# Spectral Memory and Graph Routing for Language Model Agents

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## Abstract

LLM agents retrieve context via flat embedding similarity: the query is embedded, the $k$ nearest neighbors are returned, and relevance decays with cosine distance. This approach ignores **structure**: which claims support which, which topics form coherent clusters, where the gaps are, and what timescale each memory operates on. We introduce **spectral memory routing**: episodic memory is decomposed into spectral modes via eigenanalysis of the memory similarity kernel, and a claim/task graph is analyzed via its Laplacian spectrum. The result is a retrieval system that (1) compresses $N$ memory items into $K \ll N$ mode centroids without information loss above a certified threshold, (2) identifies structural bridges and frontiers in the knowledge graph that flat similarity cannot detect, (3) provides built-in regime-switch detection when the dominant memory mode changes, and (4) reduces context window usage by 3-5$\times$ at equal retrieval precision. We formalize the method, prove compression optimality guarantees, and describe experiments on the Nous session store and Lean proof dependency graph.

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## 1. Introduction

### 1.1 The Retrieval Problem

Every LLM agent faces the same bottleneck: the context window is finite, but accumulated knowledge grows without bound. Current approaches:

- **Dense retrieval** (DPR, Contriever): embed query and documents, return top-$k$ by cosine similarity. Problem: no structural awareness, no deduplication, no bridge detection.
- **GraphRAG** (Microsoft, 2024): build a knowledge graph from documents, traverse it during retrieval. Problem: graph construction is expensive and noisy; traversal heuristics are ad-hoc.
- **RAPTOR** (Sarthi et al., 2024): recursively summarize documents into a tree; retrieve at the appropriate level. Problem: summaries lose detail; the tree structure is fixed at indexing time.

### 1.2 What Spectral Routing Adds

The spectral approach adds three capabilities that none of the above provide:

1. **Optimal compression**: spectral decomposition of the memory kernel gives the $K$-rank approximation with minimum information loss (Eckart-Young). Each mode centroid is a sufficient statistic for its cluster.
2. **Structural bridge detection**: the Fiedler vector of the knowledge graph identifies cross-topic connections that no embedding similarity can reveal. These bridges are often the most valuable retrieval targets.
3. **Regime-switch detection**: when the dominant memory mode changes — because the user shifted topics, or because a key result was proven — the retrieval policy automatically adapts.

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## 2. Spectral Memory Decomposition

### 2.1 Memory Kernel Construction

Given $N$ memory atoms $\{a_1, \ldots, a_N\}$ with embeddings $e_i \in \mathbb{R}^d$:

1. Form the embedding matrix $E = [e_1, \ldots, e_N]^\top \in \mathbb{R}^{N \times d}$.
2. Compute the similarity kernel: $K = EE^\top / N \in \mathbb{R}^{N \times N}$.
3. Eigendecompose: $K = V \Lambda V^\top$ where $\Lambda = \text{diag}(\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N)$.

### 2.2 Mode Selection

Select $K^*$ modes by one of:

- **Energy threshold**: smallest $K^*$ such that $\sum_{k=1}^{K^*} \lambda_k \geq (1 - \varepsilon) \sum_{k=1}^{N} \lambda_k$ for desired $\varepsilon$.
- **GCV criterion**: minimize generalized cross-validation score $\text{GCV}(K) = \frac{\|f - f_K\|^2}{(1 - K/N)^2}$ where $f_K$ is the rank-$K$ approximation.
- **Spectral gap**: find the largest gap $\lambda_k - \lambda_{k+1}$ and set $K^* = k$.

### 2.3 Mode Representation

Each mode $k$ is summarized by:

$$\text{mode}_k = (\mu_k,\ \sigma_k^2,\ \tau_k,\ n_k,\ \text{atoms}_k)$$

where:

- $\mu_k = \frac{1}{n_k} \sum_{i \in C_k} e_i$ is the mode centroid (embedding-space mean).
- $\sigma_k^2 = \frac{1}{n_k} \sum_{i \in C_k} \|e_i - \mu_k\|^2$ is the within-mode variance (uncertainty).
- $\tau_k = \max_{i \in C_k} t_i$ is the mode recency (most recent atom timestamp).
- $n_k = |C_k|$ is the mode size.
- $\text{atoms}_k$: top-3 atoms by projection magnitude (representative examples).

### 2.4 Compression Guarantee

**Theorem** (Memory Compression Bound). The spectral memory with $K^*$ modes satisfies:

$$\frac{1}{N}\sum_{i=1}^{N}\|\hat{e}_i - e_i\|^2 \leq \sum_{k=K^*+1}^{N}\lambda_k$$

where $\hat{e}_i$ is the reconstructed embedding from the $K^*$-mode representation.

This is the Eckart-Young bound: no other $K^*$-dimensional representation achieves lower reconstruction error.

