Spectral Overfitting

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Skeleton

Abstract

Spectral Overfitting — clean ProofEnv proof.

This paper presents 54 machine-verified theorems building on 0 established facts and 10 hypotheses. All results are formally verified in the Platonic proof kernel (88 verification units, 62 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (overfit_gap_nonneg). *Overfit Gap Nonneg.* [Platonic: overfit_gap_nonneg, domain: spectral_overfitting]

Theorem (composed_error_nonneg). *Composed Error Nonneg.* [Platonic: composed_error_nonneg, domain: spectral_overfitting]

Theorem (reg_dim_product_nonneg). *Reg Dim Product Nonneg.* [Platonic: reg_dim_product_nonneg, domain: spectral_overfitting]

Theorem (dim_gap_monotone). *Dim Gap Monotone.* [Platonic: dim_gap_monotone, domain: spectral_overfitting]

Theorem (gap_implies_test_ge_train). *Gap Implies Test Ge Train.* [Platonic: gap_implies_test_ge_train, domain: spectral_overfitting]

Theorem (sq_nonneg). *Sq Nonneg.* [Platonic: sq_nonneg, domain: spectral_overfitting]

Theorem (bias_variance_total_eq). *Bias Variance Total Eq.* [Platonic: bias_variance_total_eq, domain: spectral_overfitting]

Theorem (train_plus_gap_eq_test). *Train Plus Gap Eq Test.* [Platonic: train_plus_gap_eq_test, domain: spectral_overfitting]

Theorem (inv_n_monotone_surrogate). *Inv N Monotone Surrogate.* [Platonic: inv_n_monotone_surrogate, domain: spectral_overfitting]

Theorem (train_loss_refl_le). *Train Loss Refl Le.* [Platonic: train_loss_refl_le, domain: spectral_overfitting]

Theorem (test_loss_refl_le). *Test Loss Refl Le.* [Platonic: test_loss_refl_le, domain: spectral_overfitting]

Theorem (eff_dim_sq_nonneg). *Eff Dim Sq Nonneg.* [Platonic: eff_dim_sq_nonneg, domain: spectral_overfitting]

Theorem (reg_sq_nonneg). *Reg Sq Nonneg.* [Platonic: reg_sq_nonneg, domain: spectral_overfitting]

Theorem (bias_sq_nonneg). *Bias Sq Nonneg.* [Platonic: bias_sq_nonneg, domain: spectral_overfitting]

Theorem (var_sq_nonneg). *Var Sq Nonneg.* [Platonic: var_sq_nonneg, domain: spectral_overfitting]

Theorem (gap_abs_id). *Gap Abs Id.* [Platonic: gap_abs_id, domain: spectral_overfitting]

Theorem (add_cancel_train). *Add Cancel Train.* [Platonic: add_cancel_train, domain: spectral_overfitting]

Theorem (error_sum_sym). *Error Sum Sym.* [Platonic: error_sum_sym, domain: spectral_overfitting]

Theorem (scaled_train_nonneg). *Scaled Train Nonneg.* [Platonic: scaled_train_nonneg, domain: spectral_overfitting]

Theorem (scaled_test_nonneg). *Scaled Test Nonneg.* [Platonic: scaled_test_nonneg, domain: spectral_overfitting]

Theorem (dim_scale_nonneg). *Dim Scale Nonneg.* [Platonic: dim_scale_nonneg, domain: spectral_overfitting]

Theorem (reg_dim_sym). *Reg Dim Sym.* [Platonic: reg_dim_sym, domain: spectral_overfitting]

Theorem (double_gap_nonneg). *Double Gap Nonneg.* [Platonic: double_gap_nonneg, domain: spectral_overfitting]

Theorem (test_minus_train_sub). *Test Minus Train Sub.* [Platonic: test_minus_train_sub, domain: spectral_overfitting]

Theorem (nonneg_gap_squared). *Nonneg Gap Squared.* [Platonic: nonneg_gap_squared, domain: spectral_overfitting]

Theorem (loss_diff_triangle). *Loss Diff Triangle.* [Platonic: loss_diff_triangle, domain: spectral_overfitting]

Theorem (bias_le_error_sum). *Bias Le Error Sum.* [Platonic: bias_le_error_sum, domain: spectral_overfitting]

Theorem (var_le_error_sum). *Var Le Error Sum.* [Platonic: var_le_error_sum, domain: spectral_overfitting]

Theorem (bias_variance_decomp). *Bias Variance Decomp.* [Platonic: bias_variance_decomp, domain: spectral_overfitting]

Theorem (bias_variance_tradeoff). *Bias Variance Tradeoff.* [Platonic: bias_variance_tradeoff, domain: spectral_overfitting]

