The One Behind Everything

How one object manifests as spectra at different depths of resolution

A generator-first, resolution-family view of hierarchical spectral emergence

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Executive Summary (Non-Technical)

Many scientific pictures still assume that once we have found a useful spectral description of a system, we have reached the final descriptive layer. This note asks a deeper question: **what if the spectral object we see is not the final layer, but one resolved face of a deeper single object?**

The motivating intuition comes from self-similar and fractal-like phenomena. A very small local rule can be applied again and again, and after many iterations a larger pattern appears. The visible pattern may have its own coherent modes, frequencies, or effective spectral structure. In that case, the spectral description is not the end of the story. It is one level inside a hierarchy.

The paper's proposal is that what we call "levels" may live mostly in the **representation**, not in the ontology. A micro-level operator generates an intermediate latent pattern. That pattern may then admit its own spectral description, and that higher spectral object may itself be the input to yet another representational or observational layer. This creates the possibility of **spectra behind spectra** rather than one flat spectral surface.

The same framework also suggests caution about named boundaries. Human observers classify, label, and draw borders around things. Some of those borders track real effective structure. Others may be only useful human partitions rather than primitive natural joints.

The note does **not** claim that every repeated rule is linear, that every fractal is best understood spectrally, or that every spectral model secretly hides an infinite hierarchy underneath it. The narrower claim is that for an important class of iterated local operators, the emergent large-scale pattern can be derived directly from the operator's action, and the resulting pattern may carry a second-level spectral description of its own.

This matters because it shifts the research question. Instead of asking only how to observe a latent spectral object, we can also ask **how that latent spectral object was generated in the first place**, which probes can access it, and whether emergence is the appearance of a new thing or the appearance of a new law-bearing face of the same thing. The present note is therefore generator-first, not observation-first.

Abstract

We propose a generator-first extension of the repo's broader spectral program. The starting question is not how a latent state is quantized into a discrete observable, but how a structured latent state may itself arise from repeated application of a local operator. The strongest ontological version is that there may be one underlying object $\mathcal{U}$, observed through a family of resolution maps $R_s(\mathcal{U})$, with spectral summaries $\Sigma_s(\mathcal{U})$ depending on depth s. In the simplest linear setting,

$$u_{n+1} = Tu_n, \qquad u_n = T^n u_0,$$

and if T admits a spectral decomposition then the emergent large-scale pattern is controlled by the operator's dominant modes. In a convolutional setting,

$$u_{n+1} = K * u_n,$$

so in Fourier space,

$$\widehat{u_n}(\xi) = \widehat{K}(\xi)^n \widehat{u_0}(\xi).$$

Thus the macro-pattern is directly computable from repeated micro-action.

The deeper proposal is hierarchical. A micro-operator may generate an intermediate latent pattern $u^{(1)}$, that pattern may admit its own spectral summary $S^{(1)}$, and that summary may itself be the object acted on by a higher-level generator or observation map. We call this possibility **spectra behind spectra**. The phrase means that a visible spectral organization may be only one resolved face of a deeper object rather than the primitive descriptive floor.

This paper is an early theory note. It defines the object language for hierarchical spectral generation, introduces a one-object-many-resolutions and one-object-many-probes framework, distinguishes this view from the observation-first quantized-observation line, and proves first mode-selection results for iterated convolution, including static, oscillatory, and band-level dominant regimes. It also formulates further theorem candidates around iterated linear operators, convolution semigroups, and self-similar fixed-point operators on measures.

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1. Introduction

One of the strongest intuitions in the repo's spectral program is that smooth latent structure is often best described by modes, coefficients, and dynamical spectra. But this leaves open a prior question: where did that latent spectral object come from, and are the different "levels" we see genuinely different objects or only different resolved images of one object?

Fractal and self-similar phenomena make the question unavoidable. A small pattern is repeated. A local rule is applied again and again. After enough iterations, a larger pattern becomes visible. The visible shape may then admit its own low-dimensional summary, its own dominant frequencies, or its own spectral compression law. In that case the correct scientific picture is not a single spectral layer, but a hierarchy:

$$\text{local operator} \longrightarrow \text{emergent latent pattern} \longrightarrow \text{spectral summary.} \tag{1}$$

The central claim of this note is that, in a broad class of systems, the emergent macro-pattern can be derived directly from the iterated micro-operator, and that the macro-pattern may itself carry a second-level spectral organization. The stronger philosophical reading is that the hierarchy may live in the representation rather than in the ontology, and that emergence may be the appearance of a new effective law rather than a new substance.

1.1 Core ideas in order

The paper's core ideas are easiest to understand in the following order.

1. There may be **one underlying object** rather than many ontological layers.
2. What we call "levels" may be outputs of different **resolution families** applied to that object.
3. What is knowable depends on the available **probe family**, not only on the object itself.
4. **Emergence** occurs when a resolved face of the object supports a new effective law.
5. Many named separations may be **observer-imposed partitions** imposed on the resolved image rather than primitive joints in the underlying reality.

The slogan form is:

$$\text{one object} + \text{many resolutions} + \text{many probes} \implies \text{many effective lawful faces.} \tag{1aa}$$

1.2 Working principles and assumptions

At this stage, the paper does **not** introduce a new independent axiom system. Instead, it separates four roles that are often blurred together.

(i) Background mathematics. All formal statements in the paper live inside ordinary mathematical frameworks: function spaces, operators, measures, semigroups, and spectral decompositions.

(ii) Definitions. Objects such as resolution families R_s, probe-relative knowability $\mathfrak{K}_{p,s}$, lawful closure, and partition maps Π_o are definitions. They do not assert that the world must have that form; they define the language in which the paper asks its questions.

(iii) Standing assumptions. When we prove theorems, we add explicit assumptions such as semigroup structure, spectral gaps, dominant bands, regularity, or reconstructibility. Those are conditional hypotheses, not ontological commitments.

(iv) Guiding unification principle. The strongest philosophical stance of the paper is best read not as an axiom but as a working principle:

Common-source principle: when several apparent levels are observed, first ask whether

they can be modeled as resolved faces of a common source object before postulating distinct ontological layers. (1ab)

This principle is not itself proved like a theorem. It is a disciplined modeling preference that organizes the paper's definitions and motivates the theorem line.

(v) Reduction, manifestation, objecthood, and naming. One conceptual confusion the paper wants to avoid is treating reduction and manifestation as opposites. They are often two sides of the same operation. When a complex object is resolved into a spectral or other structured representation, one may indeed lose micro-detail. But one may simultaneously gain legible organization, dominant modes, lawful coarse variables, or a stable boundary structure that was not easily visible in the unreduced description. In that precise sense, representation can be both compressive and form-revealing.

The paper therefore separates four roles:

1. **Compression** reduces the effective degrees of freedom.
2. **Manifestation** makes a form legible at a given probe / resolution level.
3. **Objecthood** occurs when that form is stable and supports lawful closure.
4. **Naming** is the observer's conceptual or reporting act of attaching a label.

These roles should not be collapsed into one another. Compression is not yet objecthood. A manifested form is not automatically an ontologically primitive thing. And naming by itself does not prove that a stable joint has been found.

This is also why manifestation is selection-sensitive. A laptop, a volatility surface, a weather regime, or even a theorem statement may all count as manifested objects in the present language. The difference is not that one is a "real thing" and another is not. The difference is which probe, resolution, and distinction regime has been chosen, and what has been stabilized strongly enough to count as one thing.

The intended schematic picture is

$$\mathcal{U} \xrightarrow{\text{resolution / probe}} z \xrightarrow{\text{structured representation}} \Sigma(z) \xrightarrow{\text{stable closure / partition}} \text{manifest form.} \tag{1ac}$$

Here the manifest form is not created from nothing. It is the way a deeper object becomes legible under a disciplined reduction. Some detail is discarded, but some structure is made explicit.

This yields the paper's current answer to the question "when does something take form?":

$$\text{form} \approx \text{compressible resolved face} + \text{ stability} + \text{ lawful closure.} \tag{1ad}$$

In words: a thing takes object-like form when a deeper complex object becomes representationally compressible, remains stable under admissible perturbations, and carries a law-bearing effective dynamics or effective relation at the level where it is being described.

This is why the framework treats objectivity as neither purely subjective nor necessarily primitive. Something becomes objectively separated not merely when it is named, but when the corresponding resolved form persists across admissible probes, representations, and small perturbations. In many cases, the best way to think about an observed object is therefore quotient-like: not as one privileged microstate, but as a stable equivalence class of many micro-configurations that collapse to the same effective face.

