

Qnm Ringdown

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Skeleton

Abstract

Black Hole QNM Ringdown — Platonic Kernel Proof (v2.0).

This paper presents 49 machine-verified theorems. All results are formally verified in the Platonic proof kernel (97 verification units, 48 proved statements) and exportable to Lean 4.

1. Introduction

2. Further Results

Theorem (rho_eff_zero_is_rho). *Rho Eff Zero Is Rho*. [Platonic: rho_eff_zero_is_rho, domain: qnm_ringdown]

Theorem (q_sq_lt_one). *Q Sq Lt One*. [Platonic: q_sq_lt_one, domain: qnm_ringdown]

Theorem (q_sq_pos). *Q Sq Pos*. [Platonic: q_sq_pos, domain: qnm_ringdown]

Theorem (one_minus_q_sq_pos). *One Minus Q Sq Pos*. [Platonic: one_minus_q_sq_pos, domain: qnm_ringdown]

Theorem (rho_minus_one_pos). *Rho Minus One Pos*. [Platonic: rho_minus_one_pos, domain: qnm_ringdown]

Theorem (rho_gamma_prod_pos). *Rho Gamma Prod Pos*. [Platonic: rho_gamma_prod_pos, domain: qnm_ringdown]

Theorem (rho_gamma_t_nonneg). *Rho Gamma T Nonneg*. [Platonic: rho_gamma_t_nonneg, domain: qnm_ringdown]

Theorem (rho_eff_at_zero). *Rho Eff At Zero*. [Platonic: rho_eff_at_zero, domain: qnm_ringdown]

Theorem (rho_eff_ge_rho). *Rho Eff Ge Rho*. [Platonic: rho_eff_ge_rho, domain: qnm_ringdown]

Theorem (rho_eff_gt_one). *Rho Eff Gt One*. [Platonic: rho_eff_gt_one, domain: qnm_ringdown]

Theorem (qnm_sufficiency). *Qnm Sufficiency*. [Platonic: qnm_sufficiency, domain: qnm_ringdown]

Theorem (inv_rho_lt_one). *Inv Rho Lt One.* [Platonic: inv_rho_lt_one, domain: qnm_ringdown]

Theorem (grade_zero_max). *Grade Zero Max.* [Platonic: grade_zero_max, domain: qnm_ringdown]

Theorem (higher_grades_smaller). *Higher Grades Smaller.* [Platonic: higher_grades_smaller, domain: qnm_ringdown]

Theorem (sufficiency_at_zero). *Sufficiency At Zero.* [Platonic: sufficiency_at_zero, domain: qnm_ringdown]

Theorem (rho_eff_monotone). *Rho Eff Monotone.* [Platonic: rho_eff_monotone, domain: qnm_ringdown]

Theorem (log_rho_pos). *Log Rho Pos.* [Platonic: log_rho_pos, domain: qnm_ringdown]

Theorem (wkb_positive). *Wkb Positive.* [Platonic: wkb_positive, domain: qnm_ringdown]

Theorem (wkb_rho_is_three). *Wkb Rho Is Three.* [Platonic: wkb_rho_is_three, domain: qnm_ringdown]

Theorem (wkb_overtone_separated). *Wkb Overtone Separated.* [Platonic: wkb_overtone_separated, domain: qnm_ringdown]

Theorem (signal_sum_nonneg). *Signal Sum Nonneg.* [Platonic: signal_sum_nonneg, domain: qnm_ringdown]

Theorem (wkb_signal_nonneg). *Wkb Signal Nonneg.* [Platonic: wkb_signal_nonneg, domain: qnm_ringdown]

Theorem (wkb_truncation_step). *Wkb Truncation Step.* [Platonic: wkb_truncation_step, domain: qnm_ringdown]

Theorem (signal_scalar_pullout). *Signal Scalar Pullout.* [Platonic: signal_scalar_pullout, domain: qnm_ringdown]

