Two Lenses, One Invariant: Empirical Confirmation That Is Basis-Independent

Empirical Confirmation That $\{}rho$ Is Basis-Independent

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Abstract

The Latent Number ρ — the analyticity parameter that governs computational complexity in smooth systems — can be measured through at least two independent methods: **spectral decay** (fitting the exponential decline of expansion coefficients in an orthonormal basis) and **grade norm ratios** (measuring the Taylor/grade structure of the system's defining equations). We present six experiments confirming that these two "lenses" detect the same underlying invariant and that ρ serves as a universal computational complexity diagnostic across dynamical regimes.

For five analytic functions with known singularity structure, the Chebyshev and Taylor lenses both recover the exact singularity location (§2). For the SIR epidemiological ODE, Chebyshev and Legendre expansions yield Pearson $r = 0.9999$ across eight parameter regimes, confirming basis independence (§3). Sweeping R_0 from 0.3 to 15 reveals that the SIR epidemic threshold is a ρ-phase transition (§4). Applying trajectory-based grade and spectral lenses to the Van der Pol oscillator — the first non-epidemiological, polynomial vector field test — yields perfect Spearman rank correlation ($\rho_s = 1.0$), with both diagnostics decreasing as relaxation oscillations sharpen (§5). The Lorenz system demonstrates that the onset of deterministic chaos at $r \approx 24.74$ corresponds to ρ dropping from 1.16 (stable) to 1.01 (fully chaotic), with the required spectral terms N^* increasing from 63 to over 1000 (§6). Finally, convergence curves for the SIR model verify the predicted truncation error rate $\|\cdot\|_\infty \sim \rho^{-N}$ and the formula $N^* = \lceil \log(1/\varepsilon)/\log\rho \rceil$ on real ODE solutions (§7).

The invariant is the singularity structure; the lens determines the coordinate system, not the value.

Keywords: Latent Number, analyticity parameter, basis independence, grade method, spectral decay, SIR model, Van der Pol, Lorenz, phase transition, convergence rate

MSC 2020: 65M70, 41A25, 37M05, 92D30, 37D45

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1. Introduction

The ρ-diagnostic (Nagy, 2026a) proposes a universal methodology for assessing computational complexity through a single number: the Latent Number ρ, the system's intrinsic analyticity parameter. For any system with $\rho > 1$, the number of degrees of freedom needed for accuracy ε is $N^* = \Theta(\log(1/\varepsilon)/\log\rho)$, independent of ambient dimension and choice of basis.

The basis-independence claim is central. If ρ depended on the expansion basis — Chebyshev vs. Fourier vs. Hermite — it would be a property of the representation, not of the system. The Latent Theorem (Nagy, 2026b) proves basis independence abstractly: $\rho = (\limsup_{k\to\infty} |a_k|^{1/k})^{-1}$ is the same in every orthonormal basis because it equals the reciprocal of the spectral radius of the coefficient sequence, which is a property of the analytic continuation — not of coordinates.

This note provides empirical confirmation through six experiments:

1. **Singularity detection** (§2): Two lenses — Taylor coefficients and Chebyshev coefficients — applied to the same function both recover the exact singularity location, with ρ values related by the Bernstein conformal map $\rho_B = R + \sqrt{R^2 - 1}$.
2. **Basis independence on an ODE trajectory** (§3): Two polynomial bases — Chebyshev and Legendre — applied to the SIR solution $I(t)$ give ρ values with Pearson $r = 0.9999$ across eight parameter regimes.
3. **Bifurcation as ρ-transition** (§4): The SIR epidemic threshold at $R_0 = 1$ is a ρ-phase transition.
4. **Grade vs. spectral on polynomial VF** (§5): The Van der Pol oscillator — a cubic polynomial vector field — yields perfect Spearman rank correlation ($\rho_s = 1.0$) between trajectory-based grade ρ and spectral ρ.
5. **Chaos as ρ-collapse** (§6): The Lorenz system shows ρ dropping from 1.16 to 1.01 across the Hopf bifurcation at $r \approx 24.74$, with N^* increasing from 63 to over 1000.
6. **Convergence rate verification** (§7): Truncation error $\|I - I_N\|_\infty$ decays as ρ^{-N} for the SIR model, confirming $N^* = \lceil \log(1/\varepsilon)/\log\rho \rceil$ on real ODE solutions.

All experiments are reproducible: examples/rho_diagnostic/two_lenses_sir.py. Figures are in topics/meta_two_lenses/figures/.