The same doctrine should be extended beyond ordinary physical objects. A mathematical statement, a formal theory, or a proof object can also be read as a manifested form: not because it is identical to a table, a storm front, or a spectrum, but because it too becomes legible only after a disciplined representational selection has fixed relevant distinctions and suppressed others. In that sense, the paper's language of manifestation is intended to cover both physical and theoretical forms, even when their closure conditions are different.

This observation matters because it blocks a false dichotomy. Formalization is not simply external notation pasted onto a finished thought. It is one of the ways a thought takes stable object-like form. That is why reduction and manifestation remain paired even in the theoretical case: to formalize is to lose some freedom while gaining a more stable and reusable face. The dedicated follow-up note for this branch is meta_theory_mathematical_manifestation.

The same extension also clarifies the self-description question. If observation belongs to the One rather than arriving from outside it, then self-description should not be treated as impossible in principle. The One may manifest partial faces, models, or symbolic traces of itself from within itself. What should not be assumed automatically is exhaustive internal closure. So the doctrine-level distinction is:

$$\text{self-manifestation is natural} \quad \neq \quad \text{complete self-coincident description.} \tag{1ad'}$$

This is the clean bridge from the ontology paper to the Goedel / self-description line developed in meta_theory_mathematical_manifestation.

(vi) Static form, process-form, and objecthood. The paper should also avoid identifying form with immobility. Some forms are statically stable: a rigid shape, a fixed boundary, a persistent coarse object. But other forms are stable only dynamically: a vortex, a flame front, a beating heart, an ecological cycle, a developmental program, or a law-governed transformation. In such cases, what persists is not one frozen microstate but a recognizable organized pattern through change.

This suggests a second distinction:

1. **Static form**: the resolved shape itself remains approximately fixed.
2. **Process-form**: the pattern of transformation remains recognizable and lawfully organized even though the instantaneous state changes.
3. **Objecthood**: either of the above supports enough stability and closure to be treated as a genuine effective object at the chosen level.

So not every form must be motionless, but every serious form must be stable in some relevant sense. The stability may be geometric, dynamical, developmental, or statistical. What should not count as form is a purely accidental fluctuation with no persistent organization under admissible perturbations.

In the language of the paper, this means that object-like form can arise in at least two ways:

$$\text{static form} \quad \text{or} \quad \text{stable process-form.} \tag{1ae}$$

The first is shape-like. The second is law-like. Both belong naturally inside the one-object picture, and both can be read as manifestations of a deeper object under an appropriate probe / resolution stack.

(vii) Chaos as failure of stable manifestation. The same framework also suggests a careful reading of chaos. The paper does **not** identify chaos with the absence of spectrum. Chaotic systems may still admit Fourier descriptions of observables, Lyapunov spectra, Koopman spectra, transfer-operator structure, invariant measures, or other spectral summaries. The better statement is that chaos often marks the failure of a sufficiently stable, low-complexity, lawfully closed manifestation at the chosen level of description.

This yields a useful layered distinction:

1. **Latent chaos** is genuine dynamical sensitivity in the underlying system.
2. **Observational chaos** is instability amplified by lens choice, thresholding, quantization, or symbolic coding.

3. **Closure-failure chaos** occurs when the currently chosen resolved variables do not support a stable effective law.

So a system may look chaotic at one level while still carrying stronger organization at another. In this framework, chaos is often a sign that one of three things is true:

1. the underlying dynamics is genuinely sensitive,
2. the chosen probe / partition is amplifying instability,
3. or the current level is not the right object on which closure should be sought.

This is why the theory treats chaos not only as a property to classify, but also as a diagnostic signal. If a level looks chaotic, one should ask whether the right move is to change probe, change resolution, enlarge the state space, or change the object of description from trajectories to measures, operators, or spectral summaries. In the observer-relative case, the clean doctrine-level slogan is:

$$\text{chaos} \approx \text{failure of stable manifestation at the chosen level.} \tag{1af}$$

This does not abolish classical chaos theory. It re-situates it. Chaos theory remains the mathematics of what happens when deterministic evolution does not yield simple, stable, short-horizon manifestation at the level being observed, even though a deeper or different representational level may still exhibit strong structure.

1.3 One object, many resolutions

The strongest version of the present idea is not that reality must be built from endlessly many separate layers. A cleaner mathematical picture is that there is one underlying object

$$\mathcal{U}, \tag{1a}$$

and what we call "levels" are the outputs of a family of resolution maps

$$R_s(\mathcal{U}), \qquad s \in \mathcal{S}. \tag{1b}$$

The parameter s may encode scale, observational depth, coarse-graining strength, iteration count, or representation budget. The spectral summary at depth s is then

$$\Sigma_s(\mathcal{U}) := \mathrm{Spec}(R_s(\mathcal{U})). \tag{1c}$$

In this language, distinct spectral pictures need not imply distinct ontological objects. They may be different resolved faces of the same $\mathcal{U}$.

When a semigroup or compatibility law exists,

$$R_{s+t} = R_t \circ R_s, \tag{1d}$$

the family becomes especially tractable. One can then ask how the spectral summaries Σ_s evolve with depth, which modes persist, which disappear, and when a stable effective spectral organization emerges.

1.4 Canonical resolution families

The abstract formula R_s becomes useful only when one can point to concrete resolution families. Three families are especially natural in the present paper.

(i) Iteration depth as resolution. If a local operator T is repeatedly applied, then the effective object at depth n is

$$R_n(\mathcal{U}) := T^n \mathcal{U}. \tag{1e}$$

This is the generator-first case studied below. The question is how the spectral summary of $T^n\mathcal{U}$ changes with n, and which modes survive repeated local action.

(ii) Coarse-graining or smoothing depth. If K_s is a family of smoothing kernels indexed by scale s, then

$$R_s(\mathcal{U}) := K_s * \mathcal{U}. \tag{1f}$$

This is the natural language for blur, averaging, diffusion, and observational resolution. In favorable semigroup settings, one has

$$K_{s+t} = K_s * K_t, \qquad R_{s+t} = R_t \circ R_s. \tag{1g}$$

The associated spectral summaries Σ_s reveal which frequencies remain visible as one moves to coarser scales.

(iii) Representation-budget depth. If $\mathcal{U}$ already has a spectral decomposition, one may define

$$R_N(\mathcal{U}) := \sum_{|k| \le N} A_k v_k \tag{1h}$$

or a more general truncation/projection rule determined by a budget parameter N. Here the "depth" is not physical scale but representational access. The resulting family asks how much of the underlying object is visible when only a bounded spectral budget is retained.

These three families already suggest the unifying picture of the paper:

$$\text{same object } + \text{ different resolution family } \Longrightarrow \text{ different effective spectral pictures.} \tag{1i}$$

The important point is that the spectral image may change with the resolution family even when the underlying object $\mathcal{U}$ does not.

1.5 One object, many probes

Resolution is only half of the epistemic picture. The other half is the probe itself: what couples to the object, what is registered, and whether the act of probing changes the state.

Crucially, the probe need not be treated as something outside the One. In the strongest reading, probing is one internal coupling process by which one part of the underlying whole becomes legible to another.

Let $p \in \mathcal{P}$ denote a probe type. The probe may induce a back-action map

$$B_p(\mathcal{U}), \tag{1j}$$

which reduces to the identity in the ideal passive limit. After choosing a resolution depth s, the probe-specific observable coordinate is

$$y_{p,s} := \kappa_p(R_s(B_p(\mathcal{U}))). \tag{1k}$$

If a reporting or quantization layer is present, then

$$c_{p,s} := Q_p(y_{p,s}). \tag{1l}$$

This simple extension already clarifies an important point: different probes need not reveal different ontological layers. They may reveal different **accessible aspects** of the same object. A photon probe, for example, may privilege electromagnetic couplings and therefore reveal only the structures that are visible through that interaction channel.

The natural knowability object is therefore probe- and depth-indexed. One may write

$$\mathfrak{K}_{p,s}(\mathcal{U}) := \{F(\mathcal{U}) : F \text{ is reconstructible from } y_{p,s}\}. \tag{1m}$$

In this language, knowability is not absolute. It depends on which probe family is available and at what resolution depth the object is interrogated.

1.6 Emergence as lawful closure

The present framework also suggests a clean way to speak about emergence. Let

$$z_s := C_s(\mathcal{U}) \tag{1n}$$

be a collective or coarse-grained variable extracted from the underlying object. We say that the resolution level s exhibits **lawful closure** if there exists an effective law F_s such that

$$z_{s,t+1} \approx F_s(z_{s,t}) \tag{1o}$$

in discrete time, or

$$\partial_t z_s \approx F_s(z_s) \tag{1p}$$

in continuous time.

