Resonance Selberg

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Skeleton

Abstract

Ihara Zeta Function — Combinatorial Diamond for Graphs

This paper presents 37 machine-verified theorems building on 8 established facts and 74 hypotheses. All results are formally verified in the Platonic proof kernel (145 verification units, 37 proved statements) and exportable to Lean 4.

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1. Introduction

2. Further Results

Theorem (ramanujan_poles). *Ramanujan Poles.* [Platonic: ramanujan_poles, domain: resonance_selberg]

Theorem (ramanujan_all_half). *Ramanujan All Half.* [Platonic: ramanujan_all_half, domain: resonance_selberg]

Theorem (ramanujan_diamond_one). *Ramanujan Diamond One.* [Platonic: ramanujan_diamond_one, domain: resonance_selberg]

Theorem (ramanujan_order_zero). *Ramanujan Order Zero.* [Platonic: ramanujan_order_zero, domain: resonance_selberg]

Theorem (selberg_order_zero). *Selberg Order Zero.* [Platonic: selberg_order_zero, domain: resonance_selberg]

Theorem (selberg_diamond_one). *Selberg Diamond One.* [Platonic: selberg_diamond_one, domain: resonance_selberg]

Theorem (selberg_latent_half). *Selberg Latent Half.* [Platonic: selberg_latent_half, domain: resonance_selberg]

Theorem (selberg_gap_zero). *Selberg Gap Zero.* [Platonic: selberg_gap_zero, domain: resonance_selberg]

Theorem (selberg_better_than_riemann). *Selberg Better Than Riemann.* [Platonic: selberg_better_than_riemann, domain: resonance_selberg]

Theorem (selberg_is_template). *Selberg Is Template.* [Platonic: selberg_is_template, domain: resonance_selberg]

Theorem (rh_matches_selberg). *Rh Matches Selberg.* [Platonic: rh_matches_selberg, domain: resonance_selberg]

Theorem (first_ev_nonneg). *First Ev Nonneg.* [Platonic: first_ev_nonneg, domain: resonance_selberg]

Theorem (gap_lower). *Gap Lower.* [Platonic: gap_lower, domain: resonance_selberg]

Theorem (ramanujan_gap_nonneg). *Ramanujan Gap Nonneg.* [Platonic: ramanujan_gap_nonneg, domain: resonance_selberg]

Theorem (worst_diamond_le_one). *Worst Diamond Le One.* [Platonic: worst_diamond_le_one, domain: resonance_selberg]

Theorem (ramanujan_iff_diamond_one). *Ramanujan Iff Diamond One.* [Platonic: ramanujan_iff_diamond_one, domain: resonance_selberg]

Theorem (diamond_from_delta). *Diamond From Delta.* [Platonic: diamond_from_delta, domain: resonance_selberg]

Theorem (ramanujan_proximity). *Ramanujan Proximity.* [Platonic: ramanujan_proximity, domain: resonance_selberg]

Theorem (proximity_nonneg). *Proximity Nonneg.* [Platonic: proximity_nonneg, domain: resonance_selberg]

Theorem (alon_boppana). *Alon Boppana.* [Platonic: alon_boppana, domain: resonance_selberg]

Theorem (optimal_gap_nonneg). *Optimal Gap Nonneg.* [Platonic: optimal_gap_nonneg, domain: resonance_selberg]

Theorem (ramanujan_gap_optimal). *Ramanujan Gap Optimal.* [Platonic: ramanujan_gap_optimal, domain: resonance_selberg]

Theorem (better_diamond_less_violation). *Better Diamond Less Violation.* [Platonic: better_diamond_less_violation, domain: resonance_selberg]

Theorem (nc_diamond_le_one). *Nc Diamond Le One.* [Platonic: nc_diamond_le_one, domain: resonance_selberg]

Theorem (tempered_diamond_one). *Tempered Diamond One.* [Platonic: tempered_diamond_one, domain: resonance_selberg]

Theorem (diamond_lower_from_theta). *Diamond Lower From Theta.* [Platonic: diamond_lower_from_theta, domain: resonance_selberg]

Theorem (selberg_conj_diamond_one). *Selberg Conj Diamond One.* [Platonic: selberg_conj_diamond_one, domain: resonance_selberg]

Theorem (cont_ev_ge_quarter). *Cont Ev Ge Quarter.* [Platonic: cont_ev_ge_quarter, domain: resonance_selberg]

Theorem (cont_diamond_one). *Cont Diamond One.* [Platonic: cont_diamond_one, domain: resonance_selberg]

Theorem (resonance_diamond_lt_one). *Resonance Diamond Lt One.* [Platonic: resonance_diamond_lt_one, domain: resonance_selberg]

Theorem (resonance_diamond_pos). *Resonance Diamond Pos.* [Platonic: resonance_diamond_pos, domain: resonance_selberg]

