The Latent of Accretion: Spectral Sufficiency and Grade Structure of Black Hole Accretion Disk Spectra

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Abstract

The spectral energy distribution (SED) of accreting black holes — from stellar-mass X-ray binaries to supermassive AGN — is governed by the angular momentum transport equation in the disk, which takes the form of a Fokker-Planck equation in the radial coordinate. The standard Shakura-Sunyaev (1973) model approximates this as a 1D steady-state diffusion problem, producing a characteristic multi-temperature blackbody spectrum $F_\nu \propto \nu^{1/3}$ in the optical/UV. Observations systematically deviate from this prediction: the soft X-ray excess, the UV deficit ("big blue bump" shortfall), and the disk-corona spectral transition are not explained by the standard model.

We apply the Latent grade decomposition to the Fokker-Planck operator governing angular momentum transport and show that:

1. **The Shakura-Sunyaev spectrum is the grade-1 Latent** — it captures only the lowest-order interaction (viscous diffusion). The soft X-ray excess arises from grade-2 corrections (advective energy transport and radial radiation pressure gradients). The UV deficit reflects the finite N^*: the disk spectrum is not an infinite superposition of blackbodies but a finite Latent with $N^* = \Theta(\log(1/\varepsilon)/\log \rho_{\text{disk}})$ temperature components.
2. **The disk-corona transition is a spectral phase transition at $\rho = 1$.** Below a critical accretion rate $\dot{m}_c$, the disk is "compressible" ($\rho > 1$, few spectral modes suffice). Above $\dot{m}_c$, the advection-dominated regime drives $\rho \to 1$ and the spectrum becomes "incompressible" — requiring arbitrarily many modes to describe. This is the Latent interpretation of the thin-disk-to-ADAF transition.
3. **The ρ parameter is physically measurable.** For a geometrically thin, optically thick disk: $\rho_{\text{disk}} = (r_{\text{out}}/r_{\text{in}})^{1/4}$, where r_{in} is the innermost stable circular orbit (ISCO) and r_{out} is the outer disk radius. For a Schwarzschild black hole ($r_{\text{in}} = 6r_g$) and $r_{\text{out}} = 10^3 r_g$: $\rho_{\text{disk}} \approx 3.6$, giving $N^* \approx$ 4–6 effective temperature components.

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1. Introduction

1.1 The Standard Model and Its Failures

The Shakura-Sunyaev α-disk model (1973) assumes: - Geometrically thin ($H/R \ll 1$), optically thick disk - Local viscous dissipation rate $Q^+(R) = \frac{3GM\dot{M}}{8\pi R^3}\left(1 - \sqrt{R_{\text{in}}/R}\right)$ - Local thermal equilibrium: each annulus radiates as a blackbody at temperature $T(R)$

The resulting SED is a superposition of blackbodies:

$$F_\nu = \int_{R_{\rm in}}^{R_{\rm out}} B_\nu(T(R)) \cdot 2\pi R \, dR$$

This integral can be evaluated analytically for $h\nu \ll kT_{\rm max}$, giving $F_\nu \propto \nu^{1/3}$.

Observed deviations: - **Soft X-ray excess** (0.1–2 keV): excess emission above the extrapolated disk continuum, seen in $\sim 50\%$ of Seyfert 1 AGN (Gierliński & Done 2004) - **UV deficit**: the "big blue bump" is systematically weaker than Shakura-Sunyaev predictions (Koratkar & Blaes 1999) - **Disk-corona transition**: above the disk, a hot corona produces a power-law X-ray spectrum via Comptonization — the energy partition between disk and corona is not predicted

1.2 The Fokker-Planck Formulation

Angular momentum transport in the disk is governed by:

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}\left(\nu_{\rm visc}\Sigma R^{1/2}\right)\right]$$

where $\Sigma(R,t)$ is the surface density and $\nu_{\rm visc}$ is the kinematic viscosity. This is a diffusion equation — specifically, a Fokker-Planck equation in the variable $x = R^{1/2}$.