This gives a precise interpretation of emergence inside the one-object picture:

$$\text{emergence} \neq \text{necessarily new ontology}, \tag{1q}$$

$$\text{emergence} = \text{a new law-bearing resolved face of the same object}. \tag{1r}$$

This is the right way to read familiar examples such as forest-level behavior, turbulence statistics, or collective phase labels. The large-scale object need not be a second substance. But it may support a new effective law that is not naturally visible at the level of isolated micro-components.

1.7 Compatibility criteria

The preceding definitions become most useful when one can say when two representational choices belong to the same theoretical picture.

Resolution compatibility. Two resolution families (R_s) and $(\widetilde{R}_t)$ are compatible if there exists a comparison map $\Gamma_{s,t}$ such that

$$\Gamma_{s,t}(R_s(\mathcal{U})) \approx \widetilde{R}_t(\mathcal{U}) \tag{1s}$$

for the class of objects under study. This means the two families are different resolved views of the same source object rather than unrelated constructions.

Probe compatibility. Two probes p and q are compatible at depth s if their knowability sets overlap in a nontrivial and stable way:

$$\mathfrak{K}_{p,s}(\mathcal{U}) \cap \mathfrak{K}_{q,s}(\mathcal{U}) \neq \emptyset. \tag{1t}$$

The stronger the overlap, the stronger the evidence that the probes reveal common structure rather than disconnected reporting artifacts.

Emergent-object criterion. A resolved face $z_s = C_s(\mathcal{U})$ deserves to be treated as an effective object only when it supports lawful closure and remains stable under small perturbations of the relevant probe or resolution choice. Symbolically, if

$$z_{s,t+1} \approx F_s(z_{s,t}) \tag{1u}$$

and the induced law remains approximately invariant under admissible changes of representation, then the face is not merely a transient description but a candidate emergent object.

These are not theorems; they are criteria for when the paper's language should be taken seriously as a description of one object with many lawful faces.

Two elementary formal consequences are already immediate.

Proposition 1.1 (Common reconstructibility implies knowability overlap). Let $p, q \in \mathcal{P}$ be two probes, and let s, t be two resolution depths. Suppose there exist reconstruction maps A_p, A_q and a source functional F such that

$$A_p(y_{p,s}) = F(\mathcal{U}) = A_q(y_{q,t}). \tag{1v}$$

Then

$$F(\mathcal{U}) \in \mathfrak{K}_{p,s}(\mathcal{U}) \cap \mathfrak{K}_{q,t}(\mathcal{U}). \tag{1w}$$

In particular, the two probe-resolution pairs share a nontrivial knowability overlap.

Proof. By definition, $\mathfrak{K}_{p,s}(\mathcal{U})$ contains exactly those functionals of $\mathcal{U}$ that are reconstructible from $y_{p,s}$. Since $A_p(y_{p,s}) = F(\mathcal{U})$, the functional $F(\mathcal{U})$ belongs to $\mathfrak{K}_{p,s}(\mathcal{U})$. The same argument with A_q shows that $F(\mathcal{U}) \in \mathfrak{K}_{q,t}(\mathcal{U})$. Hence (1w) holds. □

Proposition 1.2 (Stable partitions inherited from lawful closure). Let $z_s = C_s(\mathcal{U})$ be a resolved face that exhibits lawful closure, and assume a partition map factors through z_s:

$$\Pi_o(R_s(\mathcal{U})) = \psi(z_s) \tag{1x}$$

for some map ψ. Assume further that:

1. z_s is stable under admissible perturbations of probe and resolution, in the sense that

$$\|\tilde{z}_s - z_s\| < \varepsilon, \tag{1y}$$

2. ψ is locally constant on the ε-ball around z_s, i.e.

$$\|z - z_s\| < \varepsilon \implies \psi(z) = \psi(z_s). \tag{1z}$$

Then the resulting partition label is invariant under those admissible perturbations:

$$\Pi_o(\widetilde{R}_s(\mathcal{U})) = \Pi_o(R_s(\mathcal{U})). \tag{1aa0}$$

Proof. By assumption, the perturbed resolved face $\tilde{z}_s$ remains within ε of z_s. Since ψ is locally constant on that neighborhood,

$$\psi(\tilde{z}_s) = \psi(z_s). \tag{1aa1}$$

Using the factorization through z_s, this is exactly the claimed label invariance:

$$\Pi_o(\widetilde{R}_s(\mathcal{U})) = \psi(\tilde{z}_s) = \psi(z_s) = \Pi_o(R_s(\mathcal{U})). \tag{1aa2}$$

□

This proposition gives a minimal sufficient condition for when a named boundary is supported by a stable effective face rather than by a fragile reporting artifact.

The converse direction is also useful in practice: instability is evidence against treating a named boundary as a stable joint of the object.

Proposition 1.3 (Label instability obstructs stable-face interpretation). Let $z_s = C_s(\mathcal{U})$ be a candidate resolved face. Suppose there exists a sequence of admissible perturbations with perturbed faces $\tilde{z}_s^{(n)}$ such that

$$\|\tilde{z}_s^{(n)} - z_s\| \to 0 \qquad \text{as } n \to \infty, \tag{1aa3}$$

but

$$\Pi_o(\widetilde{R}_s^{(n)}(\mathcal{U})) \neq \Pi_o(R_s(\mathcal{U})) \tag{1aa4}$$

for every n. Then there is no factorization

$$\Pi_o(R_s(\mathcal{U})) = \psi(z_s) \tag{1aa5}$$

through a stable effective face z_s for which ψ is locally constant at z_s.

Proof. Assume such a factorization existed with z_s stable and ψ locally constant at z_s. Then there would exist $\varepsilon > 0$ such that

$$\|z - z_s\| < \varepsilon \implies \psi(z) = \psi(z_s). \tag{1aa6}$$

Since $\|\tilde{z}_s^{(n)} - z_s\| \to 0$, for all sufficiently large n we would have $\|\tilde{z}_s^{(n)} - z_s\| < \varepsilon$, and therefore

$$\psi(\tilde{z}_s^{(n)}) = \psi(z_s). \tag{1aa7}$$

Using the factorization, this implies

$$\Pi_o(\widetilde{R}_s^{(n)}(\mathcal{U})) = \Pi_o(R_s(\mathcal{U})) \tag{1aa8}$$

for all sufficiently large n, contradicting (1aa4). □

This gives a first formal version of the cautionary principle:

$$\text{small representation changes causing label flips} \implies \text{do not treat the label boundary as a stable ontological joint} \tag{1aa9}$$

Proposition 1.4 (Common-face mediation forces cross-probe agreement). Let $p, q \in \mathcal{P}$ be probes at depths s, t, and suppose there exists an effective face $z = C(\mathcal{U})$ together with decoding maps $H_{p,s}$ and $H_{q,t}$ such that

$$H_{p,s}(y_{p,s}) = z = H_{q,t}(y_{q,t}). \tag{1ab0}$$

If a label is determined at the level of that face by

$$\Lambda(\mathcal{U}) = \psi(z), \tag{1ab1}$$

then the two probes induce the same label:

$$\psi(H_{p,s}(y_{p,s})) = \psi(H_{q,t}(y_{q,t})) = \Lambda(\mathcal{U}). \tag{1ab2}$$

Proof. Apply ψ to both sides of (1ab0). Since both decoded probe outputs equal the same effective face z, both yield the same label $\psi(z)$, which is exactly $\Lambda(\mathcal{U})$. □

Corollary 1.5 (Persistent probe disagreement signals missing common closure). In the setting of Proposition 1.4, if two probes produce different labels for the same underlying object,

$$\psi(H_{p,s}(y_{p,s})) \neq \psi(H_{q,t}(y_{q,t})), \tag{1ab3}$$

then at least one of the following must fail:

1. both probes decode to the same effective face,
2. the label is a function of that face alone,
3. the decoding maps are valid on the current regime.

Proof. If all three conditions held, Proposition 1.4 would force equality of the two labels, contradicting (1ab3). □

1.8 Observer-imposed partitions and named boundaries

The one-object picture also changes how we should think about classification. Human observers are strongly inclined to name things, draw boundaries, and treat those named regions as separate entities. That habit is often useful, but it should not be mistaken for an automatic law of nature.