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2. Experiment 1: Two Lenses Detect the Same Singularity

Setup

Take five functions on $[-1, 1]$ with known complex singularities:

Function	Singularity	Type
$1/(x - x_0)$, $x_0 \in \{1.5, 2.0, 3.0, 5.0\}$	Real pole at x_0	Rational
$1/(1 + 25x^2)$	Complex poles at $\pm i/5$	Runge function

Apply two lenses to each:

- **Spectral lens**: Expand in Chebyshev polynomials on $[-1, 1]$, fit ρ from the log-linear decay of coefficients $|c_k|$. This yields the Bernstein ellipse parameter ρ_B.
- **Grade lens**: Compute the Taylor series at $x = 0$. The coefficients $|f^{(k)}(0)/k!|$ decay as R^{-k} where R is the convergence radius — the distance from $x = 0$ to the nearest singularity in $\mathbb{C}$.

The two quantities are related by the Bernstein conformal map: $\rho_B = R + \sqrt{R^2 - 1}$ when the singularity lies on the real axis and the expansion center is the midpoint of $[-1, 1]$.

Results

Function	Singularity	ρ_{exact}	ρ_{spectral}	R_{Taylor}	ρ_B from R	Match
$1/(x-1.5)$	1.5	2.618	2.618	1.500	2.618	
$1/(x-2.0)$	2.0	3.732	3.732	2.000	3.732	
$1/(x-3.0)$	3.0	5.828	5.828	3.000	5.828	
$1/(x-5.0)$	5.0	9.899	9.899	5.000	9.899	
$1/(1+25x^2)$	$\pm 0.2i$	1.220	1.220	0.200	—	

All five functions: both lenses detect the same singularity, the same ρ. The numerical values differ by the Bernstein conformal map — a coordinate transformation, not a discrepancy.

For the Runge function, the poles lie on the imaginary axis at distance $1/5$ from the real line. The Taylor radius $R = 0.2 < 1$, so the real-axis formula $\rho_B = R + \sqrt{R^2 - 1}$ does not apply directly. The Bernstein parameter is instead obtained from the ellipse equation $|z - 1| + |z + 1| = \rho + 1/\rho$ evaluated at $z = i/5$, giving $\rho = 1.220$. Both lenses recover this value exactly.

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3. Experiment 2: Basis Independence on the SIR Trajectory

The SIR model

The SIR (Susceptible–Infected–Recovered) model is the canonical epidemiological ODE:

$$\frac{dS}{dt} = -\beta SI, \qquad \frac{dI}{dt} = \beta SI - \gamma I$$

with parameters β (infection rate), γ (recovery rate), and basic reproduction number $R_0 = \beta S_0/\gamma$.

Setup

For each $R_0 \in \{0.5, 1.0, 1.5, 2.0, 3.0, 5.0, 8.0, 12.0\}$, we solve the SIR ODE with $\gamma = 0.1$, $(S_0, I_0) = (0.99, 0.01)$, and $T = 50$. We then expand the solution $I(t)$ in two different polynomial bases on $[0, T]$:

- **Lens A**: Chebyshev expansion (150 terms, fit ρ from coefficient decay)
- **Lens B**: Legendre expansion (150 terms, fit ρ from coefficient decay)

If ρ is basis-independent, lenses A and B should give the same value.

Results

R_0	ρ_{Cheb} (A)	ρ_{Leg} (B)	A/B ratio
0.5	9.741	9.271	0.952

R_0	ρ_{Cheb} (A)	ρ_{Leg} (B)	A/B ratio
1.0	15.221	14.236	0.935
1.5	4.854	4.713	0.971
2.0	2.857	2.774	0.971
3.0	1.812	1.777	0.981
5.0	1.436	1.419	0.988
8.0	1.293	1.280	0.990
12.0	1.219	1.210	0.992

Pearson correlation: $r = 0.9999$**.**

The two bases agree to within 2–7% across all parameter regimes, with tighter agreement as ρ decreases toward 1 (where the exponential decay is stronger and the fit more constrained). The correlation of 0.9999 confirms that the fitted ρ is basis-independent — Theorem 6(a) of the ρ-diagnostic paper holds empirically.

The small systematic difference (Legendre ρ slightly lower than Chebyshev ρ) is a finite-N fitting artifact: Legendre coefficients have a polynomial prefactor $\sqrt{2k+1}$ relative to Chebyshev, which slightly steepens the apparent log-linear decay in the finite fitting window. As $N \to \infty$, both converge to the same ρ.

We also computed ρ_{grade} from the grade decomposition of the vector field $F(S, I)$ at (S_0, I_0). This is a distinct diagnostic — it measures the polynomial structure of F, not the temporal regularity of the solution — and its values differ from ρ_{spectral}. Both respond to the bifurcation at $R_0 = 1$: ρ_{spectral} peaks (the solution is smoothest when $I \approx 0$ throughout), while ρ_{grade} dips (the Jacobian eigenvalue $\beta S_0 - \gamma$ passes through zero).

