

N Body

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Skeleton

Abstract

N-Body — ProofEnv proofs aligned with NBodyGradeBound.lean + quasi archive.

This paper presents 92 machine-verified theorems. All results are formally verified in the Platonic proof kernel (187 verification units, 93 proved statements) and exportable to Lean 4.

1. Introduction

2. Further Results

Theorem (decay_rate_in_unit). *Decay Rate In Unit*. [Platonic: decay_rate_in_unit, domain: n_body]

Theorem (grade_monotone_in_epsilon). *Grade Monotone In Epsilon*. [Platonic: grade_monotone_in_epsilon, domain: n_body]

Theorem (separation_positive). *Separation Positive*. [Platonic: separation_positive, domain: n_body]

Theorem (nbody_rho_positive). *Nbody Rho Positive*. [Platonic: nbody_rho_positive, domain: n_body]

Theorem (one_minus_r_positive). *One Minus R Positive*. [Platonic: one_minus_r_positive, domain: n_body]

Theorem (grade_contraction_step). *Grade Contraction Step*. [Platonic: grade_contraction_step, domain: n_body]

Theorem (linear_is_grade_2). *Linear Is Grade 2*. [Platonic: linear_is_grade_2, domain: n_body]

Theorem (effective_grade_margin_positive). *Effective Grade Margin Positive*. [Platonic: effective_grade_margin_positive, domain: n_body]

Theorem (grade_coupling_positive). *Grade Coupling Positive*. [Platonic: grade_coupling_positive, domain: n_body]

Theorem (grade_monotone_in_epsilon_pow). *Grade Monotone In Epsilon Pow*. [Platonic: grade_monotone_in_epsilon_pow, domain: n_body]

Theorem (hill_region_constraint). *Hill Region Constraint*. [Platonic: hill_region_constraint, domain: n_body]

Theorem (kinetic_from_energy). *Kinetic From Energy*. [Platonic: kinetic_from_energy, domain: n_body]

Theorem (brake_U_equals_h). *Brake U Equals H*. [Platonic: brake_U_equals_h, domain: n_body]

Theorem (brake_I_ddot_negative). *Brake I Ddot Negative*. [Platonic: brake_I_ddot_negative, domain: n_body]

Theorem (I_ddot_negative_below_virial). *I Ddot Negative Below Virial*. [Platonic: I_ddot_negative_below_virial, domain: n_body]

Theorem (above_virial_positive_acceleration). *Above Virial Positive Acceleration*. [Platonic: above_virial_positive_acceleration, domain: n_body]

Theorem (virial_surface_zero_acceleration). *Virial Surface Zero Acceleration*. [Platonic: virial_surface_zero_acceleration, domain: n_body]

Theorem (cannot_stay_above_virial). *Cannot Stay Above Virial*. [Platonic: cannot_stay_above_virial, domain: n_body]

Theorem (cannot_stay_below_virial). *Cannot Stay Below Virial*. [Platonic: cannot_stay_below_virial, domain: n_body]

Theorem (virial_crossing_core). *Virial Crossing Core*. [Platonic: virial_crossing_core, domain: n_body]

Theorem (moeckel_solutions_violate_virial). *Moeckel Solutions Violate Virial*. [Platonic: moeckel_solutions_violate_virial, domain: n_body]

Theorem (positive_amplitude_crosses_virial). *Positive Amplitude Crosses Virial*. [Platonic: positive_amplitude_crosses_virial, domain: n_body]

Theorem (energy_partition). *Energy Partition*. [Platonic: energy_partition, domain: n_body]

Theorem (brake_conditions). *Brake Conditions*. [Platonic: brake_conditions, domain: n_body]

Theorem (moeckel_violation_pair). *Moeckel Violation Pair*. [Platonic: moeckel_violation_pair, domain: n_body]

Theorem (sundman_scaling). *Sundman Scaling*. [Platonic: sundman_scaling, domain: n_body]

Theorem (chaotic_grade3_amplification). *Chaotic Grade3 Amplification*. [Platonic: chaotic_grade3_amplification, domain: n_body]

Theorem (action_premium_positive). *Action Premium Positive*. [Platonic: action_premium_positive, domain: n_body]

Theorem (I_linear_decrease). *I Linear Decrease*. [Platonic: I_linear_decrease, domain: n_body]

Theorem (nb_total_energy_decomp). *Nb Total Energy Decomp*. [Platonic: nb_total_energy_decomp, domain: n_body]

Theorem (nb_virial_nonneg). *Nb Virial Nonneg.* [Platonic: nb_virial_nonneg, domain: n_body]

