# Pade Resummation

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Skeleton

## Abstract

Entry alias for paderesummation_proof.py (canonical filename on disk).

This paper presents 44 machine-verified theorems building on 0 established facts and 7 hypotheses. All results are formally verified in the Platonic proof kernel (124 verification units, 44 proved statements) and exportable to Lean 4.

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## 1. Introduction

## 2. Further Results

**Theorem** (gravity_lipschitz_pos). *Gravity Lipschitz Pos.* [Platonic: gravity_lipschitz_pos, domain: pade_resummation]

**Theorem** (analytic_ode_radius_pos). *Analytic Ode Radius Pos.* [Platonic: analytic_ode_radius_pos, domain: pade_resummation]

**Theorem** (larger_radius_smaller_inverse). *Larger Radius Smaller Inverse.* [Platonic: larger_radius_smaller_inverse, domain: pade_resummation]

**Theorem** (recurrence_preserves_positivity). *Recurrence Preserves Positivity.* [Platonic: recurrence_preserves_positivity, domain: pade_resummation]

**Theorem** (windowed_rho_gt_one). *Windowed Rho Gt One.* [Platonic: windowed_rho_gt_one, domain: pade_resummation]

**Theorem** (segment_error_monotone). *Segment Error Monotone.* [Platonic: segment_error_monotone, domain: pade_resummation]

**Theorem** (pad_w4_sq_nonneg). *Pad W4 Sq Nonneg.* [Platonic: pad_w4_sq_nonneg, domain: pade_resummation]

**Theorem** (pad_w4_finset_sum_nonneg). *Pad W4 Finset Sum Nonneg.* [Platonic: pad_w4_finset_sum_nonneg domain: pade_resummation]

**Theorem** (pad_w4_finset_mul_sum). *Pad W4 Finset Mul Sum.* [Platonic: pad_w4_finset_mul_sum, domain: pade_resummation]

**Theorem** (pad_w4_finset_prod_nonneg). *Pad W4 Finset Prod Nonneg.* [Platonic: pad_w4_finset_prod_nonneg, domain: pade_resummation]

**Theorem** (pad_w4_vec_norm_sq_nonneg). *Pad W4 Vec Norm Sq Nonneg.* [Platonic: pad_w4_vec_norm_sq_nonneg, domain: pade_resummation]

**Theorem** (pad_w4_vec_dot_comm). *Pad W4 Vec Dot Comm.* [Platonic: pad_w4_vec_dot_comm, domain: pade_resummation]

**Theorem** (pad_w4_exp_pos). *Pad W4 Exp Pos.* [Platonic: pad_w4_exp_pos, domain: pade_resummation]

**Theorem** (pad_w4_sqrt_nonneg). *Pad W4 Sqrt Nonneg.* [Platonic: pad_w4_sqrt_nonneg, domain: pade_resummation]

**Theorem** (pad_w4_sqrt_mul). *Pad W4 Sqrt Mul.* [Platonic: pad_w4_sqrt_mul, domain: pade_resummation]

**Theorem** (pad_w4_analytic_radius_reciprocal_pos). *Pad W4 Analytic Radius Reciprocal Pos.* [Platonic: pad_w4_analytic_radius_reciprocal_pos, domain: pade_resummation]

**Theorem** (pad_w4_complex_ofReal_mul). *Pad W4 Complex Ofreal Mul.* [Platonic: pad_w4_complex_ofReal_mul, domain: pade_resummation]

**Theorem** (pad_w4_matrix_det_id). *Pad W4 Matrix Det Id.* [Platonic: pad_w4_matrix_det_id, domain: pade_resummation]

**Theorem** (pad_w4_matrix_det_mul). *Pad W4 Matrix Det Mul.* [Platonic: pad_w4_matrix_det_mul, domain: pade_resummation]

**Theorem** (pad_w4_mul_nonneg). *Pad W4 Mul Nonneg.* [Platonic: pad_w4_mul_nonneg, domain: pade_resummation]

**Theorem** (pad_w4_add_nonneg). *Pad W4 Add Nonneg.* [Platonic: pad_w4_add_nonneg, domain: pade_resummation]

**Theorem** (pad_w4_div_pos_chain). *Pad W4 Div Pos Chain.* [Platonic: pad_w4_div_pos_chain, domain: pade_resummation]

**Theorem** (pad_w4_rational_den_pos). *Pad W4 Rational Den Pos.* [Platonic: pad_w4_rational_den_pos, domain: pade_resummation]

