

# Fine Structure

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Skeleton

## Abstract

FineStructure — Platonic kernel proofs. Source: elysium/platonic/migration\_snapshots/FineStructure/spectrum\_

This paper presents 83 machine-verified theorems. All results are formally verified in the Platonic proof kernel (160 verification units, 85 proved statements) and exportable to Lean 4.

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## 1. Introduction

## 2. Further Results

**Theorem** (e8\_min\_norm\_sq). *E8 Min Norm Sq.* [Platonic: e8\_min\_norm\_sq, domain: fine\_structure]

**Theorem** (e8\_rank). *E8 Rank.* [Platonic: e8\_rank, domain: fine\_structure]

**Theorem** (e8\_int\_roots\_val). *E8 Int Roots Val.* [Platonic: e8\_int\_roots\_val, domain: fine\_structure]

**Theorem** (e8\_roots\_integer). *E8 Roots Integer.* [Platonic: e8\_roots\_integer, domain: fine\_structure]

**Theorem** (two\_times\_eight\_sq). *Two Times Eight Sq.* [Platonic: two\_times\_eight\_sq, domain: fine\_structure]

**Theorem** (instanton\_is\_dimF4). *Instanton Is DimF4.* [Platonic: instanton\_is\_dimF4, domain: fine\_structure]

**Theorem** (cg\_num). *Cg Num.* [Platonic: cg\_num, domain: fine\_structure]

**Theorem** (cg\_sq\_gt\_one). *Cg Sq Gt One.* [Platonic: cg\_sq\_gt\_one, domain: fine\_structure]

**Theorem** (nu\_r\_in\_16). *Nu R In 16.* [Platonic: nu\_r\_in\_16, domain: fine\_structure]

**Theorem** (structural\_count). *Structural Count.* [Platonic: structural\_count, domain: fine\_structure]

**Theorem** (schwinger\_cross). *Schwinger Cross.* [Platonic: schwinger\_cross, domain: fine\_structure]

**Theorem** (shell\_ratio\_2\_1). *Shell Ratio 2 1.* [Platonic: shell\_ratio\_2\_1, domain: fine\_structure]

**Theorem** (seesaw\_exponent). *Seesaw Exponent*. [Platonic: seesaw\_exponent, domain: fine\_structure]

**Theorem** (kissing\_factor\_comm). *Kissing Factor Comm*. [Platonic: kissing\_factor\_comm, domain: fine\_structure]

**Theorem** (e8\_rank\_plus\_int\_roots). *E8 Rank Plus Int Roots*. [Platonic: e8\_rank\_plus\_int\_roots, domain: fine\_structure]

**Theorem** (hurwitz\_nontrivial\_gen). *Hurwitz Nontrivial Gen*. [Platonic: hurwitz\_nontrivial\_gen, domain: fine\_structure]

**Theorem** (mass\_gap\_exponent). *Mass Gap Exponent*. [Platonic: mass\_gap\_exponent, domain: fine\_structure]

**Theorem** (grade\_norms\_ordered). *Grade Norms Ordered*. [Platonic: grade\_norms\_ordered, domain: fine\_structure]

**Theorem** (decay\_ratio\_gt\_one). *Decay Ratio Gt One*. [Platonic: decay\_ratio\_gt\_one, domain: fine\_structure]

**Theorem** (grade\_geometric\_decay). *Grade Geometric Decay*. [Platonic: grade\_geometric\_decay, domain: fine\_structure]

**Theorem** (alpha\_in\_unit\_interval). *Alpha In Unit Interval*. [Platonic: alpha\_in\_unit\_interval, domain: fine\_structure]

**Theorem** (schwinger\_correction\_pos). *Schwinger Correction Pos*. [Platonic: schwinger\_correction\_pos, domain: fine\_structure]

**Theorem** (alpha\_sq\_lt\_alpha). *Alpha Sq Lt Alpha*. [Platonic: alpha\_sq\_lt\_alpha, domain: fine\_structure]

**Theorem** (grade\_ratio\_lt\_one). *Grade Ratio Lt One*. [Platonic: grade\_ratio\_lt\_one, domain: fine\_structure]

**Theorem** (grade2\_dominates\_tail). *Grade2 Dominates Tail*. [Platonic: grade2\_dominates\_tail, domain: fine\_structure]

**Theorem** (fermion\_state\_positive). *Fermion State Positive*. [Platonic: fermion\_state\_positive, domain: fine\_structure]

**Theorem** (gut\_chain\_dimensions). *Gut Chain Dimensions*. [Platonic: gut\_chain\_dimensions, domain: fine\_structure]