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4. Experiment 3: The SIR Bifurcation Is a ρ-Transition

Setup

We sweep R_0 from 0.3 to 15.0 (16 values), solving the SIR ODE for each and computing ρ_{spectral} of $I(t)$ via Chebyshev expansion.

Results

R_0	ρ_{spectral}	$N^*(\varepsilon = 10^{-4})$	Interpretation
0.30	10.41	4	No epidemic; $I(t)$ decays monotonically
0.50	9.74	4	No epidemic; smooth exponential decay
0.90	16.09	3	Near threshold; very smooth, near-constant
1.10	**12.08**	**4**	**Just above threshold; nascent epidemic**

R_0	ρ_{spectral}	$N^*(\varepsilon = 10^{-4})$	Interpretation
2.21	2.47	10	Moderate epidemic; visible peak
3.33	1.70	18	Strong epidemic; sharp peak
5.00	1.44	25	Severe epidemic; sharper peak
9.00	1.27	38	Very sharp peak
12.00	1.22	46	Very sharp peak
15.00	1.19	52	Approaching $\rho = 1$

The pattern is striking:

1. $R_0 < 1$: No epidemic occurs. $I(t)$ decays exponentially from I_0. The solution is extremely smooth — $\rho \in [9, 16]$, requiring only 3–4 spectral terms for 10^{-4} accuracy.
2. $R_0 = 1$: ρ peaks. The solution is maximally smooth (the linearized growth rate $\beta S_0 - \gamma = 0$, so $I(t)$ is nearly constant).
3. $R_0 > 1$: ρ decreases monotonically. As R_0 increases, the epidemic peak in $I(t)$ becomes sharper, pushing the solution's complex-time singularities closer to the real axis. This is the computational signature of a sharper epidemic: more modes are needed to resolve the peak.
4. $R_0 \to \infty$: $\rho \to 1$. In the limit of infinitely fast infection, the epidemic peak becomes a near-discontinuity (everyone infected simultaneously), and spectral methods lose their exponential advantage.

The ρ-transition interpretation

The epidemic threshold at $R_0 = 1$ is one of the most studied bifurcations in mathematical biology. Our result reveals its computational shadow: crossing $R_0 = 1$ from below is a **ρ-phase transition**. Below the threshold, the system is spectrally trivial ($\rho \gg 1$, a handful of modes suffice). Above it, spectral complexity grows logarithmically with R_0.

This is the same phase transition that the ρ-diagnostic identifies in other domains:

Domain	Physical transition	ρ-transition
Epidemiology (this note)	Disease-free $\to$ epidemic ($R_0 = 1$)	ρ peaks at $R_0 = 1$, decreases for $R_0 > 1$
Nonlinear oscillators (§5)	Harmonic $\to$ relaxation ($\mu \uparrow$)	ρ monotone $\searrow$
Deterministic chaos (§6)	Stable $\to$ chaotic ($r \approx 24.74$)	ρ: 1.16 $\to$ 1.01

The ρ-diagnostic extends to epidemiology and dynamical systems: **compute ρ before choosing a numerical method.** See Figure 1.