Theorem (bic_penalty_monotone). *Bic Penalty Monotone.* [Platonic: bic_penalty_monotone, domain: spectral_overfitting]

Theorem (interpolation_peak_nonneg). *Interpolation Peak Nonneg.* [Platonic: interpolation_peak_nonneg, domain: spectral_overfitting]

Theorem (implicit_reg_reduces_peak). *Implicit Reg Reduces Peak.* [Platonic: implicit_reg_reduces_peak, domain: spectral_overfitting]

Theorem (wave14_total_error_eq). *Wave14 Total Error Eq.* [Platonic: wave14_total_error_eq, domain: spectral_overfitting]

Theorem (wave14_bias_tail_identity). *Wave14 Bias Tail Identity.* [Platonic: wave14_bias_tail_identity, domain: spectral_overfitting]

Theorem (wave14_total_error_eq_sum). *Wave14 Total Error Eq Sum.* [Platonic: wave14_total_error_eq_sum domain: spectral_overfitting]

Theorem (wave14_cross_term_zero). *Wave14 Cross Term Zero.* [Platonic: wave14_cross_term_zero, domain: spectral_overfitting]

Theorem (wave14_prior_variance_eq_bias). *Wave14 Prior Variance Eq Bias.* [Platonic: wave14_prior_variance_eq_bias, domain: spectral_overfitting]

Theorem (wave14_variance_monotone_K). *Wave14 Variance Monotone K.* [Platonic: wave14_variance_monotone_K, domain: spectral_overfitting]

Theorem (wave14_variance_decreases_n). *Wave14 Variance Decreases N.* [Platonic: wave14_variance_decreases_n, domain: spectral_overfitting]

Theorem (wave14_error_margin_nonneg). *Wave14 Error Margin Nonneg.* [Platonic: wave14_error_margin_nonneg, domain: spectral_overfitting]

Theorem (wave14_error_margin_pos). *Wave14 Error Margin Pos.* [Platonic: wave14_error_margin_pos, domain: spectral_overfitting]

Theorem (wave14_optimal_K_approx_scalar). *Wave14 Optimal K Approx Scalar.* [Platonic: wave14_optimal_K_approx_scalar, domain: spectral_overfitting]

Theorem (wave14_ridge_filter_nonneg). *Wave14 Ridge Filter Nonneg.* [Platonic: wave14_ridge_filter_nonneg, domain: spectral_overfitting]

Theorem (wave14_ridge_filter_le_one). *Wave14 Ridge Filter Le One.* [Platonic: wave14_ridge_filter_le_one, domain: spectral_overfitting]

Theorem (wave14_bic_penalty_nonneg). *Wave14 Bic Penalty Nonneg.* [Platonic: wave14_bic_penalty_nonneg domain: spectral_overfitting]

Theorem (wave14_bic_penalty_monotone_K). *Wave14 Bic Penalty Monotone K.* [Platonic: wave14_bic_penalty_monotone_K, domain: spectral_overfitting]

Theorem (wave14_bayesian_gap_nonneg_rhs). *Wave14 Bayesian Gap Nonneg Rhs.* [Platonic: wave14_bayesian_gap_nonneg_rhs, domain: spectral_overfitting]

Theorem (wave14_minimum_rate_nonneg_scalar). *Wave14 Minimum Rate Nonneg Scalar.* [Platonic: wave14_minimum_rate_nonneg_scalar, domain: spectral_overfitting]

3. Bounds and Estimates

Theorem (variance_bound_refl). *Variance Bound Refl.* [Platonic: variance_bound_refl, domain: spectral_overfitting]

Theorem (wave14_gen_bound_tight). *Wave14 Gen Bound Tight.* [Platonic: wave14_gen_bound_tight, domain: spectral_overfitting]

Theorem (wave14_generalization_bound). *Wave14 Generalization Bound.* [Platonic: wave14_generalization_bound, domain: spectral_overfitting]

Theorem (wave14_variance_bound). *Wave14 Variance Bound.* [Platonic: wave14_variance_bound, domain: spectral_overfitting]

Theorem (wave14_gen_bound_var_term_monotone). *Wave14 Gen Bound Var Term Monotone.* [Platonic: wave14_gen_bound_var_term_monotone, domain: spectral_overfitting]

4. Formal Framework

Hypotheses

- L_train: L Train
- L_test: L Test
- eff_dim: Eff Dim
- reg: Reg
- bias_err: Bias Err
- var_err: Var Err
- total_err: Total Err
- bic_penalty: Bic Penalty
- n_samples: N Samples
- interp_peak: Interp Peak

5. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/spectral_overfitting/).

File	Role
spectral_overfitting_proof.py	
spectral_overfitting_wave14_proof.py	

6. Discussion

References