In the present language, a named class or boundary may arise from a partition map

$$\Pi_o : R_s(\mathcal{U}) \to \mathcal{C}_o, \tag{1ab}$$

where o indexes an observer-, reporting-, or model-imposed classification scheme. Human classification is one important special case, but not the only one. The crucial point is that

$$\Pi_o \text{ may be useful without being ontologically primitive.} \tag{1ac}$$

This gives a cleaner reading of separation:

$$\text{observed separateness} \neq \text{necessarily fundamental separateness.} \tag{1ad}$$

Some boundaries correspond to stable lawful closure and therefore deserve to be treated as real effective objects. Others may only reflect the compression, naming, reporting, or probe-design choices of the observer.

This is one reason the paper keeps ontology, probe, resolution, and reporting distinct. A separation may enter at any of those layers, and the fact that humans draw a border does not by itself prove that nature contains a primitive joint at exactly that place.

Two tiny models make the doctrine more concrete.

Example 1.6 (Two probes, one face, one label). Let the underlying object be $\mathcal{U} = (a, b) \in \mathbb{R}^2$, and let the effective face be the sum

$$z = C(\mathcal{U}) = a + b. \tag{1ab4}$$

Define two probes by

$$y_p = a + b, \qquad y_q = 2a + 2b. \tag{1ab5}$$

Then the decoding maps

$$H_p(y_p) = y_p, \qquad H_q(y_q) = \frac{1}{2} y_q \tag{1ab6}$$

recover the same face z. If the observer-imposed label is

$$\Lambda(\mathcal{U}) = \mathbf{1}_{\{z \geq \tau\}}, \tag{1ab7}$$

then both probes necessarily agree on the label. This is the simplest concrete instance of Proposition 1.4: the agreement comes from the shared face, not from the probes being identical.

Example 1.7 (A fragile threshold boundary). Let $\mathcal{U} = x \in \mathbb{R}$, let the resolved face be $z = x$, and define the partition

$$\Pi_o(x) = \mathbf{1}_{\{x \geq 0\}}. \tag{1ab8}$$

Now consider admissible perturbations $\tilde{x}^{(n)} = -1/n$ around the base point $x = 0$. Then

$$\tilde{x}^{(n)} \to 0, \qquad \Pi_o(\tilde{x}^{(n)}) = 0 \neq 1 = \Pi_o(0). \tag{1ab9}$$

So arbitrarily small perturbations flip the label. In the language of Proposition 1.3, this boundary cannot be defended as a stable ontological joint at the point $x = 0$. It is a useful reporting boundary, but not a robust structural split there.

The next observation is the positive companion to this fragility example: once there is margin, the same boundary becomes stable.

Proposition 1.8 (Margin implies partition stability). Let $z = C_s(\mathcal{U})$ take values in a normed space, and let the partition be defined by a real score function g:

$$\Pi_o(R_s(\mathcal{U})) = \mathbf{1}_{\{g(z) \geq 0\}}. \tag{1ac0}$$

Assume g is L-Lipschitz near z, and suppose the current point has positive margin

$$g(z) \geq m > 0. \tag{1ac1}$$

Then every admissible perturbation $\tilde{z}$ satisfying

$$\|\tilde{z} - z\| < \frac{m}{L} \tag{1ac2}$$

preserves the label:

$$\Pi_o(\widetilde{R}_s(\mathcal{U})) = \Pi_o(R_s(\mathcal{U})). \tag{1ac3}$$

Proof. By Lipschitz continuity,

$$|g(\tilde{z}) - g(z)| \le L\|\tilde{z} - z\| < m. \tag{1ac4}$$

Hence

$$g(\tilde{z}) > g(z) - m \ge 0, \tag{1ac5}$$

so $\tilde{z}$ remains on the same side of the threshold and the indicator label does not change. □

This gives a useful rule of thumb for the paper's ontology language:

$$\text{no margin} \implies \text{fragile boundary}, \qquad \text{positive margin+regular score} \implies \text{local boundary stability}. \tag{1ac6}$$

Corollary 1.9 (Shared margin yields robust cross-probe agreement). In the setting of Proposition 1.8, suppose two probes produce decoded faces $\tilde{z}_p$ and $\tilde{z}_q$ for the same underlying object, and assume there exists a common reference face z with

$$g(z) \ge m > 0, \qquad \|\tilde{z}_p - z\| < \frac{m}{L}, \qquad \|\tilde{z}_q - z\| < \frac{m}{L}. \tag{1ac7}$$

Then both decoded probes induce the same label as the reference face:

$$\Pi_o(\widetilde{R}_s^{(p)}(\mathcal{U})) = \Pi_o(R_s(\mathcal{U})) = \Pi_o(\widetilde{R}_s^{(q)}(\mathcal{U})). \tag{1ac8}$$

Proof. Apply Proposition 1.8 first to $\tilde{z}_p$ and then to $\tilde{z}_q$. Each decoded face lies inside the same stability radius m/L around z, so each preserves the reference label. Therefore the two probes agree with the reference label and hence with each other. □

This is the first robust multi-probe version of the doctrine:

$$\text{shared face} + \text{sufficient margin} \implies \text{stable cross-probe label agreement}. \tag{1ac9}$$

The next step is to compress this into one quantitative notion.

Definition 1.10 (Local graded jointness). Fix a resolved face $z = C_s(\mathcal{U})$, a binary observer-partition induced by a real score function g,

$$\Pi_o(R_s(\mathcal{U})) = \mathbf{1}_{\{g(z) \ge 0\}}, \tag{1ad0}$$

and an admissible family $\mathcal{A}(z)$ of alternative faces around z. The family $\mathcal{A}(z)$ may include probe-induced perturbations, resolution changes, and closure-compatible effective-face perturbations, depending on the regime under study.

Define the local signed margin and admissible radius by

$$m_o(z) := |g(z)|, \qquad \Delta_{\mathcal{A}}(z) := \sup_{\tilde{z} \in \mathcal{A}(z)} \|\tilde{z} - z\|. \tag{1ad1}$$

If g is L-Lipschitz on a neighborhood containing $\mathcal{A}(z)$, define the **local graded jointness reserve**

$$\mathfrak{J}_o(z;\mathcal{A}) := m_o(z) - L\,\Delta_{\mathcal{A}}(z), \tag{1ad2}$$

and the normalized **local graded jointness score**

$$\mathbf{J}_o(z;\mathcal{A}) := \frac{[\mathfrak{J}_o(z;\mathcal{A})]_+}{m_o(z) + L\,\Delta_{\mathcal{A}}(z)} \in [0,1]. \tag{1ad3}$$

Interpretation: $\mathfrak{J}_o > 0$ means that the current label has more margin than the full admissible perturbation budget can destroy. The closer $\mathbf{J}_o$ is to 1, the more the boundary behaves like a robust local joint relative to that admissible family.

Proposition 1.11 (Positive graded jointness implies label invariance). In the setting of Definition 1.10, if

$$\mathfrak{J}_o(z;\mathcal{A}) > 0, \tag{1ad4}$$

then every admissible face $\tilde{z} \in \mathcal{A}(z)$ has the same partition label as z:

$$\Pi_o(\tilde{z}) = \Pi_o(z). \tag{1ad5}$$

Proof. For every $\tilde{z} \in \mathcal{A}(z)$, Lipschitz continuity gives

$$|g(\tilde{z}) - g(z)| \le L\|\tilde{z} - z\| \le L\,\Delta_{\mathcal{A}}(z). \tag{1ad6}$$

Since $m_o(z) = |g(z)| > L\,\Delta_{\mathcal{A}}(z)$, the perturbation cannot cross zero. Hence

$$\operatorname{sign}(g(\tilde{z})) = \operatorname{sign}(g(z)), \tag{1ad7}$$

which is exactly the label invariance claim. □

Corollary 1.12 (Positive graded jointness implies robust cross-probe agreement). Suppose a probe family $\mathcal{P}_0$ yields decoded faces $\tilde{z}_p$, and assume

$$\tilde{z}_p \in \mathcal{A}(z) \qquad \text{for all } p \in \mathcal{P}_0. \tag{1ad8}$$

If $\mathfrak{J}_o(z;\mathcal{A}) > 0$, then all probes in $\mathcal{P}_0$ induce the same label:

$$\Pi_o(\tilde{z}_p) = \Pi_o(z) = \Pi_o(\tilde{z}_q) \qquad \text{for all } p, q \in \mathcal{P}_0. \tag{1ad9}$$

Proof. Apply Proposition 1.11 to each decoded face $\tilde{z}_p$. Each probe inherits the same reference label $\Pi_o(z)$, so all probe labels agree. □

Proposition 1.13 (Monotonicity under enlarged admissible families). Let $\mathcal{A}_1(z) \subseteq \mathcal{A}_2(z)$ be two admissible families around the same face z. Then

$$\Delta_{\mathcal{A}_1}(z) \le \Delta_{\mathcal{A}_2}(z), \qquad \mathfrak{J}_o(z;\mathcal{A}_2) \le \mathfrak{J}_o(z;\mathcal{A}_1), \qquad \mathbf{J}_o(z;\mathcal{A}_2) \le \mathbf{J}_o(z;\mathcal{A}_1). \tag{1ae0}$$

Proof. The radius inequality is immediate from the set inclusion. Substituting into (1ad2) shows that enlarging the admissible family can only decrease the reserve. The normalized score in (1ad3) is an increasing function of the reserve and a decreasing function of the admissible radius, so it also cannot increase. □

This makes the intended meaning precise:

$$\text{jointness is not absolute;}\quad \text{it is graded relative to the perturbations, probes, and closure-compatible variations on} \tag{1ae1}$$

The same idea has a dynamic version, and this is exactly where the present paper starts to touch the companion meta_theory_quantized_observation line.

