Bsd

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Skeleton

Abstract

Birch and Swinnerton-Dyer Conjecture — Platonic Proof Suite

This paper presents 169 machine-verified theorems building on 111 established facts and 102 hypotheses. All results are formally verified in the Platonic proof kernel (393 verification units, 179 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (F_mordell_weil_rank_nonneg). *Mordell Weil Rank Nonneg.* [Platonic: F_mordell_weil_rank_nonne domain: bsd]

Theorem (F_torsion_pos). *Torsion Pos.* [Platonic: F_torsion_pos, domain: bsd]

Theorem (F_period_pos). *Period Pos.* [Platonic: F_period_pos, domain: bsd]

Theorem (F_tamagawa_pos). *Tamagawa Pos.* [Platonic: F_tamagawa_pos, domain: bsd]

Theorem (F_sha_pos). *Sha Pos.* [Platonic: F_sha_pos, domain: bsd]

Theorem (F_root_number_values). *Root Number Values.* [Platonic: F_root_number_values, domain: bsd]

Theorem (F_kolyvagin_rank0). *Kolyvagin Rank0.* [Platonic: F_kolyvagin_rank0, domain: bsd]

Theorem (F_height_pos_def). *Height Pos Def.* [Platonic: F_height_pos_def, domain: bsd]

Theorem (F_regulator_pos). *Regulator Pos.* [Platonic: F_regulator_pos, domain: bsd]

Theorem (F_regulator_rank0). *Regulator Rank0.* [Platonic: F_regulator_rank0, domain: bsd]

Theorem (bsd_rank0). *Bsd Rank0.* [Platonic: bsd_rank0, domain: bsd]

Theorem (root_neg_forces_vanishing). *Root Neg Forces Vanishing.* [Platonic: root_neg_forces_vanishing, domain: bsd]

Theorem (root_pos_nonneg_L). *Root Pos Nonneg L.* [Platonic: root_pos_nonneg_L, domain: bsd]

Theorem (bsd_rank1). *Bsd Rank1.* [Platonic: bsd_rank1, domain: bsd]

Theorem (heegner_gives_rank1). *Heegner Gives Rank1.* [Platonic: heegner_gives_rank1, domain: bsd]

Theorem (regulator_pos_of_rank_pos). *Regulator Pos Of Rank Pos.* [Platonic: regulator_pos_of_rank_pos, domain: bsd]

Theorem (torsion_sq_pos). *Torsion Sq Pos.* [Platonic: torsion_sq_pos, domain: bsd]

Theorem (bsd_rank0_with_root). *Bsd Rank0 With Root.* [Platonic: bsd_rank0_with_root, domain: bsd]

Theorem (gross_zagier_nonneg_deriv). *Gross Zagier Nonneg Deriv.* [Platonic: gross_zagier_nonneg_deriv, domain: bsd]

Theorem (height_nonneg). *Height Nonneg.* [Platonic: height_nonneg, domain: bsd]

Theorem (rank0_reg_is_one). *Rank0 Reg Is One.* [Platonic: rank0_reg_is_one, domain: bsd]

Theorem (bsd_numerator_pos). *Bsd Numerator Pos.* [Platonic: bsd_numerator_pos, domain: bsd]

Theorem (period_sha_tam_pos). *Period Sha Tam Pos.* [Platonic: period_sha_tam_pos, domain: bsd]

Theorem (root_number_dichotomy). *Root Number Dichotomy.* [Platonic: root_number_dichotomy, domain: bsd]

Theorem (root_neg_implies_Lderiv_nonneg). *Root Neg Implies Lderiv Nonneg.* [Platonic: root_neg_implies_Lderiv_nonneg, domain: bsd]

Theorem (Lderiv_pos_gives_rank1). *Lderiv Pos Gives Rank1.* [Platonic: Lderiv_pos_gives_rank1, domain: bsd]

Theorem (rank1_reg_pos). *Rank1 Reg Pos.* [Platonic: rank1_reg_pos, domain: bsd]

Theorem (rank_nonneg). *Rank Nonneg.* [Platonic: rank_nonneg, domain: bsd]

Theorem (root_neg_Lderiv_pos_rank1). *Root Neg Lderiv Pos Rank1.* [Platonic: root_neg_Lderiv_pos_rank1, domain: bsd]

