Prime Number Theorem

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Skeleton

Abstract

Prime Number Theorem — Logical Structure

This paper presents 36 machine-verified theorems building on 0 established facts and 64 hypotheses. All results are formally verified in the Platonic proof kernel (138 verification units, 36 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (step1_341_nonneg). *Step1 341 Nonneg.* [Platonic: step1_341_nonneg, domain: prime_number_theorem]

Theorem (step2_product_ge_one). *Step2 Product Ge One.* [Platonic: step2_product_ge_one, domain: prime_number_theorem]

Theorem (step3_net_exponent_positive). *Step3 Net Exponent Positive.* [Platonic: step3_net_exponent_positiv domain: prime_number_theorem]

Theorem (step3_vanishing_small). *Step3 Vanishing Small.* [Platonic: step3_vanishing_small, domain: prime_number_theorem]

Theorem (step4_psi_positive). *Step4 Psi Positive.* [Platonic: step4_psi_positive, domain: prime_number_theorem]

Theorem (step4_psi_near_one). *Step4 Psi Near One.* [Platonic: step4_psi_near_one, domain: prime_number_theorem]

Theorem (step5_pi_near_one). *Step5 Pi Near One.* [Platonic: step5_pi_near_one, domain: prime_number_theorem]

Theorem (pnt_dependency_count). *Pnt Dependency Count.* [Platonic: pnt_dependency_count, domain: prime_number_theorem]

Theorem (three_four_one_identity). *Three Four One Identity.* [Platonic: three_four_one_identity, domain: prime_number_theorem]

Theorem (three_four_one_nonneg). *Three Four One Nonneg.* [Platonic: three_four_one_nonneg, domain: prime_number_theorem]

Theorem (three_four_one_zero_iff). *Three Four One Zero Iff.* [Platonic: three_four_one_zero_iff, domain: prime_number_theorem]

Theorem (three_four_one_strict). *Three Four One Strict.* [Platonic: three_four_one_strict, domain: prime_number_theorem]

Theorem (three_four_one_max). *Three Four One Max.* [Platonic: three_four_one_max, domain: prime_number_theorem]

Theorem (three_four_one_at_zero). *Three Four One At Zero.* [Platonic: three_four_one_at_zero, domain: prime_number_theorem]

Theorem (weighted_341_nonneg). *Weighted 341 Nonneg.* [Platonic: weighted_341_nonneg, domain: prime_number_theorem]

Theorem (weighted_341_strict). *Weighted 341 Strict.* [Platonic: weighted_341_strict, domain: prime_number_theorem]

Theorem (contradiction_exponent_positive). *Contradiction Exponent Positive.* [Platonic: contradiction_exponent_positive, domain: prime_number_theorem]

Theorem (exponent_monotone). *Exponent Monotone.* [Platonic: exponent_monotone, domain: prime_number_theorem]

Theorem (simple_zero_exponent). *Simple Zero Exponent.* [Platonic: simple_zero_exponent, domain: prime_number_theorem]

Theorem (double_zero_exponent). *Double Zero Exponent.* [Platonic: double_zero_exponent, domain: prime_number_theorem]

Theorem (zero_beats_pole). *Zero Beats Pole.* [Platonic: zero_beats_pole, domain: prime_number_theorem]

Theorem (net_order_positive). *Net Order Positive.* [Platonic: net_order_positive, domain: prime_number_theorem]

Theorem (chebyshev_pi_positive). *Chebyshev Pi Positive.* [Platonic: chebyshev_pi_positive, domain: prime_number_theorem]

Theorem (pnt_ratio_near_one). *Pnt Ratio Near One.* [Platonic: pnt_ratio_near_one, domain: prime_number_theorem]

Theorem (pnt_ratio_near_one_upper). *Pnt Ratio Near One Upper.* [Platonic: pnt_ratio_near_one_upper, domain: prime_number_theorem]

Theorem (mertens1_growth). *Mertens1 Growth.* [Platonic: mertens1_growth, domain: prime_number_theorem]

Theorem (gap_positive). *Gap Positive.* [Platonic: gap_positive, domain: prime_number_theorem]

Theorem (bertrand_postulate). *Bertrand Postulate.* [Platonic: bertrand_postulate, domain: prime_number_theorem]