**Theorem** (w4\_fs\_lt\_chain). *W4 Fs Lt Chain*. [Platonic: w4\_fs\_lt\_chain, domain: fine\_structure]

**Theorem** (w4\_fs\_complex\_I\_sq). *W4 Fs Complex I Sq*. [Platonic: w4\_fs\_complex\_I\_sq, domain: fine\_structure]

**Theorem** (w4\_fs\_sqrt\_zero). *W4 Fs Sqrt Zero*. [Platonic: w4\_fs\_sqrt\_zero, domain: fine\_structure]

**Theorem** (w4\_fs\_sqrt\_one). *W4 Fs Sqrt One*. [Platonic: w4\_fs\_sqrt\_one, domain: fine\_structure]

**Theorem** (w4\_fs\_ofReal\_zero). *W4 Fs Ofreal Zero*. [Platonic: w4\_fs\_ofReal\_zero, domain: fine\_structure]

**Theorem** (w4\_fs\_ofReal\_one). *W4 Fs Ofreal One*. [Platonic: w4\_fs\_ofReal\_one, domain: fine\_structure]

**Theorem** (fs\_alpha\_positive). *Fs Alpha Positive*. [Platonic: fs\_alpha\_positive, domain: fine\_structure]

**Theorem** (fs\_coupling\_sq\_nonneg). *Fs Coupling Sq Nonneg*. [Platonic: fs\_coupling\_sq\_nonneg, domain: fine\_structure]

**Theorem** (fs\_alpha\_in\_unit). *Fs Alpha In Unit*. [Platonic: fs\_alpha\_in\_unit, domain: fine\_structure]

**Theorem** (fs\_perturbative\_expansion\_nonneg). *Fs Perturbative Expansion Nonneg*. [Platonic: fs\_perturbative\_expansion\_nonneg, domain: fine\_structure]

**Theorem** (fs\_higher\_order\_smaller). *Fs Higher Order Smaller*. [Platonic: fs\_higher\_order\_smaller, domain: fine\_structure]

**Theorem** (fs\_instanton\_correction\_nonneg). *Fs Instanton Correction Nonneg*. [Platonic: fs\_instanton\_correction\_nonneg, domain: fine\_structure]

**Theorem** (fs\_w11\_inv\_mono\_positive\_reals). *Fs W11 Inv Mono Positive Reals*. [Platonic: fs\_w11\_inv\_mono\_positive\_reals, domain: fine\_structure]

**Theorem** (fs\_w11\_sub\_lt\_iff). *Fs W11 Sub Lt Iff*. [Platonic: fs\_w11\_sub\_lt\_iff, domain: fine\_structure]

**Theorem** (fs\_w11\_div\_pos\_of\_pos). *Fs W11 Div Pos Of Pos*. [Platonic: fs\_w11\_div\_pos\_of\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_recip\_mul\_pos). *Fs W11 Recip Mul Pos*. [Platonic: fs\_w11\_recip\_mul\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_one\_minus\_mul\_nonpos\_pos). *Fs W11 One Minus Mul Nonpos Pos*. [Platonic: fs\_w11\_one\_minus\_mul\_nonpos\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_sq\_pos\_of\_pos). *Fs W11 Sq Pos Of Pos*. [Platonic: fs\_w11\_sq\_pos\_of\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_mul\_pos\_of\_pos\_pos). *Fs W11 Mul Pos Of Pos Pos*. [Platonic: fs\_w11\_mul\_pos\_of\_pos\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_mul\_lt\_mul\_pos\_left). *Fs W11 Mul Lt Mul Pos Left*. [Platonic: fs\_w11\_mul\_lt\_mul\_pos\_left, domain: fine\_structure]

**Theorem** (fs\_w11\_mul\_le\_mul\_nonneg\_left). *Fs W11 Mul Le Mul Nonneg Left*. [Platonic: fs\_w11\_mul\_le\_mul\_nonneg\_left, domain: fine\_structure]

**Theorem** (fs\_w11\_div\_lt\_self\_of\_one\_lt\_den). *Fs W11 Div Lt Self Of One Lt Den*. [Platonic: fs\_w11\_div\_lt\_self\_of\_one\_lt\_den, domain: fine\_structure]

**Theorem** (fs\_w11\_af\_denom\_pos). *Fs W11 Af Denom Pos*. [Platonic: fs\_w11\_af\_denom\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_add\_pos). *Fs W11 Add Pos.* [Platonic: fs\_w11\_add\_pos, domain: fine\_structure]

**Theorem** (fs\_w11\_nonneg\_mul\_nonneg). *Fs W11 Nonneg Mul Nonneg.* [Platonic: fs\_w11\_nonneg\_mul\_nonneg, domain: fine\_structure]

**Theorem** (fs\_w11\_eqr\_refl\_sample). *Fs W11 Eqr Refl Sample.* [Platonic: fs\_w11\_eqr\_refl\_sample, domain: fine\_structure]

**Theorem** (beta\_function\_sign\_qed). *Beta Function Sign Qed.* [Platonic: beta\_function\_sign\_qed, domain: fine\_structure]

**Theorem** (running\_coupling\_denominator\_pos). *Running Coupling Denominator Pos.* [Platonic: running\_coupling\_denominator\_pos, domain: fine\_structure]