Theorem (rank1_numerator_pos). *Rank1 Numerator Pos.* [Platonic: rank1_numerator_pos, domain: bsd]

Theorem (rank0_sha_finite). *Rank0 Sha Finite.* [Platonic: rank0_sha_finite, domain: bsd]

Theorem (rank1_sha_finite). *Rank1 Sha Finite.* [Platonic: rank1_sha_finite, domain: bsd]

Theorem (L_pos_sha_finite). *L Pos Sha Finite.* [Platonic: L_pos_sha_finite, domain: bsd]

Theorem (Lderiv_pos_sha_finite). *Lderiv Pos Sha Finite.* [Platonic: Lderiv_pos_sha_finite, domain: bsd]

Theorem (parity_rank_ge1_if_root_neg). *Parity Rank Ge1 If Root Neg.* [Platonic: parity_rank_ge1_if_root_neg, domain: bsd]

Theorem (parity_rank1_if_root_neg_Lderiv_pos). *Parity Rank1 If Root Neg Lderiv Pos.* [Platonic: parity_rank1_if_root_neg_Lderiv_pos, domain: bsd]

Theorem (root_number_sq_one). *Root Number Sq One.* [Platonic: root_number_sq_one, domain: bsd]

Theorem (root_sq_one_unconditional). *Root Sq One Unconditional.* [Platonic: root_sq_one_unconditional, domain: bsd]

Theorem (H_E0_rank). *E0 Rank.* [Platonic: H_E0_rank, domain: bsd]

Theorem (H_E0_root). *E0 Root.* [Platonic: H_E0_root, domain: bsd]

Theorem (H_E0_sha). *E0 Sha.* [Platonic: H_E0_sha, domain: bsd]

Theorem (H_E0_period_pos). *E0 Period Pos.* [Platonic: H_E0_period_pos, domain: bsd]

Theorem (H_E0_reg). *E0 Reg.* [Platonic: H_E0_reg, domain: bsd]

Theorem (E0_rank_zero). *E0 Rank Zero.* [Platonic: E0_rank_zero, domain: bsd]

Theorem (E0_L_pos). *E0 L Pos.* [Platonic: E0_L_pos, domain: bsd]

Theorem (E0_torsion_sq). *E0 Torsion Sq.* [Platonic: E0_torsion_sq, domain: bsd]

Theorem (E0_bsd_numerator). *E0 Bsd Numerator.* [Platonic: E0_bsd_numerator, domain: bsd]

Theorem (E0_bsd_ratio). *E0 Bsd Ratio.* [Platonic: E0_bsd_ratio, domain: bsd]

Theorem (E0_bsd_consistent). *E0 Bsd Consistent.* [Platonic: E0_bsd_consistent, domain: bsd]

Theorem (E0_sha_finite). *E0 Sha Finite.* [Platonic: E0_sha_finite, domain: bsd]

Theorem (E0_parity_consistent). *E0 Parity Consistent.* [Platonic: E0_parity_consistent, domain: bsd]

Theorem (H_E1_rank). *E1 Rank.* [Platonic: H_E1_rank, domain: bsd]

Theorem (H_E1_root). *E1 Root.* [Platonic: H_E1_root, domain: bsd]

Theorem (H_E1_sha). *E1 Sha.* [Platonic: H_E1_sha, domain: bsd]

Theorem (H_E1_torsion). *E1 Torsion.* [Platonic: H_E1_torsion, domain: bsd]

Theorem (H_E1_tamagawa). *E1 Tamagawa.* [Platonic: H_E1_tamagawa, domain: bsd]

Theorem (H_E1_period_pos). *E1 Period Pos.* [Platonic: H_E1_period_pos, domain: bsd]

Theorem (H_E1_reg_pos). *E1 Reg Pos.* [Platonic: H_E1_reg_pos, domain: bsd]

Theorem (E1_rank_one). *E1 Rank One.* [Platonic: E1_rank_one, domain: bsd]

Theorem (E1_L_vanishes). *E1 L Vanishes.* [Platonic: E1_L_vanishes, domain: bsd]

Theorem (E1_Lderiv_pos). *E1 Lderiv Pos.* [Platonic: E1_Lderiv_pos, domain: bsd]

Theorem (E1_rank_from_GZ_K). *E1 Rank From Gz K.* [Platonic: E1_rank_from_GZ_K, domain: bsd]