Theorem (nb_action_nonneg). *Nb Action Nonneg.* [Platonic: nb_action_nonneg, domain: n_body]

Theorem (nb_galerkin_truncation_error). *Nb Galerkin Truncation Error.* [Platonic: nb_galerkin_truncation_error, domain: n_body]

Theorem (w4_nb_lt_dimension_chain). *W4 Nb Lt Dimension Chain.* [Platonic: w4_nb_lt_dimension_chain, domain: n_body]

Theorem (w4_nb_sqrt_zero). *W4 Nb Sqrt Zero.* [Platonic: w4_nb_sqrt_zero, domain: n_body]

Theorem (w4_nb_sqrt_one). *W4 Nb Sqrt One.* [Platonic: w4_nb_sqrt_one, domain: n_body]

Theorem (w4_nb_triple_product_six). *W4 Nb Triple Product Six.* [Platonic: w4_nb_triple_product_six, domain: n_body]

Theorem (w4_nb_four_factor_twentyfour). *W4 Nb Four Factor Twentyfour.* [Platonic: w4_nb_four_factor_twentyfour, domain: n_body]

Theorem (hill_region_constraint). *Hill Region Constraint.* [Platonic: hill_region_constraint, domain: n_body]

Theorem (kinetic_from_energy). *Kinetic From Energy.* [Platonic: kinetic_from_energy, domain: n_body]

Theorem (grade_decay_rate_lt_one). *Grade Decay Rate Lt One.* [Platonic: grade_decay_rate_lt_one, domain: n_body]

Theorem (grade_tensor_coeff_pos). *Grade Tensor Coeff Pos.* [Platonic: grade_tensor_coeff_pos, domain: n_body]

Theorem (nekhoroshev_exponent_pos). *Nekhoroshev Exponent Pos.* [Platonic: nekhoroshev_exponent_pos, domain: n_body]

Theorem (galerkin_energy_conservation). *Galerkin Energy Conservation.* [Platonic: galerkin_energy_conservation, domain: n_body]

Theorem (full_exponent). *Full Exponent.* [Platonic: full_exponent, domain: n_body]

Theorem (reduced_exponent). *Reduced Exponent.* [Platonic: reduced_exponent, domain: n_body]

Theorem (symmetry_improves_exponent). *Symmetry Improves Exponent.* [Platonic: symmetry_improves_exponent, domain: n_body]

Theorem (inverse_eps_positive). *Inverse Eps Positive.* [Platonic: inverse_eps_positive, domain: n_body]

Theorem (epsilon_half_positive). *Epsilon Half Positive.* [Platonic: epsilon_half_positive, domain: n_body]

Theorem (mul_self_nonneg_of_nonneg). *Mul Self Nonneg Of Nonneg.* [Platonic: mul_self_nonneg_of_nonneg, domain: n_body]

Theorem (sq_nonneg_real). *Sq Nonneg Real.* [Platonic: sq_nonneg_real, domain: n_body]

Theorem (add_lt_add_left). *Add Lt Add Left*. [Platonic: add_lt_add_left, domain: n_body]

Theorem (le_add_of_nonneg_right). *Le Add Of Nonneg Right*. [Platonic: le_add_of_nonneg_right, domain: n_body]

Theorem (lt_add_of_pos_right). *Lt Add Of Pos Right*. [Platonic: lt_add_of_pos_right, domain: n_body]

Theorem (mul_lt_mul_pos_right). *Mul Lt Mul Pos Right*. [Platonic: mul_lt_mul_pos_right, domain: n_body]

Theorem (sub_pos_of_lt). *Sub Pos Of Lt*. [Platonic: sub_pos_of_lt, domain: n_body]

Theorem (both_positive_implies_total_gt_component). *Both Positive Implies Total Gt Component*. [Platonic: both_positive_implies_total_gt_component, domain: n_body]

Theorem (decomp_total_positive_from_component). *Decomp Total Positive From Component*. [Platonic: decomp_total_positive_from_component, domain: n_body]

Theorem (ratio_positive_from_positive_components). *Ratio Positive From Positive Components*. [Platonic: ratio_positive_from_positive_components, domain: n_body]

Theorem (ratio_lt_one_from_positive_complement). *Ratio Lt One From Positive Complement*. [Platonic: ratio_lt_one_from_positive_complement, domain: n_body]

Theorem (r_squared_in_unit_interval). *R Squared In Unit Interval*. [Platonic: r_squared_in_unit_interval, domain: n_body]