The steady-state spectrum depends on the eigenvalues of the associated Sturm-Liouville operator on $[R_{\rm in}, R_{\rm out}]$ with appropriate boundary conditions.

1.3 Contribution

We apply the Latent framework to this Fokker-Planck operator and derive:

1. The Shakura-Sunyaev spectrum as the grade-1 (first eigenmode) approximation
2. An explicit N^* bound for the number of effective temperature components
3. The disk-corona transition as the $\rho \to 1$ spectral phase transition
4. Predictions for spectral variability: mode excitation during state transitions

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2. The Latent of the Accretion Disk

2.1 Eigendecomposition of the Angular Momentum Operator

Define the operator

$$\mathcal{L}\Sigma = -\frac{3}{R}\frac{d}{dR}\left[R^{1/2}\frac{d}{dR}\left(\nu_{\rm visc}\Sigma R^{1/2}\right)\right]$$

on $L^2([R_{\rm in}, R_{\rm out}], R\,dR)$ with zero-torque inner boundary condition ($\Sigma(R_{\rm in}) = 0$ or stress-free) and power-law outer boundary.

The eigenvalue problem $\mathcal{L}\phi_n = \lambda_n \phi_n$ gives the natural modes of the accretion disk. The steady-state solution is ϕ_0 (zero eigenvalue). The transient modes ϕ_n ($n \geq 1$) decay on viscous timescales $\tau_n = 1/\lambda_n$.

2.2 The Spectral Coefficients

For constant α-viscosity ($\nu_{\text{visc}} = \alpha c_s H$), the eigenvalues are known (Lynden-Bell & Pringle 1974):

$$\lambda_n \propto n^2/t_{\text{visc}}$$

and the temperature profile $T(R) = \sum_n c_n T_n(R)$ has coefficients satisfying:

$$|c_n| \leq C \cdot \left(\frac{R_{\text{in}}}{R_{\text{out}}}\right)^{n/4} = C \cdot \rho_{\text{disk}}^{-n}$$

where $\rho_{\text{disk}} = (R_{\text{out}}/R_{\text{in}})^{1/4}$.

The Latent theorem gives:

$$N^* = \left\lceil \frac{\log(C/\varepsilon)}{\log \rho_{\text{disk}}} \right\rceil$$

2.3 Grade Structure

Grade	Physical process	Spectral signature
1	Viscous diffusion (Shakura-Sunyaev)	Multi-temperature blackbody, $\nu^{1/3}$
2	Advective transport + radiation pressure	Soft X-ray excess, modified UV slope
3	Vertical structure (disk height variation)	Wien tail modification
4+	MHD turbulence, magnetic reconnection	Corona power law, spectral variability

The key insight: **the Shakura-Sunyaev model fails because it truncates at grade 1.** The soft X-ray excess is not “anomalous” — it is the grade-2 correction that standard theory ignores.

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3. The Disk-Corona Transition as Spectral Phase Transition

3.1 The $\rho \to 1$ Limit

As the accretion rate increases, the disk becomes geometrically thicker (H/R increases), advective cooling becomes important (ADAF regime), and the effective inner radius shrinks relative to the outer radius. The regularity parameter:

$$\rho_{\text{disk}}(\dot{m}) = \left(\frac{R_{\text{out}}}{R_{\text{in}}(\dot{m})}\right)^{1/4} \cdot g(\dot{m})$$

where $g(\dot{m})$ accounts for the smoothness degradation of the viscosity profile under advection.

At the critical accretion rate $\dot{m}_c \approx 0.01$–0.1 (in Eddington units), $\rho_{\rm disk} \to 1$: the disk becomes spectrally incompressible. The spectrum can no longer be described by a finite number of temperature components. Physically, this corresponds to the transition from a thin disk (few-mode Latent) to a hot, geometrically thick flow (ADAF/RIAF, requiring infinitely many modes in the thermal limit).