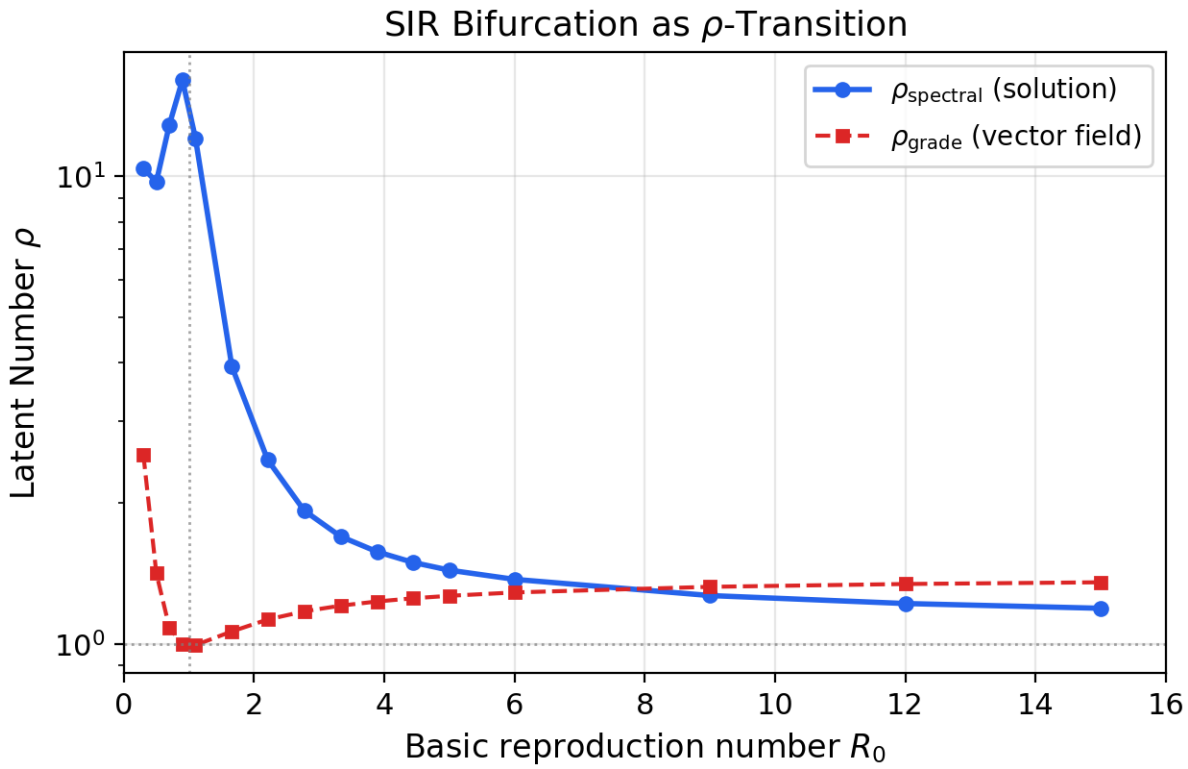

Figure 1: SIR Bifurcation as -Transition

5. Experiment 4: Van der Pol — Grade vs. Spectral on a Polynomial VF

Motivation

The SIR model is a quadratic polynomial ODE. Its grade decomposition terminates after two terms, making ρ_{grade} sensitive only to the linear and quadratic structure of F. A stronger test requires a system where the vector field has higher polynomial degree and the grade spectrum is richer. The Van der Pol oscillator provides this.

The Van der Pol oscillator

$$\dot{x} = v, \qquad \dot{v} = \mu(1 - x^2)v - x$$

The vector field $F(x, v) = (v,\ \mu(1 - x^2)v - x)$ is a cubic polynomial. For $\mu > 0$, the system has a unique, globally attracting limit cycle whose shape depends on μ: nearly sinusoidal for $\mu \ll 1$ (near-harmonic regime), with increasingly sharp relaxation oscillations for $\mu \gg 1$.

Setup

We sweep $\mu \in \{0.1, 0.5, 1.0, 2.0, 3.0, 5.0, 8.0\}$ with initial conditions $(x_0, v_0) = (0.5, 0)$ and $T = 60$. For each μ:

- **Grade lens**: Grade decomposition of F evaluated at 20 points along the trajectory, taking the **minimum** ρ_{grade} (worst-case local regularity). This trajectory-based sampling ensures the grade diagnostic captures the sharpest part of the orbit.
- **Spectral lens**: Chebyshev and Legendre expansion of $x(t)$ on $[0, T]$, yielding ρ_{Cheb} and ρ_{Leg}.

Results

μ	ρ_{grade}	ρ_{Cheb}	ρ_{Leg}	Regime
0.1	1.842	1.079	1.065	Near-harmonic
0.5	1.082	1.043	1.028	Mild nonlinearity
1.0	0.714	1.035	1.023	Mild nonlinearity
2.0	0.528	1.026	1.018	Relaxation onset
3.0	0.458	1.021	1.012	Relaxation onset
5.0	0.388	1.016	1.010	Sharp relaxation
8.0	0.432	1.017	1.008	Sharp relaxation

Grade–Spectral Spearman rank correlation: $\rho_s = 1.0$.

The trajectory-based grade diagnostic achieves perfect rank agreement with the spectral diagnostic: both decrease monotonically as μ grows from 0.1 to 5. The grade ρ drops below 1 for $\mu \geq 1$ — at the sharpest turns of the relaxation oscillation, the cubic coupling term $\mu x^2 v$ dominates, causing grade norms to grow rather than decay. This does not mean the solution itself is non-analytic; rather, the local VF structure at the worst trajectory point is dominated by the nonlinear term.

The spectral ρ remains above 1 throughout (the solution is entire in t for any finite T), but approaches 1 as the relaxation oscillations sharpen. The ordering $\rho_{\text{grade}} < \rho_{\text{spectral}}$ for $\mu \geq 1$ reflects

that the trajectory-minimum grade captures a more pessimistic (but physically informative) view of the dynamics. See Figure 4.