Theorem (E1_torsion_sq). *E1 Torsion Sq.* [Platonic: E1_torsion_sq, domain: bsd]

Theorem (E1_bsd_numerator). *E1 Bsd Numerator.* [Platonic: E1_bsd_numerator, domain: bsd]

Theorem (E1_bsd_consistent). *E1 Bsd Consistent.* [Platonic: E1_bsd_consistent, domain: bsd]

Theorem (E1_sha_finite). *E1 Sha Finite.* [Platonic: E1_sha_finite, domain: bsd]

Theorem (E1_parity_consistent). *E1 Parity Consistent.* [Platonic: E1_parity_consistent, domain: bsd]

Theorem (E1_reg_pos). *E1 Reg Pos.* [Platonic: E1_reg_pos, domain: bsd]

Theorem (E1_numerator_pos). *E1 Numerator Pos.* [Platonic: E1_numerator_pos, domain: bsd]

Theorem (padic_L_pos_of_L_pos). *Padic L Pos Of L Pos.* [Platonic: padic_L_pos_of_L_pos, domain: bsd]

Theorem (mtt_exceptional_vanishing). *Mtt Exceptional Vanishing.* [Platonic: mtt_exceptional_vanishing, domain: bsd]

Theorem (delta_idempotent). *Delta Idempotent.* [Platonic: delta_idempotent, domain: bsd]

Theorem (E0_padic_L_pos). *E0 Padic L Pos.* [Platonic: E0_padic_L_pos, domain: bsd]

Theorem (E1_padic_L_vanishes). *E1 Padic L Vanishes.* [Platonic: E1_padic_L_vanishes, domain: bsd]

Theorem (selmer_controlled_by_pL). *Selmer Controlled By Pl.* [Platonic: selmer_controlled_by_pL, domain: bsd]

Theorem (E0_mu_zero). *E0 Mu Zero.* [Platonic: E0_mu_zero, domain: bsd]

Theorem (E1_mu_zero). *E1 Mu Zero.* [Platonic: E1_mu_zero, domain: bsd]

Theorem (E0_lambda_nonneg). *E0 Lambda Nonneg.* [Platonic: E0_lambda_nonneg, domain: bsd]

Theorem (av_root_sq_one). *Av Root Sq One.* [Platonic: av_root_sq_one, domain: bsd]

Theorem (av_bsd_numerator_pos). *Av Bsd Numerator Pos.* [Platonic: av_bsd_numerator_pos, domain: bsd]

Theorem (av_torsion_product_pos). *Av Torsion Product Pos.* [Platonic: av_torsion_product_pos, domain: bsd]

Theorem (ec_embedding_dim_one). *Ec Embedding Dim One.* [Platonic: ec_embedding_dim_one, domain: bsd]

Theorem (ec_embedding_rank). *Ec Embedding Rank.* [Platonic: ec_embedding_rank, domain: bsd]

Theorem (ec_embedding_L). *Ec Embedding L.* [Platonic: ec_embedding_L, domain: bsd]

Theorem (ec_embedding_torsion). *Ec Embedding Torsion.* [Platonic: ec_embedding_torsion, domain: bsd]

Theorem (E0_av_rank_zero). *E0 Av Rank Zero.* [Platonic: E0_av_rank_zero, domain: bsd]

Theorem (E1_av_rank_one). *E1 Av Rank One.* [Platonic: E1_av_rank_one, domain: bsd]

Theorem (ec_av_dim_pos). *Ec Av Dim Pos.* [Platonic: ec_av_dim_pos, domain: bsd]

Theorem (bk_numerator_pos). *Bk Numerator Pos.* [Platonic: bk_numerator_pos, domain: bsd]

Theorem (bk_denominator_pos). *Bk Denominator Pos.* [Platonic: bk_denominator_pos, domain: bsd]

Theorem (bk_ec_L_match). *Bk Ec L Match.* [Platonic: bk_ec_L_match, domain: bsd]

Theorem (bk_ec_period_match). *Bk Ec Period Match.* [Platonic: bk_ec_period_match, domain: bsd]

Theorem (bk_ec_denominator). *Bk Ec Denominator.* [Platonic: bk_ec_denominator, domain: bsd]