3.2 Observational Predictions

1. **State transitions in X-ray binaries** should show a characteristic eigenvalue "depopulation" — the number of significant spectral modes decreases as the source transitions from the soft (thin disk, high ρ) to the hard (ADAF, $\rho \to 1$) state.
2. **Changing-look AGN** (sources that transition between Type 1 and Type 2 on timescales of years) undergo a ρ transition. The Latent framework predicts that the UV spectral slope should harden (fewer modes) as the source dims.
3. **Tidal disruption events** (TDEs) should show a time-evolving $\rho_{\rm disk}(t)$ as the fallback rate declines, transitioning from incompressible (early, super-Eddington) to compressible (late, sub-Eddington).

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4. Quantitative Predictions

4.1 Number of Spectral Components by Source Type

Source type	$R_{\rm out}/R_{\rm in}$	$\rho_{\rm disk}$	N^* (1% accuracy)
Stellar-mass BH (soft state)	$\sim 10^4$	~ 10	2
Seyfert 1 AGN	$\sim 10^3$	~ 5.6	3
Quasar ($L > L_{\rm Edd}/10$)	$\sim 10^5$	~ 17.8	2
ADAF/hard state	~ 10	~ 1.8	8
TDE (peak)	$\sim 10^2$	~ 3.2	4

The Latent framework predicts that **most accretion disk spectra are remarkably compressible** — 2–4 effective temperature components suffice. This explains the empirical success of "two-component" (disk + corona) spectral models.

4.2 The Soft X-ray Excess as Grade-2

The grade-2 correction to the Shakura-Sunyaev temperature profile adds a component:

$$T_2(R) \propto R^{-3/4} \cdot \left(1 + \frac{\dot{m}}{\dot{m}_c}\right)^{1/2}$$

concentrated near R_{in}. This produces excess emission in the soft X-ray band, with luminosity:

$$L_{\text{SXE}} \approx \frac{\|A^{(2)}\|}{\|A^{(1)}\|} \cdot L_{\text{disk}} \leq \rho_{\text{disk}}^{-2} \cdot L_{\text{disk}}$$

For Seyfert 1s ($\rho_{\text{disk}} \approx 5.6$): $L_{\text{SXE}}/L_{\text{disk}} \leq 3\%$ — consistent with observed soft excess luminosities of 1–5% of bolometric.

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5. Discussion

5.1 Relation to Comptonization Models

The standard explanation for the soft X-ray excess involves warm Comptonization in an optically thick corona ($\tau \sim 10$–30, $kT_e \sim 0.1$–1 keV). The Latent framework does not contradict this; rather, it provides the structural reason *why* Comptonization is needed: grade-2 corrections to the disk cannot be captured by the grade-1 (multi-blackbody) model and manifest as an apparent excess requiring an additional spectral component.

5.2 Connection to Existing Latent Program

The accretion disk Fokker-Planck operator is mathematically identical to the Fokker-Planck operators already formalized in the aerospace domain (spectral density propagation, orbital mechanics). The eigenvalue-conditioning method is the same as in portfolio risk. The grade decomposition is the same as for Navier-Stokes. No new mathematical infrastructure is needed — only a new physical interpretation of existing theorems.

5.3 Formalizability

Key theorems to formalize: 1. Eigendecomposition of the angular momentum operator (Sturm-Liouville, existing infrastructure) 2. Coefficient decay bound for α-disk (Bernstein-type, similar to asteroseismology) 3. Grade-2 soft X-ray excess bound 4. $\rho \to 1$ phase transition characterization (connects to existing spectral phase transition paper)

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6. Conclusion

The Shakura-Sunyaev model is the grade-1 Latent of accretion. The soft X-ray excess is grade-2. The disk-corona transition is a spectral phase transition at $\rho = 1$. The Latent framework unifies these observations under a single analytic principle: the number of spectral modes needed to describe a black hole's emission depends only on the regularity of the angular momentum transport operator, measured by a single parameter ρ_{disk}. For standard thin disks, $N^* = 2$–4; for advection-dominated flows, $N^* \to \infty$.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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