Theorem (double_contraction). *Double Contraction*. [Platonic: double_contraction, domain: n_body]

Theorem (contraction_preserves_positivity). *Contraction Preserves Positivity*. [Platonic: contraction_preserves_positivity, domain: n_body]

Theorem (error_accumulation_two_windows). *Error Accumulation Two Windows*. [Platonic: error_accumulation_two_windows, domain: n_body]

Theorem (truncation_residual_positive). *Truncation Residual Positive*. [Platonic: truncation_residual_positive, domain: n_body]

Theorem (truncation_ratio_in_unit). *Truncation Ratio In Unit*. [Platonic: truncation_ratio_in_unit, domain: n_body]

Theorem (kam_codim_separates_iff_one). *Kam Codim Separates Iff One*. [Platonic: kam_codim_separates_iff_one, domain: n_body]

Theorem (kam_high_codim_no_separation). *Kam High Codim No Separation*. [Platonic: kam_high_codim_no_separation, domain: n_body]

Theorem (word_length_positive_from_segments). *Word Length Positive From Segments*. [Platonic: word_length_positive_from_segments, domain: n_body]

Theorem (total_grade_monotone_in_segments). *Total Grade Monotone In Segments*. [Platonic: total_grade_monotone_in_segments, domain: n_body]

Theorem (grade_linear_in_word_length). *Grade Linear In Word Length*. [Platonic: grade_linear_in_word_length, domain: n_body]

Theorem (three_body_6dof). *Three Body 6dof*. [Platonic: three_body_6dof, domain: n_body]

Theorem (three_body_reduced_dof). *Three Body Reduced Dof*. [Platonic: three_body_reduced_dof, domain: n_body]

3. Bounds and Estimates

Theorem (step_chain_error_bound). *Step Chain Error Bound*. [Platonic: step_chain_error_bound, domain: n_body]

Theorem (grade_tensor_bound_sq). *Grade Tensor Bound Sq*. [Platonic: grade_tensor_bound_sq, domain: n_body]

Theorem (bounded_kinetic). *Bounded Kinetic*. [Platonic: bounded_kinetic, domain: n_body]

Theorem (virial_oscillation_bound). *Virial Oscillation Bound*. [Platonic: virial_oscillation_bound, domain: n_body]

Theorem (bounded_virial_full_straddle). *Bounded Virial Full Straddle*. [Platonic: bounded_virial_full_straddle, domain: n_body]

Theorem (grade_bound_sandwich). *Grade Bound Sandwich*. [Platonic: grade_bound_sandwich, domain: n_body]

Theorem (decay_chain_bound). *Decay Chain Bound*. [Platonic: decay_chain_bound, domain: n_body]

Theorem (idot_bound_after_brake). *Idot Bound After Brake*. [Platonic: idot_bound_after_brake, domain: n_body]

Theorem (bounded_kinetic). *Bounded Kinetic*. [Platonic: bounded_kinetic, domain: n_body]

Theorem (grade_decay_rate_bound). *Grade Decay Rate Bound*. [Platonic: grade_decay_rate_bound, domain: n_body]

Theorem (galerkin_residual_bound). *Galerkin Residual Bound*. [Platonic: galerkin_residual_bound, domain: n_body]

Theorem (grade2_bounded_by_total). *Grade2 Bounded By Total*. [Platonic: grade2_bounded_by_total, domain: n_body]

Theorem (grade3_bounded_by_total). *Grade3 Bounded By Total*. [Platonic: grade3_bounded_by_total, domain: n_body]

Theorem (geometric_sum_bound_two_terms). *Geometric Sum Bound Two Terms*. [Platonic: geometric_sum_bound_two_terms, domain: n_body]

4. Stability Results

Theorem (nb_nekhoroshev_stability_time). *Nb Nekhoroshev Stability Time*. [Platonic: nb_nekhoroshev_stability_time, domain: n_body]

Theorem (nekhoroshev_stability_time_pos). *Nekhoroshev Stability Time Pos*. [Platonic: nekhoroshev_stability_time_pos, domain: n_body]

Theorem (stability_at_1e4). *Stability At 1e4*. [Platonic: stability_at_1e4, domain: n_body]

Theorem (stability_time_grows). *Stability Time Grows*. [Platonic: stability_time_grows, domain: n_body]

5. Convergence Results

Theorem (convergence_doubles). *Convergence Doubles*. [Platonic: convergence_doubles, domain: n_body]

6. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/n_body/).

File	Role
n_body_proof.py	

7. Discussion

References