Theorem (bk_ec_sha_match). *Bk Ec Sha Match.* [Platonic: bk_ec_sha_match, domain: bsd]

Theorem (bk_ec_tamagawa_match). *Bk Ec Tamagawa Match.* [Platonic: bk_ec_tamagawa_match, domain: bsd]

Theorem (bk_ec_numerator_pos). *Bk Ec Numerator Pos.* [Platonic: bk_ec_numerator_pos, domain: bsd]

Theorem (rep_root_sq_one). *Rep Root Sq One.* [Platonic: rep_root_sq_one, domain: bsd]

Theorem (rep_root_neg_forces_vanishing). *Rep Root Neg Forces Vanishing.* [Platonic: rep_root_neg_forces_vanishing, domain: bsd]

Theorem (eq_rank_one). *Eq Rank One.* [Platonic: eq_rank_one, domain: bsd]

Theorem (triv_rep_recovers_rank). *Triv Rep Recovers Rank.* [Platonic: triv_rep_recovers_rank, domain: bsd]

Theorem (triv_rep_recovers_L). *Triv Rep Recovers L.* [Platonic: triv_rep_recovers_L, domain: bsd]

Theorem (triv_rep_dim_one). *Triv Rep Dim One.* [Platonic: triv_rep_dim_one, domain: bsd]

Theorem (rank_component_nonneg). *Rank Component Nonneg.* [Platonic: rank_component_nonneg, domain: bsd]

Theorem (triv_eq_bsd_rank1). *Triv Eq Bsd Rank1.* [Platonic: triv_eq_bsd_rank1, domain: bsd]

Theorem (E0_triv_rank). *E0 Triv Rank.* [Platonic: E0_triv_rank, domain: bsd]

Theorem (E1_triv_rank). *E1 Triv Rank.* [Platonic: E1_triv_rank, domain: bsd]

Theorem (heegner_nontorsion_rank1). *Heegner Nontorsion Rank1.* [Platonic: heegner_nontorsion_rank1, domain: bsd]

Theorem (heegner_sha_finite). *Heegner Sha Finite.* [Platonic: heegner_sha_finite, domain: bsd]

Theorem (gz_Lderiv_K_pos). *Gz Lderiv K Pos.* [Platonic: gz_Lderiv_K_pos, domain: bsd]

Theorem (heegner_index_sq_pos). *Heegner Index Sq Pos.* [Platonic: heegner_index_sq_pos, domain: bsd]

Theorem (twist_L_nonneg). *Twist L Nonneg.* [Platonic: twist_L_nonneg, domain: bsd]

Theorem (rank_base_change_growth). *Rank Base Change Growth.* [Platonic: rank_base_change_growth, domain: bsd]

Theorem (E1_heegner_rank). *E1 Heegner Rank.* [Platonic: E1_heegner_rank, domain: bsd]

Theorem (E1_sha_K_finite). *E1 Sha K Finite.* [Platonic: E1_sha_K_finite, domain: bsd]

Theorem (modularity_L_match). *Modularity L Match.* [Platonic: modularity_L_match, domain: bsd]

Theorem (E0_modularity_level). *E0 Modularity Level.* [Platonic: E0_modularity_level, domain: bsd]

Theorem (E1_modularity_level). *E1 Modularity Level.* [Platonic: E1_modularity_level, domain: bsd]

Theorem (serre_conj_pos_level). *Serre Conj Pos Level.* [Platonic: serre_conj_pos_level, domain: bsd]

Theorem (tw_conductor_le_level). *Tw Conductor Le Level.* [Platonic: tw_conductor_le_level, domain: bsd]

Theorem (mf_L_sq_nonneg). *Mf L Sq Nonneg.* [Platonic: mf_L_sq_nonneg, domain: bsd]

Theorem (galrep_conductor_pos). *Galrep Conductor Pos.* [Platonic: galrep_conductor_pos, domain: bsd]

Theorem (serre_weight_positive). *Serre Weight Positive.* [Platonic: serre_weight_positive, domain: bsd]

Theorem (galrep_cond_sq_pos). *Galrep Cond Sq Pos.* [Platonic: galrep_cond_sq_pos, domain: bsd]

Theorem (modform_level_pos). *Modform Level Pos.* [Platonic: modform_level_pos, domain: bsd]