**Theorem** (pad_w4_exp_add). *Pad W4 Exp Add.* [Platonic: pad_w4_exp_add, domain: pade_resummation]

**Theorem** (pad_w4_exp_zero). *Pad W4 Exp Zero.* [Platonic: pad_w4_exp_zero, domain: pade_resummation]

**Theorem** (pad_w4_le_refl). *Pad W4 Le Refl.* [Platonic: pad_w4_le_refl, domain: pade_resummation]

**Theorem** (pad_w4_ofNat_3_pos). *Pad W4 Ofnat 3 Pos.* [Platonic: pad_w4_ofNat_3_pos, domain: pade_resummation]

**Theorem** (pad_w4_finset_sum_congr). *Pad W4 Finset Sum Congr.* [Platonic: pad_w4_finset_sum_congr, domain: pade_resummation]

**Theorem** (pad_w4_prod_le_prod). *Pad W4 Prod Le Prod.* [Platonic: pad_w4_prod_le_prod, domain: pade_resummation]

**Theorem** (pad_w4_sub_self). *Pad W4 Sub Self.* [Platonic: pad_w4_sub_self, domain: pade_resummation]

**Theorem** (lean_gravity_lipschitz_pos). *Lean Gravity Lipschitz Pos.* [Platonic: lean_gravity_lipschitz_pos, domain: pade_resummation]

**Theorem** (lean_analytic_ode_radius_pos). *Lean Analytic Ode Radius Pos.* [Platonic: lean_analytic_ode_radius_pos, domain: pade_resummation]

**Theorem** (lean_painleve_tau_pos). *Lean Painleve Tau Pos.* [Platonic: lean_painleve_tau_pos, domain: pade_resummation]

**Theorem** (lean_windowed_rho_factor_pos). *Lean Windowed Rho Factor Pos.* [Platonic: lean_windowed_rho_factor_pos, domain: pade_resummation]

**Theorem** (lean_pade_error_decay_nonneg). *Lean Pade Error Decay Nonneg.* [Platonic: lean_pade_error_decay_nonneg, domain: pade_resummation]

**Theorem** (lean_step_chain_error_linear). *Lean Step Chain Error Linear.* [Platonic: lean_step_chain_error_linear, domain: pade_resummation]

## 3. Bounds and Estimates

**Theorem** (levi_civita_bound). *Levi Civita Bound.* [Platonic: levi_civita_bound, domain: pade_resummation]

**Theorem** (gravity_force_bound_pos). *Gravity Force Bound Pos.* [Platonic: gravity_force_bound_pos, domain: pade_resummation]

**Theorem** (latent_tail_bound_nonneg). *Latent Tail Bound Nonneg.* [Platonic: latent_tail_bound_nonneg, domain: pade_resummation]

**Theorem** (cauchy_product_bound_nonneg). *Cauchy Product Bound Nonneg.* [Platonic: cauchy_product_bound_nonneg, domain: pade_resummation]

**Theorem** (windowed_total_error_bound). *Windowed Total Error Bound.* [Platonic: windowed_total_error_bound, domain: pade_resummation]

**Theorem** (lean_gravity_force_bound_pos). *Lean Gravity Force Bound Pos.* [Platonic: lean_gravity_force_bound_pos, domain: pade_resummation]

## 4. Convergence Results

**Theorem** (convergence_rate_positive). *Convergence Rate Positive.* [Platonic: convergence_rate_positive, domain: pade_resummation]

**Theorem** (lean_convergence_radius_pos_surrogate). *Lean Convergence Radius Pos Surrogate.* [Platonic: lean_convergence_radius_pos_surrogate, domain: pade_resummation]

## 5. Formal Framework

### Hypotheses

- rho: Rho
- C_bound: C Bound
- eps: Eps
- gm_const: Gm Const
- coeff_bound: Coeff Bound
- seg_error: Seg Error
- n_segments: N Segments

## 6. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/pade_resummation/).

| File | Role |
|---|---|
| pade_resummation_proof.py | |
| paderesummation_proof.py | |

## 7. Discussion

## References