Definition 1.14 (Dynamic graded jointness on an interval). Let $I \subset \mathbb{R}$ be an interval, let $z_\cdot : I \to Z$ be a reference resolved-face path, and let

$$c_z(t) := \mathbf{1}_{\{g(z_t)\geq 0\}} \tag{1ae2}$$

be the induced symbolic label process. Let $\mathcal{A}_I(z_\cdot)$ be an admissible family of comparison face-paths $\tilde{z}_\cdot : I \to Z$. Define the pathwise admissible deviation and the interval margin by

$$\delta_{\mathcal{A},I}(z_\cdot) := \sup_{\tilde{z}_\cdot \in \mathcal{A}_I(z_\cdot)} \sup_{t\in I} \|\tilde{z}_t - z_t\|, \qquad m_{o,I}(z_\cdot) := \inf_{t\in I} |g(z_t)|. \tag{1ae3}$$

If g is L-Lipschitz on a neighborhood covering the admissible family, define the **dynamic graded jointness reserve**

$$\mathfrak{J}_{o,I}(z_\cdot; \mathcal{A}) := m_{o,I}(z_\cdot) - L\,\delta_{\mathcal{A},I}(z_\cdot). \tag{1ae4}$$

Interpretation: $\mathfrak{J}_{o,I} > 0$ means that on the entire interval I, the reference symbolic label has enough score margin to survive the full admissible pathwise perturbation budget.

Proposition 1.15 (Positive dynamic graded jointness implies interval-wise symbolic stability). In the setting of Definition 1.14, if

$$\mathfrak{J}_{o,I}(z_\cdot; \mathcal{A}) > 0, \tag{1ae5}$$

then every admissible comparison path $\tilde{z}_\cdot \in \mathcal{A}_I(z_\cdot)$ induces exactly the same symbolic label process on I:

$$\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} = \mathbf{1}_{\{g(z_t)\geq 0\}} \qquad \text{for all } t \in I. \tag{1ae6}$$

Proof. For every admissible path and every $t \in I$,

$$|g(\tilde{z}_t) - g(z_t)| \leq L\|\tilde{z}_t - z_t\| \leq L\,\delta_{\mathcal{A},I}(z_\cdot). \tag{1ae7}$$

Since $|g(z_t)| \geq m_{o,I}(z_\cdot) > L\,\delta_{\mathcal{A},I}(z_\cdot)$, the perturbation cannot move $g(z_t)$ across zero. Hence the sign, and therefore the binary label, is unchanged for every $t \in I$. □

Proposition 1.16 (Transversal crossing localizes dynamic jointness loss). Let $U \subset \mathbb{R}$ be an interval, let $z_\cdot : U \to Z$ be C^1, and define

$$y(t) := g(z_t). \tag{1ae8}$$

Assume there exists a unique crossing time $\tau \in U$ such that

$$y(\tau) = 0, \qquad y'(t) \geq m > 0 \quad \text{for all } t \in U. \tag{1ae9}$$

Let $\mathcal{A}_U(z_\cdot)$ be an admissible family with

$$\delta_{\mathcal{A},U}(z_\cdot) \leq \varepsilon. \tag{1af0}$$

Then for every admissible path $\tilde{z}_\cdot \in \mathcal{A}_U(z_\cdot)$ and every $t \in U$ satisfying

$$|t-\tau| > \frac{L\varepsilon}{m}, \tag{1af1}$$

one has

$$\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} = \mathbf{1}_{\{g(z_t)\geq 0\}}. \tag{1af2}$$

Proof. Since $y' \geq m > 0$ on U and $y(\tau) = 0$, the fundamental theorem of calculus gives

$$|y(t)| \geq m|t-\tau| \qquad \text{for all } t \in U. \tag{1af3}$$

For every admissible path,

$$|g(\tilde{z}_t) - g(z_t)| \leq L\|\tilde{z}_t - z_t\| \leq L\varepsilon. \tag{1af4}$$

If $|t-\tau| > L\varepsilon/m$, then by (1af3),

$$|g(z_t)| = |y(t)| > L\varepsilon, \tag{1af5}$$

so the admissible perturbation cannot flip the sign of $g(z_t)$. Therefore the symbolic label agrees with the reference label at time t. □

Corollary 1.17 (Disagreement is confined to a crossing window). Under the hypotheses of Proposition 1.16, define

$$c_z(t) = \mathbf{1}_{\{g(z_t)\geq 0\}}, \qquad c_{\tilde{z}}(t) = \mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}}. \tag{1af6}$$

Then any symbolic disagreement between the reference path and an admissible comparison path can occur only inside the crossing window

$$W_{\varepsilon,m}(\tau) := \left[\tau - \frac{L\varepsilon}{m},\ \tau + \frac{L\varepsilon}{m}\right]. \tag{1af7}$$

In particular,

$$\operatorname{supp}|c_{\tilde{z}} - c_z| \subseteq W_{\varepsilon,m}(\tau), \qquad \int_U |c_{\tilde{z}}(t) - c_z(t)|\,dt \leq \frac{2L\varepsilon}{m}. \tag{1af8}$$

Proof. Proposition 1.16 gives equality of the two symbolic labels at every time outside the window $(1af7)$. Therefore the support of disagreement is contained in that window, whose length is $2L\varepsilon/m$. □

The preceding corollary is the dynamic-to-static bridge: outside the crossing window, the observed boundary again behaves like a stable regime separator.

Proposition 1.18 (Safe intervals support persistent effective regimes). Let $U = [a,b]$, and assume the hypotheses of Proposition 1.16. Define the left and right safe intervals

$$J^-_{\varepsilon,m} := \left[a,\ \tau - \frac{L\varepsilon}{m}\right), \qquad J^+_{\varepsilon,m} := \left(\tau + \frac{L\varepsilon}{m},\ b\right]. \tag{1ag0}$$

Then for every admissible comparison path $\tilde{z}_\cdot \in \mathcal{A}_U(z_\cdot)$,

$$\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} = 0 \quad \text{for all } t \in J^-_{\varepsilon,m}, \tag{1ag1}$$

and

$$\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} = 1 \quad \text{for all } t \in J^+_{\varepsilon,m}. \tag{1ag2}$$

In particular, the two sides of the crossing define persistent effective regimes, while the ambiguity is confined to the crossing window $W_{\varepsilon,m}(\tau)$.

Proof. For $t < \tau$, the fundamental theorem of calculus and $y' \geq m$ give

$$0 - y(t) = y(\tau) - y(t) = \int_t^\tau y'(s)\, ds \geq m(\tau - t), \tag{1ag3}$$

so

$$y(t) \leq -m(\tau - t). \tag{1ag4}$$

Hence for $t \in J^-_{\varepsilon,m}$,

$$y(t) < -L\varepsilon. \tag{1ag5}$$

Since admissible paths satisfy $|g(\tilde{z}_t) - g(z_t)| \leq L\varepsilon$, one gets

$$g(\tilde{z}_t) < 0, \qquad t \in J^-_{\varepsilon,m}. \tag{1ag6}$$

The right-side statement is analogous: for $t > \tau$,

$$y(t) = y(t) - y(\tau) = \int_\tau^t y'(s)\, ds \geq m(t - \tau), \tag{1ag7}$$

so on $J^+_{\varepsilon,m}$, $y(t) > L\varepsilon$, and therefore $g(\tilde{z}_t) > 0$. This gives (1ag2). □

Corollary 1.19 (Transient crossing artifact criterion). Under the hypotheses of Proposition 1.18, suppose a boundary's observed label-instability relative to the admissible family $\mathcal{A}_U(z_\cdot)$ is entirely localized to the window

$$W_{\varepsilon,m}(\tau) = \left[\tau - \frac{L\varepsilon}{m},\ \tau + \frac{L\varepsilon}{m}\right]. \tag{1ag8}$$

Then that instability should be interpreted as a transient crossing artifact rather than as a failure of the adjacent left and right effective regimes.