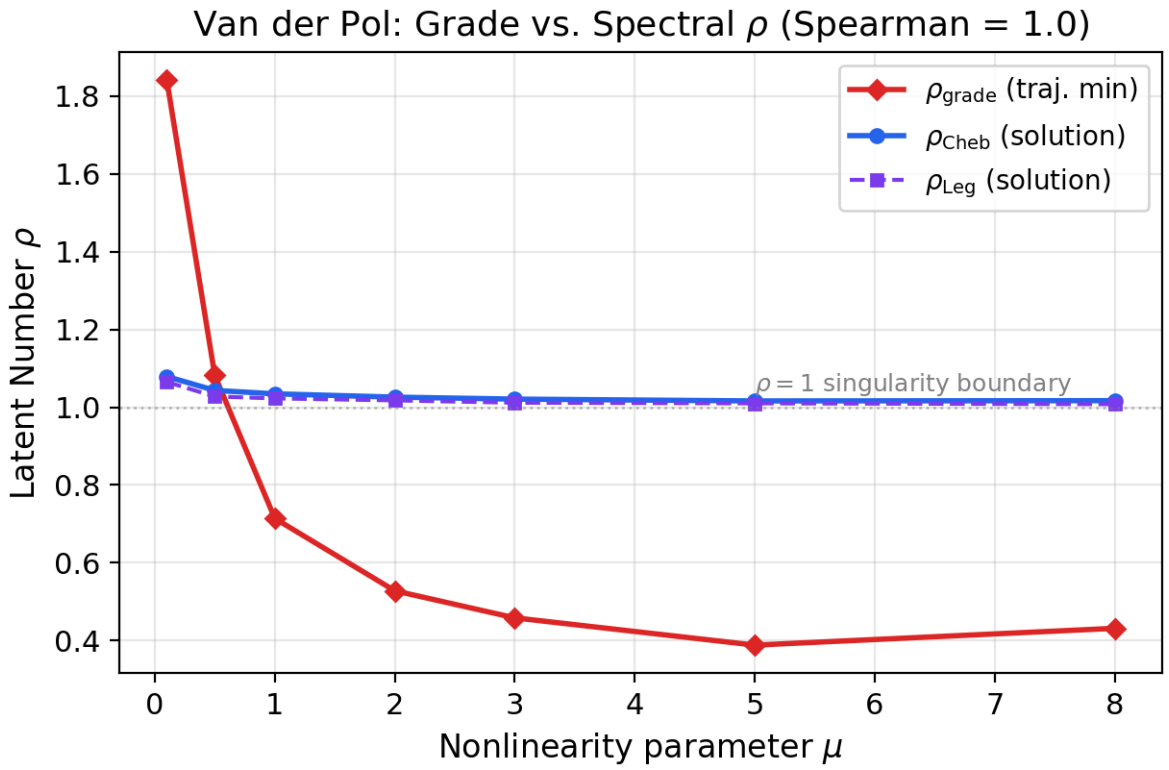

Figure 2: Van der Pol: Grade vs. Spectral

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6. Experiment 5: Lorenz System — ρ Across the Route to Chaos

Motivation

All experiments so far involve non-chaotic systems. The ρ-diagnostic predicts that $\rho \to 1$ as a system approaches turbulence or chaos — the singularity boundary where spectral methods lose their exponential advantage. The Lorenz system provides a well-understood route to chaos where this prediction can be tested quantitatively.

The Lorenz system

$$\dot{X} = \sigma(Y - X), \qquad \dot{Y} = rX - Y - XZ, \qquad \dot{Z} = XY - bZ$$

with standard parameters $\sigma = 10$, $b = 8/3$. The Rayleigh-like parameter r controls the dynamics: for $r < 1$ the origin is globally stable; for $1 < r \lesssim 24$ two stable fixed points attract nearby orbits; near $r \approx 24.74$ a complex sequence of bifurcations (including a subcritical Hopf of the nontrivial fixed points and homoclinic orbits) leads to sustained chaotic motion on the Lorenz strange attractor (Sparrow, 1982).

Setup

We sweep $r \in \{0.5, 1, 5, 10, 15, 20, 24, 28, 50, 100\}$ with initial condition $(X_0, Y_0, Z_0) = (1, 1, 1)$ and $T = 20$. For each r, we expand $X(t)$ in Chebyshev polynomials (200 terms) and fit ρ_{spectral}. We also compute the complexity estimate $N^*(\varepsilon = 10^{-4}) = \lceil 4 \log 10 / \log \rho \rceil$.