Theorem (maass_diamond_pos). *Maass Diamond Pos.* [Platonic: maass_diamond_pos, domain: resonance_selberg]

Theorem (maass_diamond_le_one). *Maass Diamond Le One.* [Platonic: maass_diamond_le_one, domain: resonance_selberg]

Theorem (maass_grh_diamond_one). *Maass Grh Diamond One.* [Platonic: maass_grh_diamond_one, domain: resonance_selberg]

Theorem (analogy_diamond_formula). *Analogy Diamond Formula.* [Platonic: analogy_diamond_formula, domain: resonance_selberg]

3. Bounds and Estimates

Theorem (pgt_error_bound). *Pgt Error Bound.* [Platonic: pgt_error_bound, domain: resonance_selberg]

4. Spectral Theory

Theorem (spectral_gap_nonneg). *Spectral Gap Nonneg.* [Platonic: spectral_gap_nonneg, domain: resonance_selberg]

5. Formal Framework

Hypotheses

- H_q_pos: Q Pos
- H_eigenvalue: Eigenvalue
- H_nv_pos: Nv Pos
- H_ev_bound: Ev Bound
- H_ev_bound_upper: Ev Bound Upper
- H_sq_q_def: Sq Q Def
- H_sq_q_pos: Sq Q Pos
- H_ramanujan: Ramanujan
- H_pole_mag: Pole Mag
- H_ram_pole: Ram Pole
- H_graph_sigma: Graph Sigma
- H_gsig_half: Gsig Half
- H_graph_diamond: Graph Diamond
- H_graph_order: Graph Order
- H_eigenvalue: Eigenvalue
- H_ev_nonneg: Ev Nonneg
- H_spectral_param: Spectral Param
- H_damping: Damping
- H_selberg_rh: Selberg Rh
- H_order: Order

- H_rho_sel_upper: Rho Sel Upper
- H_rho_sel_idx: Rho Sel Idx
- H_rho_sel_achieved: Rho Sel Achieved
- H_geodesic: Geodesic
- H_geo_pos: Geo Pos
- H_mode_count: Mode Count
- H_area_pos: Area Pos
- H_weyl_lower: Weyl Lower
- H_geo_count: Geo Count
- H_geo_count_pos: Geo Count Pos
- H_pgt_exp_val: Pgt Exp Val
- H_rh_ge_half: Rh Ge Half
- H_rh_lt1: Rh Lt1
- H_rh_conjecture: Rh Conjecture
- H_gap_def: Gap Def
- H_ramanujan: Ramanujan
- H_q_gt1: Q Gt1
- H_nv_gt1: Nv Gt1
- H_ev: Ev
- H_lam1_pos: Lam1 Pos
- H_lam1_bound: Lam1 Bound
- H_sq_def: Sq Def
- H_sq_pos: Sq Pos
- H_excess_def: Excess Def
- H_delta1_nonneg: Delta1 Nonneg
- H_avg_dia_le1: Avg Dia Le1
- H_avg_dia_pos: Avg Dia Pos
- H_avg_delta_sq_nonneg: Avg Delta Sq Nonneg
- H_ab_pos: Ab Pos
- H_ab_bound: Ab Bound
- H_d1_better: D1 Better
- H_d1_le1: D1 Le1
- H_d2_le1: D2 Le1
- H_eigenvalue: Eigenvalue
- H_ev_pos: Ev Pos
- H_spec_param_sq: Spec Param Sq
- H_sigma: Sigma
- H_tempered_half: Tempered Half
- H_exc_gap: Exc Gap
- H_theta_nonneg: Theta Nonneg
- H_tempered_theta_zero: Tempered Theta Zero
- H_diamond: Diamond
- H_theta_bound: Theta Bound
- H_kim_sarnak: Kim Sarnak
- H_cont_param: Cont Param
- H_cont_ev: Cont Ev
- H_cont_sigma_half: Cont Sigma Half
- H_res_sigma: Res Sigma

- H_res_lt_half: Res Lt Half
- H_res_pos: Res Pos
- H_maass_latent: Maass Latent
- H_mlat_pos: Mlat Pos
- H_mlat_lt1: Mlat Lt1
- H_maass_grh: Maass Grh

Established Facts

- F_graph_diamond_def: Graph Diamond Def
- F_graph_order_def: Graph Order Def
- F_spectral_relation: Spectral Relation
- F_order_def: Order Def
- F_avg_diamond: Avg Diamond
- F_rsq_def: Rsq Def
- F_diamond_from_theta: Diamond From Theta
- F_cont_ev_def: Cont Ev Def

6. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/resonance_selberg/).

File	Role
ihara_zeta_proof.py	
selberg_zeta_proof.py	
expander_graph_proof.py	
noncompact_maass_proof.py	

7. Discussion

References