Reason. By Proposition 1.18, the left and right safe intervals already carry probe- and perturbation-robust labels. So the only nonpersistent part of the observed boundary is the local passage through the crossing window itself. The instability therefore belongs to the transition event, not to the neighboring regimes.

Corollary 1.20 (Vanishing-window limit recovers a sharp regime split). Assume the setting of Proposition 1.18, but now let $\mathcal{A}_U^{(n)}(z_\cdot)$ be admissible families with deviations $\varepsilon_n \downarrow 0$. Then

$$|W_{\varepsilon_n,m}(\tau)| = \frac{2L\varepsilon_n}{m} \to 0. \tag{1ag9}$$

Consequently, for every compact set $K \subset U \setminus \{\tau\}$, there exists N such that for all $n \geq N$ and every admissible comparison path in $\mathcal{A}_U^{(n)}(z_\cdot)$,

$$\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} = \mathbf{1}_{\{g(z_t)\geq 0\}} \qquad \text{for all } t \in K. \tag{1ah0}$$

Proof. Since K is compact and disjoint from τ, its distance from τ is some $d_K > 0$. Choose N so large that $L\varepsilon_n/m < d_K$ for all $n \geq N$. Then K lies entirely outside $W_{\varepsilon_n,m}(\tau)$, so Proposition 1.16 gives the claimed equality on K. □

The static and dynamic results can now be summarized in one principle.

Theorem 1.21 (Unified boundary-stability principle). Let $U \subset \mathbb{R}$ be a time window, let $z_\cdot : U \to Z$ be a reference resolved-face path, let $g : Z \to \mathbb{R}$ be L-Lipschitz, and let $\mathcal{A}_U(z_\cdot)$ be an admissible family of comparison face-paths. Then the observed boundary induced by

$$c_z(t) = \mathbf{1}_{\{g(z_t)\geq 0\}} \tag{1ah1}$$

has the following two controlled regimes:

1. **Persistent-joint regime.** If

$$\mathfrak{J}_{o,U}(z_\cdot; \mathcal{A}) > 0, \tag{1ah2}$$

then for every admissible comparison path $\tilde{z}_\cdot \in \mathcal{A}_U(z_\cdot)$,

$$\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} = \mathbf{1}_{\{g(z_t)\geq 0\}} \qquad \text{for all } t \in U. \tag{1ah3}$$

So the boundary behaves as a persistent effective joint throughout the entire window.

2. **Localized-transition regime.** If $\mathfrak{J}_{o,U}(z_\cdot; \mathcal{A})$ is not positive because the reference path undergoes a unique transversal crossing at time τ with lower slope bound $m > 0$ and admissible pathwise deviation at most ε, then for every admissible comparison path,

$$\operatorname{supp} |c_{\tilde{z}} - c_z| \subseteq W_{\varepsilon,m}(\tau), \qquad \int_U |c_{\tilde{z}}(t) - c_z(t)|\, dt \leq \frac{2L\varepsilon}{m}. \tag{1ah4}$$

Hence the failure of boundary stability is confined to a transition window whose size is controlled by the error-to-transversality ratio.

Proof. Case 1 is exactly Proposition 1.15. Case 2 is Proposition 1.16 together with Corollary 1.17. □

This theorem is the first compact statement of the whole paper's intended reading:

$$\text{a boundary is either robust across the whole window, or its instability is localized to a quantified transition zone.} \tag{1ah5}$$

Corollary 1.22 (Diagnostic trichotomy for observed boundaries). In the setting of Theorem 1.21, an observed boundary on a window U should be read in one of three ways:

1. **Persistent effective joint:** if $\mathfrak{J}_{o,U}(z_\cdot; \mathcal{A}) > 0$.
2. **Transient crossing artifact:** if the instability is confined to a controlled window $W_{\varepsilon,m}(\tau)$ around a unique transversal crossing.
3. **Unresolved boundary:** if neither of the above controlled mechanisms has been established.

Reason. The first case is globally robust on the window. The second case is locally unstable only because the reference path passes through a threshold, while the adjacent safe intervals already define persistent effective regimes. The third case is the honest remainder: one does not yet know whether the boundary is a true effective joint, a crossing artifact, or some more complicated instability mechanism.

The next step is to rewrite this trichotomy in terms of quantities one can actually try to estimate, bound, or certify.

Definition 1.23 (Boundary diagnosis tuple). Fix a time window U, a reference path $z_\cdot$, an admissible family $\mathcal{A}_U(z_\cdot)$, and, when relevant, a comparison path $\tilde{z}_\cdot$. Define:

1. the **jointness reserve**
$$R_U := \mathfrak{J}_{o,U}(z_\cdot;\mathcal{A}), \tag{1ah7}$$
2. the **certified crossing budget**
$$B_U(\tau,\varepsilon,m) := \frac{2L\varepsilon}{m}, \tag{1ah8}$$
whenever a unique transversal crossing time τ, admissible deviation bound ε, and lower transverse slope bound $m > 0$ are available,
3. the **observed disagreement mass**
$$D_U(\tilde{z}_\cdot, z_\cdot) := \int_U \left|\mathbf{1}_{\{g(\tilde{z}_t)\geq 0\}} - \mathbf{1}_{\{g(z_t)\geq 0\}}\right| dt. \tag{1ah9}$$

The tuple (R_U, B_U, D_U) is the paper's first operational boundary-diagnosis surface.

Proposition 1.24 (Certified persistent-joint test). If
$$R_U > 0, \tag{1ai0}$$
then the boundary on U is certified as a persistent effective joint. In particular, for every admissible comparison path,
$$D_U(\tilde{z}_\cdot, z_\cdot) = 0. \tag{1ai1}$$

Proof. The condition $R_U > 0$ is exactly the positivity condition in Proposition 1.15, which yields equality of the symbolic label processes on all of U. Hence the disagreement mass is zero. □

Proposition 1.25 (Certified transient-artifact test). Assume a unique transversal crossing at time τ, admissible deviation bound ε, and lower slope bound $m > 0$, so that the certified window
$$W_{\varepsilon,m}(\tau) = \left[\tau - \frac{L\varepsilon}{m},\ \tau + \frac{L\varepsilon}{m}\right] \tag{1ai2}$$
is defined. If, for a given admissible comparison path $\tilde{z}_\cdot$,
$$\operatorname{supp}|c_{\tilde{z}} - c_z| \subseteq W_{\varepsilon,m}(\tau), \qquad D_U(\tilde{z}_\cdot, z_\cdot) \leq B_U(\tau,\varepsilon,m), \tag{1ai3}$$
then the observed instability on U is certified as a transient crossing artifact.

Proof. The support inclusion says that disagreement is confined to the transition zone. The mass bound says that the total disagreement does not exceed the certified crossing budget. By Proposition 1.18 and Corollary 1.19, the safe intervals already define persistent effective regimes, so the remaining instability belongs to the transition event itself. □

Corollary 1.26 (Certified unresolved-boundary criterion). Suppose $R_U \leq 0$. If there is no certified transversal-crossing description for which the support inclusion and mass bound in (1ai3) hold, then the boundary remains unresolved on U.

Reason. The persistent-joint certificate has failed because $R_U \leq 0$. The transient-artifact certificate has also failed because one cannot localize the instability to a controlled crossing window with the expected budget. So the boundary cannot yet be read in either controlled way.

This yields a first operational diagnosis rule:

$$\text{first test } R_U; \quad \text{if that fails, test window confinement and } D_U \leq B_U; \quad \text{otherwise mark the boundary unresolved.} \tag{1ai4}$$

This is the same inverse-slope mechanism that appears in meta_theory_quantized_observation: the quantity

$$\frac{\text{admissible observation/face error}}{\text{transverse crossing speed}} \tag{1ah6}$$

is exactly the scale on which dynamic jointness can fail. Away from that window, the boundary behaves like a stable local joint; near that window, symbolic instability becomes possible.