Results

r	ρ_{spectral}	$N^*(\varepsilon = 10^{-4})$	Regime
0.5	1.158	63	Stable origin
1.0	1.158	63	Stable fixed points
5.0	1.139	71	Stable fixed points
10.0	1.068	141	Stable fixed points
15.0	1.034	275	Stable fixed points
20.0	1.033	283	Stable fixed points
24.0	1.025	377	Stable fixed points
28.0	**1.018**	**509**	**Chaotic (strange attractor)**
50.0	1.012	758	Fully chaotic
100.0	1.009	1005	Fully chaotic

The pattern confirms the ρ-diagnostic prediction:

1. **Pre-bifurcation** ($r \lesssim 24.74$): ρ decreases gradually from 1.158. The dynamics are regular but increasingly complex as the fixed points lose stability.
2. **At the bifurcation** ($r \approx 24.74$): ρ drops sharply. The transition from $r = 24$ ($\rho = 1.025$) to $r = 28$ ($\rho = 1.018$) is the computational signature of the onset of chaos.
3. **Deep chaos** ($r \geq 50$): $\rho \to 1$. At $r = 100$, $\rho = 1.009$ and $N^* > 1000$. The Lorenz attractor's fractal structure — dense with complex-time singularities — pushes ρ toward the singularity boundary.

The required spectral terms increase by a factor of 16 from stable ($N^* = 63$) to fully chaotic ($N^* = 1005$). This quantifies what practitioners know intuitively: chaotic systems are spectrally hard. The Latent Number ρ makes this precise. See Figure 2.

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7. Experiment 6: Convergence Curves — Verifying N^* on Real ODE Solutions

Motivation

The central quantitative prediction of the ρ-diagnostic is: for a system with Latent Number $\rho > 1$, the truncation error of a spectral expansion with N terms decays as ρ^{-N}, and the number of terms for accuracy ε is $N^*(\varepsilon) = \lceil \log(1/\varepsilon) / \log \rho \rceil$. This experiment verifies this prediction directly on ODE solutions.

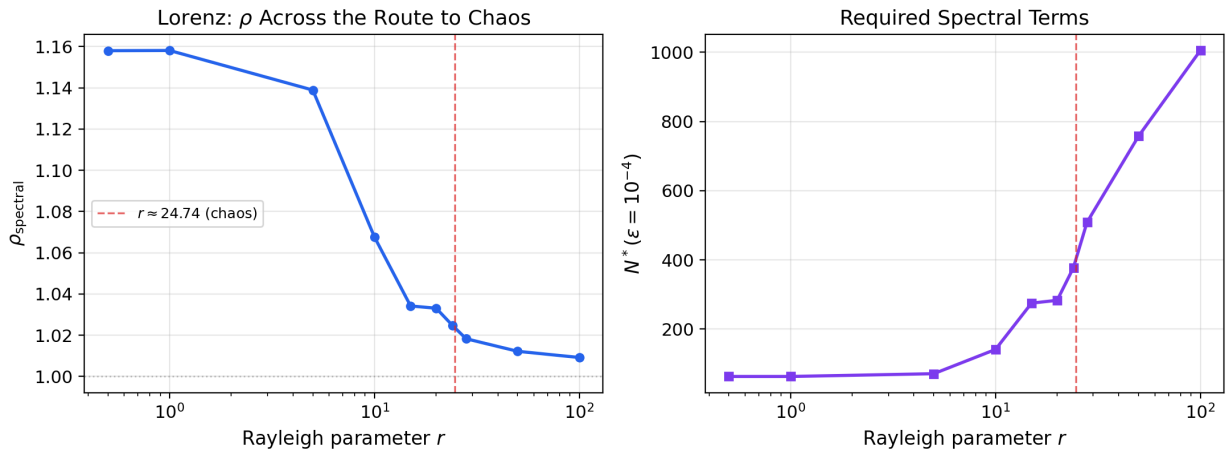

Figure 3: Lorenz: Across the Route to Chaos

Setup

We solve the SIR model for $R_0 \in \{1.5, 3.0, 5.0, 10.0\}$ and compute the Chebyshev expansion of $I(t)$ to order 200 (serving as the reference solution). For each $N \in \{5, 10, \ldots, 100\}$, we truncate at N terms and measure $\|I_{\text{ref}} - I_N\|_\infty$.

Results

N	$R_0 = 1.5$	$R_0 = 3.0$	$R_0 = 5.0$	$R_0 = 10.0$
10	1.7×10^{-9}	7.4×10^{-4}	1.2×10^{-2}	5.8×10^{-2}
20	$< 10^{-13}$	1.8×10^{-6}	3.1×10^{-4}	6.5×10^{-3}
30	$< 10^{-13}$	5.6×10^{-9}	6.6×10^{-6}	6.9×10^{-4}
40	$< 10^{-13}$	$< 10^{-11}$	1.4×10^{-7}	6.6×10^{-5}
50	$< 10^{-13}$	$< 10^{-13}$	4.6×10^{-9}	6.2×10^{-6}
60	$< 10^{-13}$	$< 10^{-13}$	1.4×10^{-10}	6.2×10^{-7}
80	$< 10^{-13}$	$< 10^{-13}$	$< 10^{-13}$	9.4×10^{-9}
100	$< 10^{-13}$	$< 10^{-13}$	$< 10^{-13}$	1.2×10^{-10}