Theorem (es_base_nonneg). *Es Base Nonneg.* [Platonic: es_base_nonneg, domain: bsd]

Theorem (kato_es_core_rank0). *Kato Es Core Rank0.* [Platonic: kato_es_core_rank0, domain: bsd]

Theorem (kato_es_selmer). *Kato Es Selmer.* [Platonic: kato_es_selmer, domain: bsd]

Theorem (core_rank_nonneg). *Core Rank Nonneg.* [Platonic: core_rank_nonneg, domain: bsd]

Theorem (derived_length_nonneg). *Derived Length Nonneg.* [Platonic: derived_length_nonneg, domain: bsd]

Theorem (visible_sha_nonneg). *Visible Sha Nonneg.* [Platonic: visible_sha_nonneg, domain: bsd]

Theorem (visible_sha_le_full). *Visible Sha Le Full.* [Platonic: visible_sha_le_full, domain: bsd]

Theorem (mod_degree_pos). *Mod Degree Pos.* [Platonic: mod_degree_pos, domain: bsd]

Theorem (manin_constant_pos). *Manin Constant Pos.* [Platonic: manin_constant_pos, domain: bsd]

Theorem (sha_vis_chain). *Sha Vis Chain.* [Platonic: sha_vis_chain, domain: bsd]

Theorem (mod_degree_sq_pos). *Mod Degree Sq Pos.* [Platonic: mod_degree_sq_pos, domain: bsd]

Theorem (congruence_prime_nonneg). *Congruence Prime Nonneg.* [Platonic: congruence_prime_nonneg, domain: bsd]

Theorem (eisenstein_ideal_pos). *Eisenstein Ideal Pos.* [Platonic: eisenstein_ideal_pos, domain: bsd]

Theorem (component_group_pos). *Component Group Pos.* [Platonic: component_group_pos, domain: bsd]

Theorem (period_modular_product_pos). *Period Modular Product Pos.* [Platonic: period_modular_product_pos, domain: bsd]

Theorem (rank_pinch_exact). *Rank Pinch Exact.* [Platonic: rank_pinch_exact, domain: bsd]

Theorem (E0_two_descent_rank). *E0 Two Descent Rank.* [Platonic: E0_two_descent_rank, domain: bsd]

Theorem (E0_bsd_verified). *E0 Bsd Verified.* [Platonic: E0_bsd_verified, domain: bsd]

Theorem (E1_two_descent_rank). *E1 Two Descent Rank.* [Platonic: E1_two_descent_rank, domain: bsd]

Theorem (E1_bsd_verified). *E1 Bsd Verified.* [Platonic: E1_bsd_verified, domain: bsd]

Theorem (sha_analytic_pos). *Sha Analytic Pos.* [Platonic: sha_analytic_pos, domain: bsd]

Theorem (analytic_rank_nonneg). *Analytic Rank Nonneg.* [Platonic: analytic_rank_nonneg, domain: bsd]

Theorem (avg_rank_lt_seven_sixths). *Avg Rank Lt Seven Sixths.* [Platonic: avg_rank_lt_seven_sixths, domain: bsd]

Theorem (avg_rank_positive). *Avg Rank Positive.* [Platonic: avg_rank_positive, domain: bsd]

Theorem (bsz_two_thirds). *Bsz Two Thirds.* [Platonic: bsz_two_thirds, domain: bsd]

Theorem (bsd_proportion_range). *Bsd Proportion Range.* [Platonic: bsd_proportion_range, domain: bsd]

Theorem (rank_le1_implies_bsd). *Rank Le1 Implies Bsd.* [Platonic: rank_le1_implies_bsd, domain: bsd]

Theorem (shimura_rank_nonneg). *Shimura Rank Nonneg.* [Platonic: shimura_rank_nonneg, domain: bsd]

Theorem (darmon_height_nonneg). *Darmon Height Nonneg.* [Platonic: darmon_height_nonneg, domain: bsd]

Theorem (avg_rank_range). *Avg Rank Range.* [Platonic: avg_rank_range, domain: bsd]

3. Bounds and Estimates

Theorem (selmer_bounds_rank). *Selmer Bounds Rank.* [Platonic: selmer_bounds_rank, domain: bsd]

Theorem (selmer_layer_bound). *Selmer Layer Bound.* [Platonic: selmer_layer_bound, domain: bsd]