**Theorem** (asymptotic\_freedom\_coupling\_decrease). *Asymptotic Freedom Coupling Decrease.* [Platonic: asymptotic\_freedom\_coupling\_decrease, domain: fine\_structure]

**Theorem** (coupling\_perturbative\_regime). *Coupling Perturbative Regime.* [Platonic: coupling\_perturbative\_regime, domain: fine\_structure]

**Theorem** (weinberg\_angle\_pythagorean). *Weinberg Angle Pythagorean.* [Platonic: weinberg\_angle\_pythagorean, domain: fine\_structure]

**Theorem** (weinberg\_coupling\_ratio). *Weinberg Coupling Ratio.* [Platonic: weinberg\_coupling\_ratio, domain: fine\_structure]

**Theorem** (z\_mass\_from\_w\_mass\_ratio). *Z Mass From W Mass Ratio.* [Platonic: z\_mass\_from\_w\_mass\_ratio, domain: fine\_structure]

**Theorem** (anomalous\_moment\_hierarchy). *Anomalous Moment Hierarchy.* [Platonic: anomalous\_moment\_hierarchy, domain: fine\_structure]

**Theorem** (anomalous\_moment\_sum\_positive). *Anomalous Moment Sum Positive.* [Platonic: anomalous\_moment\_sum\_positive, domain: fine\_structure]

**Theorem** (charge\_screening\_reduces). *Charge Screening Reduces.* [Platonic: charge\_screening\_reduces, domain: fine\_structure]

**Theorem** (vacuum\_polarization\_tensor\_nonneg). *Vacuum Polarization Tensor Nonneg.* [Platonic: vacuum\_polarization\_tensor\_nonneg, domain: fine\_structure]

**Theorem** (effective\_coupling\_from\_screening). *Effective Coupling From Screening.* [Platonic: effective\_coupling\_from\_screening, domain: fine\_structure]

**Theorem** (gut\_unification\_point). *Gut Unification Point.* [Platonic: gut\_unification\_point, domain: fine\_structure]

**Theorem** (inverse\_coupling\_linear\_running). *Inverse Coupling Linear Running.* [Platonic: inverse\_coupling\_linear\_running, domain: fine\_structure]

**Theorem** (e8\_lattice\_norm\_sq\_even). *E8 Lattice Norm Sq Even.* [Platonic: e8\_lattice\_norm\_sq\_even, domain: fine\_structure]

**Theorem** (root\_system\_dual\_coxeter). *Root System Dual Coxeter.* [Platonic: root\_system\_dual\_coxeter, domain: fine\_structure]

**Theorem** (e6\_dim). *E6 Dim.* [Platonic: e6\_dim, domain: fine\_structure]

**Theorem** (e7\_dim). *E7 Dim.* [Platonic: e7\_dim, domain: fine\_structure]

**Theorem** (e8\_dim). *E8 Dim.* [Platonic: e8\_dim, domain: fine\_structure]

**Theorem** (exceptional\_chain\_dim\_mono). *Exceptional Chain Dim Mono.* [Platonic: exceptional\_chain\_dim\_mono, domain: fine\_structure]

**Theorem** (dirac\_product\_nonneg). *Dirac Product Nonneg.* [Platonic: dirac\_product\_nonneg, domain: fine\_structure]

**Theorem** (charge\_quantization\_half\_integer). *Charge Quantization Half Integer.* [Platonic: charge\_quantization\_half\_integer, domain: fine\_structure]

**Theorem** (lamb\_shift\_decomposition). *Lamb Shift Decomposition.* [Platonic: lamb\_shift\_decomposition, domain: fine\_structure]

**Theorem** (ckm\_row\_unitarity). *Ckm Row Unitarity.* [Platonic: ckm\_row\_unitarity, domain: fine\_structure]

**Theorem** (ckm\_jarlskog\_nonneg). *Ckm Jarlskog Nonneg.* [Platonic: ckm\_jarlskog\_nonneg, domain: fine\_structure]

**Theorem** (rg\_monotonicity). *Rg Monotonicity.* [Platonic: rg\_monotonicity, domain: fine\_structure]

### 3. Bounds and Estimates

**Theorem** (schwinger\_alpha\_over\_two\_pi\_bound). *Schwinger Alpha Over Two Pi Bound.* [Platonic: schwinger\_alpha\_over\_two\_pi\_bound, domain: fine\_structure]

**Theorem** (radiative\_correction\_bounded). *Radiative Correction Bounded.* [Platonic: radiative\_correction\_bounded, domain: fine\_structure]

### 4. Convergence Results

**Theorem** (coupling\_convergence\_indicator). *Coupling Convergence Indicator.* [Platonic: coupling\_convergence\_indicator, domain: fine\_structure]

### 5. Stability Results

**Theorem** (rg\_fixed\_point\_stability). *Rg Fixed Point Stability.* [Platonic: rg\_fixed\_point\_stability, domain: fine\_structure]

### 6. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/fine\_structure/).

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File	Role
finestructure_proof.py	

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## 7. Discussion

## References