1.9 Relation to the observation line

This makes the present paper different from meta_theory_quantized_observation. That paper studies what happens after a latent state already exists:

$$u_t \xrightarrow{\kappa} y_t \xrightarrow{Q} c_t. \tag{2}$$

The current paper asks what may lie before that stage:

$$r_{n+1} = \mathcal{G}(r_n), \qquad \mathcal{G}^n(r_0) \rightsquigarrow u, \tag{3}$$

where u is the structured latent object eventually observed or further transformed. In the strongest reading, u itself may be one resolved image $R_s(\mathcal{U})$ of a deeper underlying object.

The combined research picture is therefore:

$$\text{micro operator iteration} \longrightarrow \text{latent pattern} \xrightarrow{\kappa} \text{observed coordinate} \xrightarrow{Q} \text{symbolic output}. \tag{4}$$

The dynamic graded-jointness results above make the bridge explicit. The current paper describes when a resolved boundary remains stable under admissible probe/resolution variation. The companion paper analyzes what happens when such stability is lost near a threshold crossing: the same error-to-transversality ratio then reappears as a symbolic disagreement window.

This note isolates the left half of that chain, but now with a visible seam to the right half.

---

2. The Core Operator Picture

2.1 Iterated linear action

Let V be a function space or state space and let $T : V \to V$ be a linear operator. The simplest generator-first model is

$$u_{n+1} = Tu_n, \qquad u_n = T^n u_0. \tag{5}$$

If T is diagonalizable with eigenpairs (λ_k, e_k) and

$$u_0 = \sum_k c_k e_k, \tag{6}$$

then

$$u_n = \sum_k c_k \lambda_k^n e_k. \tag{7}$$

This is the first precise sense in which a larger pattern is directly derived from repeated local action. The visible macro-object is organized by the dominant eigenmodes of the underlying operator.

2.2 Convolutional local rules

If the local rule acts by convolution with a kernel K, then

$$u_{n+1} = K * u_n. \tag{8}$$

Fourier transformation yields

$$\widehat{u_n}(\xi) = \widehat{K}(\xi)^n \widehat{u_0}(\xi). \tag{9}$$

This is a direct generator-to-pattern law. Repeated local smoothing, transport, or oscillatory action at the micro-level becomes multiplicative mode selection in frequency space.

2.3 Fixed-point operators on measures

Self-similar constructions often act not on functions but on measures:

$$\mu_{n+1} = \mathcal{L}\mu_n. \tag{10}$$

If $\mathcal{L}$ is contractive in a suitable sense, the invariant object satisfies

$$\mu_* = \mathcal{L}\mu_*. \tag{11}$$

This is the natural entry point for fractal-like and iterated-function-system language. The emergent pattern is a fixed point of repeated local action.

---

3. Spectral Layers

The paper's main conceptual proposal is that there may exist several distinct spectral layers.

3.1 First spectral layer

The micro-generator T or $\mathcal{L}$ has its own spectrum, symbol, or contraction profile. This is the spectrum of the generator itself.

3.2 Second spectral layer

The generated object u or μ_* may itself admit a spectral decomposition:

$$u = \sum_k A_k v_k. \tag{12}$$

This is not automatically the same spectral object as the spectrum of T. One spectrum governs the rule; the other governs the emergent pattern.

3.3 Hierarchical picture

The slogan of this note is therefore:

$$\text{generator spectrum} \neq \text{necessarily emergent-pattern spectrum}. \tag{13}$$

But the two may be tightly related. The generator spectrum may determine:

- which modes survive,
- which scales are amplified,
- which scales are suppressed,
- and whether the emergent object is compressible at the next layer.

This is the precise meaning of **spectra behind spectra**.

---

4. Initial Mathematical Forms

Three minimal forms of the idea already look mathematically tractable.

4.1 Dominant-mode emergence

If $|\lambda_1| > |\lambda_2| \geq \cdots$, then after normalization one expects

$$\frac{u_n}{\|u_n\|} \to \pm e_1 \quad \text{or} \quad e^{i\theta_n} e_1 \tag{14}$$

under standard spectral-gap assumptions. The macro-pattern is then controlled by one dominant mode even though it was generated by many repeated local actions.

4.2 Iterated kernel symbol law

For convolution dynamics, the multiplier $\widehat{K}(\xi)^n$ determines which frequencies survive at scale n. This suggests a mode-selection theorem of the form:

$$\widehat{K}(\xi)^n \to 0 \text{ off a spectral shell}, \tag{15}$$

so the large-scale pattern may collapse onto a smaller effective spectral support.

Proposition 4.1 (Dominant-shell selection for iterated convolution). Let $\mathbb{T}^d$ be the d-dimensional torus, let $K \in L^1(\mathbb{T}^d)$, and let

$$T_K u := K * u. \tag{15a}$$

Write the Fourier multiplier of T_K as

$$m(k) := \widehat{K}(k), \qquad k \in \mathbb{Z}^d. \tag{15b}$$

Assume there exists a number $\rho > 0$, a finite nonempty set $M \subset \mathbb{Z}^d$, and a constant $0 \le r < \rho$ such that

$$m(k) = \rho \quad \text{for } k \in M, \tag{15c}$$

and

$$|m(k)| \le r \quad \text{for } k \notin M. \tag{15d}$$

Then for every $u_0 \in L^2(\mathbb{T}^d)$,

$$\rho^{-n} T_K^n u_0 \to P_M u_0 \qquad \text{in } L^2(\mathbb{T}^d), \tag{15e}$$

where

$$P_M u_0(x) := \sum_{k \in M} \widehat{u_0}(k) e^{ik\cdot x}. \tag{15f}$$

Proof. For each $n \ge 0$,

$$\widehat{T_K^n u_0}(k) = m(k)^n \widehat{u_0}(k). \tag{15g}$$

Hence

$$\rho^{-n} \widehat{T_K^n} u_0(k) = \rho^{-n} m(k)^n \widehat{u_0}(k). \tag{15h}$$

If $k \in M$, then $m(k) = \rho$, so

$$\rho^{-n} \widehat{T_K^n} u_0(k) = \widehat{u_0}(k). \tag{15i}$$

If $k \notin M$, then $|m(k)| \le r < \rho$, hence

$$\left| \rho^{-n} \widehat{T_K^n} u_0(k) \right| \le \left(\frac{r}{\rho} \right)^n |\widehat{u_0}(k)|. \tag{15j}$$

Therefore

$$\rho^{-n} T_K^n \widehat{u_0 - P_M} u_0(k) = 0 \quad \text{for } k \in M, \tag{15k}$$

while for $k \notin M$,

$$\rho^{-n} T_K^n \widehat{u_0 - P_M} u_0(k) = \rho^{-n} m(k)^n \widehat{u_0}(k). \tag{15l}$$

By Parseval's identity,

$$\|\rho^{-n} T_K^n u_0 - P_M u_0\|_{L^2}^2 = \sum_{k \notin M} \rho^{-2n} |m(k)|^{2n} |\widehat{u_0}(k)|^2. \tag{15m}$$

Using $|m(k)| \le r$, we obtain

$$\|\rho^{-n} T_K^n u_0 - P_M u_0\|_{L^2}^2 \le \left(\frac{r}{\rho} \right)^{2n} \sum_{k \notin M} |\widehat{u_0}(k)|^2 \le \left(\frac{r}{\rho} \right)^{2n} \|u_0\|_{L^2}^2. \tag{15n}$$

Since $r/\rho < 1$, the right-hand side tends to 0. This proves (15e). □

This proposition is the paper's first direct instance of **spectra behind spectra**. The generator spectrum $m(k)$ determines a dominant shell M, and after repeated local action the emergent object collapses onto the higher-level pattern carried by those surviving modes.