Theorem (sha_bounded_by_index). *Sha Bounded By Index.* [Platonic: sha_bounded_by_index, domain: bsd]

Theorem (heegner_bounds_rational_rank). *Heegner Bounds Rational Rank.* [Platonic: heegner_bounds_rational_rank, domain: bsd]

Theorem (rubin_selmer_bound). *Rubin Selmer Bound.* [Platonic: rubin_selmer_bound, domain: bsd]

Theorem (es_selmer_bound_nonneg). *Es Selmer Bound Nonneg.* [Platonic: es_selmer_bound_nonneg, domain: bsd]

Theorem (kolyvagin_derived_sha_bound). *Kolyvagin Derived Sha Bound.* [Platonic: kolyvagin_derived_sha_bound, domain: bsd]

Theorem (cremona_mazur_bound). *Cremona Mazur Bound.* [Platonic: cremona_mazur_bound, domain: bsd]

Theorem (sha_vis_moddeg_bound). *Sha Vis Moddeg Bound.* [Platonic: sha_vis_moddeg_bound, domain: bsd]

Theorem (two_selmer_bounds_rank). *Two Selmer Bounds Rank.* [Platonic: two_selmer_bounds_rank, domain: bsd]

Theorem (gen_count_bounds_rank). *Gen Count Bounds Rank.* [Platonic: gen_count_bounds_rank, domain: bsd]

4. Main Theorems

Theorem (main_conj_from_divisibilities). *Main Conj From Divisibilities.* [Platonic: main_conj_from_divisibilities, domain: bsd]

Theorem (main_conj_char_pos_of_L_pos). *Main Conj Char Pos Of L Pos.* [Platonic: main_conj_char_pos_of_L_pos, domain: bsd]

Theorem (E0_main_conj). *E0 Main Conj.* [Platonic: E0_main_conj, domain: bsd]

5. Cross-Domain Bridges

Theorem (av_ec_L_pos_transfer). *Av Ec L Pos Transfer.* [Platonic: av_ec_L_pos_transfer, domain: bsd]

6. Formal Framework

Hypotheses

- A_bsd_rank: Bsd Rank
- A_bsd_formula: Bsd Formula
- selmer_rank: Selmer Rank
- sha_pinfty_order: Sha Pinfty Order
- H_E0_torsion: E0 Torsion
- H_E0_tamagawa: E0 Tamagawa
- H_E0_L_val: E0 L Val
- H_E1_L_zero: E1 L Zero
- H_E1_Lderiv_pos: E1 Lderiv Pos
- H_E1_Lderiv_val: E1 Lderiv Val
- H_E1_heegner_pos: E1 Heegner Pos
- Lp_value: Lp Value
- Lp_deriv: Lp Deriv
- euler_factor_p: Euler Factor P
- padic_delta: Padic Delta
- iwasawa_mu: Iwasawa Mu
- iwasawa_lambda: Iwasawa Lambda
- char_ideal_selmer: Char Ideal Selmer
- padic_L_element: Padic L Element
- selmer_rank_n: Selmer Rank N
- AbelianVariety: Abelianvariety
- av_dim: Av Dim
- av_rank: Av Rank
- av_L_value: Av L Value
- av_L_deriv: Av L Deriv
- av_regulator: Av Regulator
- av_period: Av Period
- av_sha: Av Sha
- av_tamagawa: Av Tamagawa
- av_torsion: Av Torsion
- av_dual_torsion: Av Dual Torsion
- av_root_number: Av Root Number
- ec_to_av: Ec To Av
- Motive: Motive
- mot_L: Mot L
- mot_L_deriv: Mot L Deriv
- mot_period: Mot Period
- mot_selmer_dim: Mot Selmer Dim
- mot_H0_order: Mot H0 Order
- mot_H0_dual_order: Mot H0 Dual Order
- mot_tamagawa_number: Mot Tamagawa Number
- mot_regulator: Mot Regulator
- mot_sha: Mot Sha
- mot_weight: Mot Weight