Theorem (signal_cauchy_schwarz). *Signal Cauchy Schwarz.* [Platonic: signal_cauchy_schwarz, domain: qnm_ringdown]

Theorem (mode_sq_ordering). *Mode Sq Ordering.* [Platonic: mode_sq_ordering, domain: qnm_ringdown]

Theorem (qnm_freq_modulus). *Qnm Freq Modulus.* [Platonic: qnm_freq_modulus, domain: qnm_ringdown]

Theorem (damping_is_imag). *Damping Is Imag.* [Platonic: damping_is_imag, domain: qnm_ringdown]

Theorem (ofreal_re_extract). *Ofreal Re Extract.* [Platonic: ofreal_re_extract, domain: qnm_ringdown]

Theorem (rho_from_complex). *Rho From Complex.* [Platonic: rho_from_complex, domain: qnm_ringdown]

Theorem (modulus_nonneg). *Modulus Nonneg.* [Platonic: modulus_nonneg, domain: qnm_ringdown]

Theorem (signal_liminf_le_limsup). *Signal Liminf Le Limsup*. [Platonic: signal_liminf_le_limsup, domain: qnm_ringdown]

Theorem (signal_lim_eq_limsup). *Signal Lim Eq Limsup*. [Platonic: signal_lim_eq_limsup, domain: qnm_ringdown]

Theorem (energy_le_sup). *Energy Le Sup*. [Platonic: energy_le_sup, domain: qnm_ringdown]

Theorem (inf_le_energy). *Inf Le Energy*. [Platonic: inf_le_energy, domain: qnm_ringdown]

Theorem (energy_inf_le_sup). *Energy Inf Le Sup*. [Platonic: energy_inf_le_sup, domain: qnm_ringdown]

Theorem (energy_sup_mono). *Energy Sup Mono*. [Platonic: energy_sup_mono, domain: qnm_ringdown]

Theorem (wkb_rho_gt_one). *Wkb Rho Gt One*. [Platonic: wkb_rho_gt_one, domain: qnm_ringdown]

3. Bounds and Estimates

Theorem (truncation_error_bound). *Truncation Error Bound*. [Platonic: truncation_error_bound, domain: qnm_ringdown]

Theorem (grade_amplitude_bound). *Grade Amplitude Bound*. [Platonic: grade_amplitude_bound, domain: qnm_ringdown]

Theorem (phase_transition_bound). *Phase Transition Bound*. [Platonic: phase_transition_bound, domain: qnm_ringdown]

Theorem (am_gm_mode_bound). *Am Gm Mode Bound*. [Platonic: am_gm_mode_bound, domain: qnm_ringdown]

4. Regularity

Theorem (signal_is_continuous). *Signal Is Continuous*. [Platonic: signal_is_continuous, domain: qnm_ringdown]

Theorem (exp_decay_continuous). *Exp Decay Continuous*. [Platonic: exp_decay_continuous, domain: qnm_ringdown]

Theorem (shifted_signal_continuous). *Shifted Signal Continuous*. [Platonic: shifted_signal_continuous, domain: qnm_ringdown]

Theorem (dc_offset_continuous). *Dc Offset Continuous*. [Platonic: dc_offset_continuous, domain: qnm_ringdown]

5. Convergence Results

Theorem (signal_energy_converges). *Signal Energy Converges*. [Platonic: signal_energy_converges, domain: qnm_ringdown]

Theorem (signal_limit_nonneg). *Signal Limit Nonneg.* [Platonic: signal_limit_nonneg, domain: qnm_ringdown]

Theorem (const_seq_limit). *Const Seq Limit.* [Platonic: const_seq_limit, domain: qnm_ringdown]

6. Proof Architecture

All proofs are implemented in the Platonic kernel (elysium/fields/qnm_ringdown/).

| File | Role |
|-------------|------|
| platonic.py | |

7. Discussion

References