Theorem (li_correction_positive). *Li Correction Positive.* [Platonic: li_correction_positive, domain: prime_number_theorem]

Theorem (rh_error_sublinear). *Rh Error Sublinear.* [Platonic: rh_error_sublinear, domain: prime_number_theorem]

3. Bounds and Estimates

Theorem (zfr_boundary_in_strip). *Zfr Boundary In Strip.* [Platonic: zfr_boundary_in_strip, domain: prime_number_theorem]

Theorem (wider_zfr_tighter_bound). *Wider Zfr Tighter Bound.* [Platonic: wider_zfr_tighter_bound, domain: prime_number_theorem]

Theorem (general_cosine_bound). *General Cosine Bound.* [Platonic: general_cosine_bound, domain: prime_number_theorem]

Theorem (ratio_bounded_below). *Ratio Bounded Below.* [Platonic: ratio_bounded_below, domain: prime_number_theorem]

Theorem (ratio_bounded_above). *Ratio Bounded Above.* [Platonic: ratio_bounded_above, domain: prime_number_theorem]

Theorem (pnt_gap_bound). *Pnt Gap Bound.* [Platonic: pnt_gap_bound, domain: prime_number_theorem]

4. Formal Framework

Hypotheses

- H_c_ge: C Ge
- H_c_le: C Le
- H_partial_sum: Partial Sum
- H_ps_nonneg: Ps Nonneg
- H_lp_is_ps: Lp Is Ps
- H_m_ge1: M Ge1
- H_eps_pos: Eps Pos
- H_eps_lt1: Eps Lt1
- H_psi_pos: Psi Pos
- H_pnt_err_nn: Pnt Err Nn
- H_pnt_lower: Pnt Lower
- H_pnt_upper: Pnt Upper
- H_pi_pos: Pi Pos
- H_red_nn: Red Nn
- H_red_small: Red Small
- H_pi_from_psi: Pi From Psi
- H_cos_ge: Cos Ge
- H_cos_le: Cos Le
- H_w_pos: W Pos
- H_ct_ge: Ct Ge
- H_ct_le: Ct Le
- H_eps_pos: Eps Pos
- H_eps_small: Eps Small

- H_m_ge1: M Ge1
- H_zfr_pos: Zfr Pos
- H_zfr_lt1: Zfr Lt1
- H_zfr2_pos: Zfr2 Pos
- H_wider: Wider
- H_a_pos: A Pos
- H_dom: Dom
- H_dom_neg: Dom Neg
- H_pi_pos: Pi Pos
- H_xlnx_pos: Xlnx Pos
- H_cl_pos: Cl Pos
- H_cl_lt1: Cl Lt1
- H_cu_gt1: Cu Gt1
- H_cheb_lower: Cheb Lower
- H_cheb_upper: Cheb Upper
- H_ratio_def: Ratio Def
- H_ratio_lower: Ratio Lower
- H_ratio_upper: Ratio Upper
- H_cu_lt2: Cu Lt2
- H_eps_pos: Eps Pos
- H_pnt_lower: Pnt Lower
- H_pnt_upper: Pnt Upper
- H_logx_pos: Logx Pos
- H_mc1_pos: Mc1 Pos
- H_mertens1_lower: Mertens1 Lower
- H_mertens1_upper: Mertens1 Upper
- H_logx_large: Logx Large
- H_pn_pos: Pn Pos
- H_pnext_gt: Pnext Gt
- H_delta_pos: Delta Pos
- H_gap_small: Gap Small
- H_nn_pos: Nn Pos
- H_pi_n_bound: Pi N Bound
- H_pi_2n_bound: Pi 2n Bound
- H_bertrand_const: Bertrand Const
- H_li_pos: Li Pos
- H_li_gt_xlnx: Li Gt Xlnx
- H_sqrtx_pos: Sqrtx Pos
- H_rh_error_nn: Rh Error Nn
- H_rh_error_bound: Rh Error Bound
- H_log_lt_sqrt: Log Lt Sqrt

5. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/prime_number_theorem/).

File	Role
pnt_structure_proof.py	
pnt_algebraic_proof.py	
pnt_consequences_proof.py	

6. Discussion

References