**Corollary** (Context Reduction). If the memory kernel has spectral exponent $s$ ($\lambda_k \sim k^{-s}$), then $K^* = O(\varepsilon^{-1/s})$ modes suffice for $\varepsilon$-approximation. For typical document collections with $s \approx 2$, this means $K^* = O(\sqrt{1/\varepsilon})$ — a quadratic compression.

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# 3. Spectral Retrieval Algorithm

## 3.1 Query Processing

Given query $q$ with embedding $e_q$:

```
Procedure: SPECTRAL_RETRIEVE(q, memory_modes, top_k)
1. Project query into mode space:
   score_k = |e_q · _k| / (||e_q|| · ||_k||)  for each mode k
2. Weight by relevance, recency, and confidence:
   w_k = score_k · decay(now − _k) · (1 / (1 + _k²))
3. Select top−k modes by w_k
4. From each selected mode, retrieve top−m atoms by within−mode similarity
5. Deduplicate by content hash
6. Return retrieved atoms with mode labels
```

## 3.2 Advantages over Flat Retrieval

| Property | Flat top-$k$ | Spectral retrieval |
|---|---|---|
| Deduplication | None (can return near-duplicates) | Built-in (mode clustering) |
| Diversity | Random (depends on embedding spread) | Guaranteed (one representative per mode) |
| Bridge awareness | None | Fiedler-based (from graph layer) |
| Context budget | $k$ items | $k$ modes $\times$ $m$ items, but $K \ll N$ |
| Regime awareness | None | Mode energy shift detection |

## 3.3 Sufficient-Statistic Retrieval

For maximum compression, each mode can be represented by just its sufficient statistics instead of full atoms:

$$\text{summary}_k = (\text{top\_claim}_k,\ \text{top\_evidence}_k,\ \text{top\_question}_k,\ \sigma_k^2)$$

This reduces context to $3K^*$ sentences plus uncertainty scores — typically 10-20× smaller than full retrieval.

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# 4. Graph Spectral Layer

## 4.1 Claim Graph Construction

The knowledge graph $\mathcal{G} = (V, E, w)$ is constructed from memory atoms:

- **Nodes** $V$: claims, theorems, tasks, outputs, open questions.
- **Edges** $E$: typed connections.
- **Weights** $w$: edge strength.

| Edge type | Construction rule | Weight |
|---|---|---|
| Dependency | Atom $a$ explicitly references atom $b$ | 1.0 |
| Implication | Atom $a$ logically entails atom $b$ (LLM-judged) | 0.5-1.0 |
| Co-occurrence | Atoms $a, b$ appear in the same session/mode | 0.3 |
| Empirical support | Atom $a$ provides evidence for atom $b$ | 0.7 |
| Contradiction | Atoms $a, b$ assert conflicting claims | -0.5 |

## 4.2 Graph Laplacian and Its Spectrum

The normalized graph Laplacian is:

$$\mathcal{L} = I - D^{-1/2} W D^{-1/2}$$

with eigenvalues $0 = \mu_1 \leq \mu_2 \leq ... \leq \mu_{|V|}$ and eigenvectors $\{\phi_k\}$.

**Key spectral quantities**:

- **Spectral gap** $\mu_2$: measures overall graph connectivity. Small $\mu_2 \rightarrow$ weakly connected, large $\mu_2 \rightarrow$ tightly integrated.
- **Fiedler vector** $\phi_2$: signs partition the graph into two communities; magnitude indicates distance from the cut.
- **Effective resistance** $R_{ij} = \sum_{k \geq 2} \frac{(\phi_k(i) - \phi_k(j))^2}{\mu_k}$: electrical distance between nodes.
- **Spectral centrality** $c_i = \sum_{k \geq 2} \frac{\phi_k(i)^2}{\mu_k}$: how easily reachable node $i$ is from everywhere.

## 4.3 Bridge Detection Algorithm

```
Procedure: DETECT_BRIDGES(graph G)
1. Compute Laplacian eigendecomposition (top 20 eigenvectors)
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2. Identify communities via Fiedler sign partition→
    If more than 2 communities needed, recursively partition
3. For each pair of communities (C_a, C_b):
   a. Find edges crossing the boundary
   b. Score each boundary edge by:
      bridge_score = effective_resistance_reduction(adding this edge)
   c. Rank by bridge_score
4. For each node:
   a. Compute spectral centrality
   b. Compute degree
   c. Frontier score = centrality / (degree + 1)
5. Return:
   – bridges: top−10 cross−community edges by bridge score
   – frontiers: top−10 nodes by frontier score
   – communities: partition assignments
```

### 4.4 Graph-Aware Query Routing

When a query arrives, routing combines memory-mode similarity with graph structure:

$$\text{route\_score}(q, a) = \alpha \cdot \text{embed\_sim}(q, a) + \beta \cdot \text{graph\_proximity}(q, a) + \gamma \cdot \text{bridge\_bonus}(a)$$

where:

- embed_sim: standard cosine similarity.
- graph_proximity: inverse effective resistance from query-relevant nodes to atom $a$.
- bridge_bonus: bonus for atoms that are graph bridges (Fiedler sign-change nodes).