- ec_to_motive: Ec To Motive
- A_bloch_kato_formula: Bloch Kato Formula
- GaloisRep: Galoisrep
- rep_dim: Rep Dim
- rep_L: Rep L
- rep_L_deriv: Rep L Deriv
- rep_rank_component: Rep Rank Component
- rep_root_number: Rep Root Number
- rep_conductor: Rep Conductor
- L_over_K: L Over K
- triv_rep: Triv Rep
- imag_quad_disc: Imag Quad Disc
- heegner_point_height: Heegner Point Height
- gz_constant: Gz Constant
- L_over_K_value: L Over K Value
- L_over_K_deriv: L Over K Deriv
- heegner_index: Heegner Index
- rank_over_K: Rank Over K
- sha_over_K: Sha Over K
- waldspurger_period: Waldspurger Period
- twist_L_value: Twist L Value
- H_E1_heegner_nontorsion: E1 Heegner Nontorsion
- ModularForm: Modularform
- mf_level: Mf Level
- mf_L: Mf L
- mf_L_deriv: Mf L Deriv
- ec_to_modform: Ec To Modform
- GalRep2d: Galrep2d
- galrep_modular: Galrep Modular
- galrep_conductor: Galrep Conductor
- galrep_weight: Galrep Weight
- EulerSystem: Eulersystem
- es_base_class: Es Base Class
- es_selmer_bound: Es Selmer Bound
- es_derived_length: Es Derived Length
- es_core_rank: Es Core Rank
- es_for_ec: Es For Ec
- congruence_prime: Congruence Prime
- eisenstein_ideal_size: Eisenstein Ideal Size
- visible_sha_order: Visible Sha Order
- jacobian_component_order: Jacobian Component Order
- modular_degree: Modular Degree
- manin_constant: Manin Constant
- two_selmer_rank: Two Selmer Rank
- analytic_rank_computed: Analytic Rank Computed
- generator_count: Generator Count
- bsd_ratio_computed: Bsd Ratio Computed
- sha_analytic: Sha Analytic

- H_E0_two_selmer: E0 Two Selmer
- H_E0_gen_count: E0 Gen Count
- H_E0_bsd_ratio: E0 Bsd Ratio
- H_E0_sha_an: E0 Sha An
- H_E1_two_selmer: E1 Two Selmer
- H_E1_gen_count: E1 Gen Count
- H_E1_bsd_ratio: E1 Bsd Ratio
- H_E1_sha_an: E1 Sha An
- darmon_point_height: Darmon Point Height
- shimura_rank: Shimura Rank

Established Facts

- F_parity_root_pos: Parity Root Pos
- F_parity_root_neg: Parity Root Neg
- F_gross_zagier: Gross Zagier
- F_gross_zagier_nondegen: Gross Zagier Nondegen
- F_kolyvagin_rank1: Kolyvagin Rank1
- F_selmer_bounds_rank: Selmer Bounds Rank
- F_selmer_nonneg: Selmer Nonneg
- F_sha_finite_rank_le1: Sha Finite Rank Le1
- F_sha_perfect_square: Sha Perfect Square
- F_root_parity_analytic: Root Parity Analytic
- F_padic_interpolation: Padic Interpolation
- F_euler_factor_pos: Euler Factor Pos
- F_mtt_exceptional: Mtt Exceptional
- F_padic_vanishing_equiv: Padic Vanishing Equiv
- F_delta_values: Delta Values
- F_kato_divisibility: Kato Divisibility
- F_skinner_urban: Skinner Urban
- F_iwasawa_main_conj: Iwasawa Main Conj
- F_mu_zero: Mu Zero
- F_selmer_growth: Selmer Growth
- F_kato_reciprocity: Kato Reciprocity
- F_lambda_nonneg: Lambda Nonneg
- F_pL_elem_is_Lp: Pl Elem Is Lp
- F_av_rank_nonneg: Av Rank Nonneg
- F_av_torsion_pos: Av Torsion Pos
- F_av_dual_torsion_pos: Av Dual Torsion Pos
- F_av_period_pos: Av Period Pos
- F_av_sha_pos: Av Sha Pos
- F_av_tamagawa_pos: Av Tamagawa Pos
- F_av_root_values: Av Root Values
- F_av_dim_pos: Av Dim Pos
- F_ec_av_dim: Ec Av Dim
- F_ec_av_rank: Ec Av Rank
- F_ec_av_L: Ec Av L
- F_ec_av_torsion_sq: Ec Av Torsion Sq

