, evolve, and observe. Knowability is the study of that possibility; Spectra are the language in which that possibility is resolved.