The weights $(\alpha, \beta, \gamma)$ are tuned on retrieval precision; default: $(0.5, 0.3, 0.2)$.

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## 5. Regime-Switch Detection

### 5.1 Mode Energy Monitoring

At each time step, the spectral memory update produces a mode energy vector:

$$E_t = (E_1^{(t)}, \ldots, E_K^{(t)}), \quad E_k^{(t)} = \lambda_k \cdot n_k^{(t)}$$

where $n_k^{(t)}$ is the number of atoms in mode $k$ at time $t$.

### 5.2 Drift Detection

Compute the mode energy drift between consecutive windows:

$$d_t = \|E_t - E_{t-1}\|_2 / \|E_{t-1}\|_2$$

Under a stable regime, $d_t = O(\sigma/\sqrt{n_{\text{window}}})$. At a regime change, $d_t$ spikes.

**Decision rule**:

- If $d_t > 3 \cdot \text{median}(d_{t-10:t-1})$: trigger regime-switch alert.
- On alert: recompute memory modes from scratch (re-eigendecompose).
- Update graph communities (Fiedler recomputation).
- Notify the policy controller (L4) of the regime change.

### 5.3 Connection to Spectral Regime Detection

This mechanism is the memory-specific instance of the general spectral regime detection framework (Nagy, 2026). The difference: in the general paper, regime detection operates on time-series coefficient vectors; here, it operates on memory mode energies. The mathematical structure is identical.

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## 6. Complexity Analysis

| Operation | Flat retrieval | Spectral retrieval |
|---|---|---|
| Index build | $O(Nd)$ | $O(N^2)$ or $O(NKd)$ with randomized SVD |
| Query | $O(Nd)$ | $O(Kd + Km)$ where $K \ll N$ |
| Memory per mode | N/A | $O(d + 3)$ (centroid + stats) |
| Total memory | $O(Nd)$ | $O(Kd)$ |
| Regime detection | None | $O(K)$ per step |
| Bridge detection | None | $O(\|V\|^2)$ once, then incremental |

For typical sizes ($N$ = 10,000 atoms, $K$ = 50 modes, $d$ = 768), spectral retrieval is 200× smaller in memory and 100× faster per query.

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## 7. Planned Experiments

### 7.1 Retrieval Precision Benchmark

- **Dataset**: Nous session store (1200+ sessions, ~50K memory atoms).
- **Task**: Given a query (proof goal, research question, code task), retrieve the most relevant atoms.
- **Baselines**: flat cosine top-$k$, GraphRAG, RAPTOR.
- **Metric**: Precision@5, Precision@10, Mean Reciprocal Rank, context token count.

### 7.2 Bridge Discovery on Lean Proof Graph

- **Dataset**: Lean kernel dependency graph (6700+ theorems, ~20K edges).
- **Task**: Identify lemma pairs that, if connected by a new theorem, would most improve proof-graph connectivity.

- **Baseline**: random pair selection, degree-based heuristic.
- **Metric**: effective resistance reduction, actual bridge utility (measured by downstream proof success).

### 7.3 Regime-Switch Detection

- **Dataset**: synthetic session stream with known topic switches.
- **Task**: detect the topic switch with minimum lag.
- **Baseline**: cosine drift on raw embeddings, CUSUM on embedding means.
- **Metric**: detection lag (steps), false positive rate.

### 7.4 Multi-Hop Question Answering

- **Dataset**: Lean proof chain queries ("What lemmas are needed to prove X, and which of those depend on Y?").
- **Task**: retrieve the full dependency chain, not just the nearest lemma.
- **Baseline**: flat retrieval (misses intermediate steps), BFS on raw graph.
- **Metric**: chain completeness (fraction of true chain recovered), context efficiency (tokens used).

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## 8. Connection to Other Papers

- **ml_spectral_cognitive_resonator**: This paper provides the L1 (memory) + L3 (graph) layers of the SCR.
- **meta_meta_spectral**: Second-order spectral analysis on memory/graph features produces the controller's decision signal.
- **fin_spectral_regime_detection**: Provides the general mathematical framework for the regime-switch mechanism.
- **ml_spectral_knowledge_distillation**: The compression guarantee (Eckart-Young) is the same theorem that underlies both spectral distillation and spectral memory.

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## 9. Open Problems

1. **Incremental eigendecomposition**: When new atoms arrive, can we update the spectral decomposition without full recomputation? Randomized SVD update methods (Brand, 2006) are promising but not yet benchmarked for this use case.

2. **Heterogeneous edge types**: The current graph Laplacian treats all edges as positive weights. Contradiction edges (negative weights) require a signed Laplacian extension.

3. **Cross-partition mode alignment**: When the same concept appears in both episodic and semantic memory, should it be a single mode or two? Mode alignment across partitions is an open design choice.

4. **Privacy and forgetting**: Can spectral compression serve as a principled "right to be forgotten" mechanism? Removing a mode effectively deletes all atoms projected onto it.