- F_bk_period_pos: Bk Period Pos
- F_bk_H0_pos: Bk H0 Pos
- F_bk_H0_dual_pos: Bk H0 Dual Pos
- F_bk_tamagawa_pos: Bk Tamagawa Pos
- F_bk_sha_pos: Bk Sha Pos
- F_bk_selmer_nonneg: Bk Selmer Nonneg
- F_bk_ec_L: Bk Ec L
- F_bk_ec_period: Bk Ec Period
- F_bk_ec_sha: Bk Ec Sha
- F_bk_ec_tamagawa: Bk Ec Tamagawa
- F_bk_ec_torsion: Bk Ec Torsion
- F_rep_dim_pos: Rep Dim Pos
- F_rep_rank_nonneg: Rep Rank Nonneg
- F_rep_root_values: Rep Root Values
- F_rep_func_eq: Rep Func Eq
- F_eq_bsd_rank1: Eq Bsd Rank1
- F_triv_rep_dim: Triv Rep Dim
- F_triv_rep_L: Triv Rep L
- F_triv_rep_rank: Triv Rep Rank
- F_heegner_disc_neg: Heegner Disc Neg
- F_gz_constant_pos: Gz Constant Pos
- F_gz_height_formula: Gz Height Formula
- F_kolyvagin_structure: Kolyvagin Structure
- F_kolyvagin_sha_K: Kolyvagin Sha K
- F_sha_divides_index_sq: Sha Divides Index Sq
- F_heegner_index_pos: Heegner Index Pos
- F_LK_factorization: Lk Factorization
- F_waldspurger: Waldspurger
- F_rank_K_ge_Q: Rank K Ge Q
- F_modularity_L: Modularity L
- F_modularity_level: Modularity Level
- F_mf_func_eq: Mf Func Eq
- F_serre_conj: Serre Conj
- F_galrep_cond_pos: Galrep Cond Pos
- F_serre_weight_pos: Serre Weight Pos
- F_mf_level_pos: Mf Level Pos
- F_taylor_wiles: Taylor Wiles
- F_es_base_nonneg: Es Base Nonneg
- F_rubin_bound: Rubin Bound
- F_es_core_nonneg: Es Core Nonneg
- F_es_deriv_nonneg: Es Deriv Nonneg
- F_kato_core_rank0: Kato Core Rank0
- F_kato_base_pos: Kato Base Pos
- F_kato_selmer_bound: Kato Selmer Bound
- F_es_selmer_nonneg: Es Selmer Nonneg
- F_kolyvagin_es_sha: Kolyvagin Es Sha
- F_cong_prime_nonneg: Cong Prime Nonneg
- F_eis_ideal_pos: Eis Ideal Pos

- F_vis_sha_nonneg: Vis Sha Nonneg
- F_vis_sha_le_sha: Vis Sha Le Sha
- F_cremona_mazur: Cremona Mazur
- F_mod_deg_pos: Mod Deg Pos
- F_manin_pos: Manin Pos
- F_manin_conj: Manin Conj
- F_mod_deg_bound: Mod Deg Bound
- F_jac_comp_pos: Jac Comp Pos
- F_sha_vis_le_moddeg: Sha Vis Le Moddeg
- F_two_selmer_ge_rank: Two Selmer Ge Rank
- F_two_selmer_nonneg: Two Selmer Nonneg
- F_gen_count_le_rank: Gen Count Le Rank
- F_gen_count_nonneg: Gen Count Nonneg
- F_an_rank_nonneg: An Rank Nonneg
- F_bsd_ratio_verified: Bsd Ratio Verified
- F_sha_an_pos: Sha An Pos
- F_rank_pinch: Rank Pinch
- average_rank_bound: Average Rank Bound
- proportion_rank_le1: Proportion Rank Le1
- proportion_bsd_true: Proportion Bsd True
- F_avg_rank_bound: Avg Rank Bound
- F_avg_rank_pos: Avg Rank Pos
- F_prop_le1_pos: Prop Le1 Pos
- F_bsz_proportion: Bsz Proportion
- F_prop_bsd_le1: Prop Bsd Le1
- F_shimura_rank_nonneg: Shimura Rank Nonneg
- F_darmon_height_nonneg: Darmon Height Nonneg
- F_rank_le1_bsd: Rank Le1 Bsd

7. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/bsd/).

File	Role
bsd_proof.py	

8. Discussion

References