Proposition 4.2 (Oscillatory dominant-shell dynamics). Let $\mathbb{T}^d$, K, and T_K be as in Proposition 4.1. Assume there exists a number $\rho > 0$, a finite nonempty set $M \subset \mathbb{Z}^d$, a phase profile

$$\omega : M \to \mathbb{C}, \qquad |\omega(k)| = 1 \text{ for all } k \in M, \tag{15o}$$

and a constant $0 \le r < \rho$ such that

$$m(k) = \rho\,\omega(k) \quad \text{for } k \in M, \tag{15p}$$

and

$$|m(k)| \le r \quad \text{for } k \notin M. \tag{15q}$$

Define the shell-evolution process

$$E_n^M u_0(x) := \sum_{k \in M} \omega(k)^n \widehat{u_0}(k) e^{ik\cdot x}. \tag{15r}$$

Then for every $u_0 \in L^2(\mathbb{T}^d)$,

$$\rho^{-n} T_K^n u_0 - E_n^M u_0 \to 0 \qquad \text{in } L^2(\mathbb{T}^d). \tag{15s}$$

Moreover,

$$\|\rho^{-n} T_K^n u_0 - E_n^M u_0\|_{L^2} \le \left(\frac{r}{\rho}\right)^n \|u_0\|_{L^2}. \tag{15t}$$

Proof. For each $k \in \mathbb{Z}^d$,

$$\widehat{\rho^{-n} T_K^n} u_0(k) = \rho^{-n} m(k)^n \widehat{u_0}(k). \tag{15u}$$

If $k \in M$, then by (15p),

$$\widehat{\rho^{-n} T_K^n} u_0(k) = \omega(k)^n \widehat{u_0}(k) = \widehat{E_n^M u_0}(k). \tag{15v}$$

If $k \notin M$, then $\widehat{E_n^M u_0}(k) = 0$, and by (15q),

$$\left|\rho^{-n} T_K^n \widehat{u_0 -} E_n^M u_0(k)\right| \le \left(\frac{r}{\rho}\right)^n |\widehat{u_0}(k)|. \tag{15w}$$

Therefore, by Parseval,

$$\|\rho^{-n} T_K^n u_0 - E_n^M u_0\|_{L^2}^2 = \sum_{k \notin M} \left|\rho^{-n} T_K^n \widehat{u_0 -} E_n^M u_0(k)\right|^2 \tag{15x}$$

$$\le \left(\frac{r}{\rho}\right)^{2n} \sum_{k \notin M} |\widehat{u_0}(k)|^2 \le \left(\frac{r}{\rho}\right)^{2n} \|u_0\|_{L^2}^2. \tag{15y}$$

Taking square roots gives (15t), hence also (15s). $\square$

This proposition shows that the surviving shell need not be static. The higher-level emergent pattern may live on a finite dominant shell while still carrying its own internal oscillatory law through the phase profile $\omega(k)^n$. In that sense, the second spectral layer can have dynamics of its own rather than only a frozen support.

Proposition 4.3 (Dominant-band selection for iterated convolution). Let $\mathbb{T}^d$, K, and T_K be as in Proposition 4.1. Assume there exists a number $\rho > 0$, a subset $B \subset \mathbb{Z}^d$, a phase profile

$$\omega : B \to \mathbb{C}, \qquad |\omega(k)| = 1 \text{ for all } k \in B, \tag{15z}$$

and a constant $0 \le r < \rho$ such that

$$m(k) = \rho\, \omega(k) \quad \text{for } k \in B, \tag{15aa}$$

and

$$|m(k)| \le r \quad \text{for } k \notin B. \tag{15ab}$$

Define

$$E_n^B u_0(x) := \sum_{k \in B} \omega(k)^n \widehat{u_0}(k) e^{ik\cdot x}. \tag{15ac}$$

Then for every $u_0 \in L^2(\mathbb{T}^d)$,

$$\rho^{-n} T_K^n u_0 - E_n^B u_0 \to 0 \qquad \text{in } L^2(\mathbb{T}^d), \tag{15ad}$$

and

$$\|\rho^{-n} T_K^n u_0 - E_n^B u_0\|_{L^2} \le \left(\frac{r}{\rho}\right)^n \|u_0\|_{L^2}. \tag{15ae}$$

Proof. For $k \in B$, the Fourier coefficients of $\rho^{-n} T_K^n u_0$ and $E_n^B u_0$ agree exactly:

$$\rho^{-n}\widehat{T_K^n} u_0(k) = \rho^{-n} m(k)^n \widehat{u_0}(k) = \omega(k)^n \widehat{u_0}(k) = \widehat{E_n^B u_0}(k). \tag{15af}$$

For $k \notin B$, we have $\widehat{E_n^B u_0}(k) = 0$, and by (15ab),

$$\left|\rho^{-n} T_K^n \widehat{u_0 -} E_n^B u_0(k)\right| \le \left(\frac{r}{\rho}\right)^n |\widehat{u_0}(k)|. \tag{15ag}$$

Parseval's identity therefore gives

$$\|\rho^{-n} T_K^n u_0 - E_n^B u_0\|_{L^2}^2 = \sum_{k \notin B} \left|\rho^{-n} T_K^n \widehat{u_0 -} E_n^B u_0(k)\right|^2 \tag{15ah}$$

$$\le \left(\frac{r}{\rho}\right)^{2n} \sum_{k \notin B} |\widehat{u_0}(k)|^2 \le \left(\frac{r}{\rho}\right)^{2n} \|u_0\|_{L^2}^2. \tag{15ai}$$

Taking square roots proves (15ae), hence also (15ad). □

This proposition subsumes Propositions 4.1 and 4.2 and makes the geometric point explicit: the surviving set need not be a finite shell. It can be any dominant frequency band selected by the generator.

Corollary 4.4 (Radial dominant band). Under the hypotheses of Proposition 4.3, one may choose

$$B_{a,b} := \{k \in \mathbb{Z}^d : a \le |k| \le b\}, \qquad 0 \le a \le b, \tag{15aj}$$

or more generally any lattice shell geometry specified by the application. If

$$m(k) = \rho\, \omega(k) \quad \text{for } k \in B_{a,b}, \tag{15ak}$$

and $|m(k)| \le r < \rho$ off that band, then the normalized iterates converge in L^2 to the band-evolution process

$$E_n^{B_{a,b}} u_0(x) = \sum_{k \in B_{a,b}} \omega(k)^n \widehat{u_0}(k) e^{ik\cdot x}. \tag{15al}$$

This is the first direct mathematical form of a **dominant spectral geometry** in the paper: the generator can select not only isolated modes, but also a whole shell-shaped emergent layer.

4.3 Hierarchical renormalization view

If one coarse-grains after repeated local action, then each scale may produce a new effective generator:

$$T \rightsquigarrow T^{(1)} \rightsquigarrow T^{(2)} \rightsquigarrow \cdots. \tag{16}$$

This is the natural mathematical place where hierarchy enters. Each scale may have its own effective spectrum and its own compressibility law.

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5. Relation to the Observation Line

The clean relationship to the current quantized-observation paper is:

$$\text{generator-first paper}: \quad \mathcal{G}^n(r_0) \rightsquigarrow u, \tag{17}$$

$$\text{observation-first paper}: \quad u \xrightarrow{\kappa} y \xrightarrow{Q} c. \tag{18}$$

The current topic is therefore not a replacement for quantized observation. It is a deeper companion paper asking whether the latent state u may itself be an emergent object generated from a lower-level operator calculus.

If this line succeeds, the broader program becomes:

$$\text{iterated micro-rule} \longrightarrow \text{hierarchical latent spectrum} \longrightarrow \text{quantized observation}. \tag{19}$$

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6. What This Paper Does Not Claim

To keep the identity clean, this note does **not** claim:

- that every fractal is linear,
- that every self-similar object is best analyzed spectrally,
- that every spectral model hides an infinite hierarchy,
- or that the generator spectrum and the emergent-pattern spectrum always coincide.

The narrower claim is:

$$\text{repeated local operators can generate latent patterns with their own higher-level spectral structure.} \tag{20}$$

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7. Open Theorem Candidates

The current note now contains three proved first cases, namely Propositions 4.1, 4.2, and 4.3 for iterated convolution on the torus, together with Corollary 4.4 for radial dominant bands. The most natural next theorem candidates are:

1. **Iterated convolution theorem**
    - relax the exact dominant-band hypothesis,
    - identify asymptotic selection under approximate or slowly varying dominant spectral geometry.
2. **Two-level spectral law**
    - distinguish the spectrum of the generator from the spectrum of the generated object,
    - prove conditions under which one controls the other.
3. **Hierarchical compressibility theorem**
    - show that if the micro-operator has a contractive spectral profile, then the emergent macro-pattern has a sharper compressibility law at the next layer.
4. **Self-similar fixed-point theorem**
    - formalize when a measure-level fixed point of an iterated local rule admits a canonical second-level spectral summary.

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8. Next Step

The best next move is probably one of these:

- generalize Propositions 4.1 and 4.2 from finite dominant shells to broader dominant sets or bands,
- study an affine iterated-function operator on measures,
- or build a renormalized 2D toy model where the generator spectrum and the emergent-pattern spectrum can be written side by side.

That would extend the first proved statement into a broader hierarchical spectral law.