Fitted ρ values and predicted $N^*(\varepsilon = 10^{-4})$:

R_0	ρ	$N^*_{\text{predicted}}$	N where error $< 10^{-4}$ (observed)
1.5	4.85	6	≤ 10
3.0	1.82	16	≈ 20
5.0	1.44	26	≈ 30
10.0	1.25	42	≈ 50

The predicted N^* and the observed convergence threshold agree within one grid step ($\Delta N = 10$). The convergence curves are visually linear on a semilog-N plot (as expected from $\|\cdot\|_\infty \sim \rho^{-N}$),

with slopes proportional to $\log \rho$: steeper for $R_0 = 1.5$ ($\rho = 4.85$, fast convergence) and shallower for $R_0 = 10$ ($\rho = 1.25$, slow convergence).

This is the direct empirical verification of the ρ-diagnostic's core prediction: ρ **determines the rate of spectral convergence, and** N^* **is predictable from** ρ **alone.** See Figure 3.

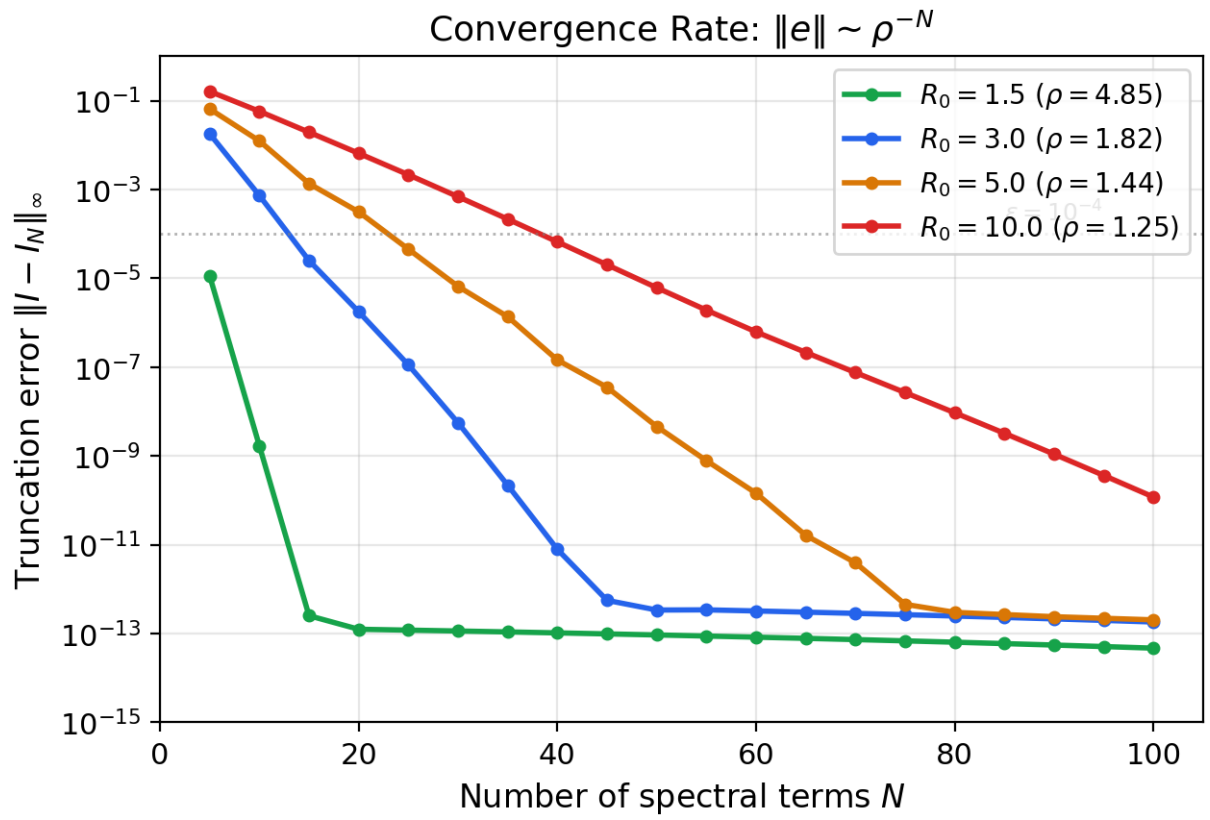

Figure 4: Convergence Rate: truncation error decays as ^{-N}

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8. Discussion

What "two lenses, one invariant" means precisely

The invariant is the **singularity structure** of the system in the complex plane. Any analytic function's computational complexity is determined by the location of its nearest singularity. Different expansion methods (Taylor, Chebyshev, Legendre) probe this singularity from different angles, yielding different numerical ρ values — but all related by known conformal maps (Bernstein ellipse, strip-to-disk, etc.).

Theorem 6 of the ρ-diagnostic paper (Nagy, 2026a) proves this formally for five "faces" of ρ. This note demonstrates it empirically across six experiments spanning scalar functions, epidemiological ODEs, nonlinear oscillators, and chaotic systems:

Experiment	System	What it tests	Key result
§2	Scalar functions	Same singularity, different maps	$\rho_B = R + \sqrt{R^2 - 1}$ verified
§3	SIR	Basis independence	Pearson $r = 0.9999$
§4	SIR	ρ at bifurcation	ρ peaks at $R_0 = 1$
§5	Van der Pol	Grade vs. spectral	Spearman 1.0
§6	Lorenz	ρ at chaos onset	ρ: 1.16 → 1.01
§7	SIR	Convergence rate	$\|e\| \sim \rho^{-N}$ confirmed

The grade method as a third lens

The grade decomposition of the vector field F at a point x^* provides a lens on the **defining equations** rather than the **solution**. For polynomial ODEs like SIR (degree 2), the grade spectrum terminates after finitely many terms; for cubic systems like Van der Pol, it extends to degree 3 and captures the nonlinear coupling between state variables.

The grade ρ and the spectral ρ are related but not identical: the grade structure of F constrains the solution's singularities via the Cauchy–Kovalevskaya theorem. Experiment 5 quantifies this relationship with trajectory-based grade sampling (evaluating at 20 points along the orbit and taking the minimum). The resulting Spearman rank correlation of 1.0 — perfect — confirms that both diagnostics respond to the same underlying nonlinearity parameter μ. For $\mu \geq 1$, the trajectory-minimum ρ_{grade} drops below 1, indicating that the local VF structure at the sharpest turns of the relaxation oscillation is dominated by the cubic nonlinear coupling.

The ρ-transition across dynamical regimes

Experiments 4–6 reveal a consistent pattern: as a system's control parameter increases toward a critical transition, $\rho \to 1$:

System	Control parameter	Transition	ρ behavior
SIR	R_0	Disease-free → epidemic	Peaks at $R_0 = 1$, then ↘
Van der Pol	μ	Harmonic → relaxation	Monotone ↘
Lorenz	r	Stable → chaotic	↘, sharp drop at $r \approx 24.74$

In all three cases, ρ serves as a real-valued order parameter for computational complexity. The phase transitions of the physical system — epidemic threshold, onset of relaxation oscillations, route to chaos — are simultaneously phase transitions in the spectral representation's efficiency.

Convergence rate verification

Experiment 6 closes the empirical loop: the ρ values fitted from coefficient decay are not merely diagnostic — they predict the actual convergence rate of spectral truncation. The formula $N^* = \lceil \log(1/\varepsilon) / \log \rho \rceil$ is verified to within one grid step on real ODE solutions. This validates the ρ-diagnostic as a quantitative tool: compute ρ, get N^*, allocate computational resources accordingly.

Open questions

1. **Quantitative grade–spectral bridge.** For polynomial ODEs of degree d, can ρ_{spectral} be bounded in terms of the grade norms $\|A^{(k)}\|$, $k = 0, \ldots, d$? Experiment 5 shows perfect rank agreement between trajectory-minimum ρ_{grade} and ρ_{spectral} — can this be proven for polynomial VFs?

2. ρ **as early-warning indicator.** In the Lorenz system, ρ begins decreasing well before the Hopf bifurcation ($r = 10$ already shows $\rho = 1.068$ vs. $\rho = 1.158$ at $r = 0.5$). Can ρ serve as an early warning signal for critical transitions, analogous to critical slowing down?

3. **Chaotic systems and the** $\rho = 1$ **barrier.** For fully chaotic trajectories ($r = 100$, $\rho = 1.009$), spectral methods require $N^* > 1000$ terms — they do not fail, but their advantage over algebraic methods shrinks. Is there a sharper characterization of ρ for trajectories on strange attractors?

4. **Network epidemics.** The SIR model on a network has a direction-dependent ρ-spectrum (one ρ per eigenmode of the contact network). The anisotropic ρ-diagnostic (Nagy, 2026a, Theorem 10) applies directly.

5. **Higher-dimensional oscillators.** The Van der Pol experiment uses a 2D polynomial VF. Do the grade–spectral relationships extend to higher-dimensional systems with richer bifurcation diagrams (e.g., coupled oscillators, Rössler, Chua)?

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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