The Spectral Generator of the N-Body Latent: Connecting Padé Poles, Koopman Eigenvalues, and Dynamical Classification

Connecting Padé Poles, Koopman Eigenvalues, and Dynamical Classification

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Abstract

The N-body Latent [Nagy 2026g, 2026i] provides an exact, finite representation of gravitational trajectories via the generating function $G(z; \mathbf{v}_0)$. The Universal Spectral Representation Theorem [Nagy 2026b] shows that for stochastic systems, the Fokker–Planck spectral generator M — a grade-2 Latent — is a sufficient statistic for the entire process. But what plays the role of M for the deterministic N-body problem?

We answer this by connecting the Latent framework to the **Koopman operator** of Hamiltonian dynamics. The Koopman generator $\mathcal{L}$ is the deterministic counterpart of the Fokker–Planck generator: it advances observables forward in time and its spectrum encodes the full dynamical structure. We prove four results that establish the connection:

1. **Padé–Koopman correspondence** (Theorem 1). The poles of the Padé approximant to $G(z; \mathbf{v}_0)$ converge to the singularities of the Koopman resolvent. For regular orbits, these approximate the discrete Koopman eigenvalues; for chaotic orbits, they approximate the continuous spectrum.

2. **Spectral complexity theorem** (Theorem 2). The spectral entropy of the Padé pole distribution characterizes the topological entropy of the orbit. Low entropy   regular (quasi-periodic); high entropy   chaotic.

3. **Quantitative Sundman theorem** (Theorem 3). The Padé convergence rate ρ equals $\exp(2\pi\tau_{\min}/T)$, where $\tau_{\min}$ is the distance from the Koopman spectrum to the real axis in complex time — determined by the nearest collision singularity.

4. **Classification theorem** (Theorem 4). The orbit classification (bounded/ejection/collision) is encoded in the topology of the Koopman spectrum, which is approximated by the Padé pole landscape.

The deterministic spectral generator $\mathcal{L}$ is a grade-2 Latent in the hierarchy of [Nagy 2026e], completing the parallel:

	Stochastic (USRT)	Deterministic (this paper)
Grade-1 Latent	Density $p(x) \to$ spectral coefficients	Trajectory $\mathbf{q}(t) \to$ Fourier modes Λ_n
Grade-2 Latent	Fokker–Planck generator M	Koopman generator $\mathcal{L}$
Grade-3 Latent	Time-varying generator $M(t)$	Meta-Latent over IC space

All four results are supported by numerical evidence from 30,000+ three-body orbit computations. The spectral generator framework unifies the N-body Latent, the USRT, and the Latent of Latents into a single hierarchy.

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1. Introduction

1.1 The Missing Link

The Latent framework has established three separate results for the N-body problem:

- **The trajectory Latent** [Nagy 2026g]: the generating function $G(z; \mathbf{v}_0) = \sum \Lambda_n z^n$ provides an exact, finite representation of any collision-free trajectory.
- **The spectral generator** [Nagy 2026b, §8.1]: for stochastic dynamics, the Fokker–Planck generator matrix M is a grade-2 Latent — a sufficient statistic for the entire process.
- **The meta-Latent** [Nagy 2026j]: the mapping IC $\mapsto \Lambda(\text{IC})$ from initial conditions to orbit Latents has its own finite representation — the Latent of Latents.

What is missing is the connection between these three objects. Specifically: **what is the grade-2 Latent (the "spectral generator") of a deterministic Hamiltonian system?** The Fokker–Planck equation does not apply — there is no diffusion. Yet there must be an object that encodes how the Latent evolves in time and how it depends on initial conditions.

1.2 The Koopman Operator

The answer comes from ergodic theory. For any deterministic dynamical system with flow Φ^t, the **Koopman operator** $\mathcal{K}^t$ acts on observables:

$$(\mathcal{K}^t f)(x) = f(\Phi^t(x))$$

It is the dual of the Perron–Frobenius operator (which advances densities). For Hamiltonian systems, $\mathcal{K}^t$ is unitary on $L^2(\text{phase space}, d\mu_{\text{Liouville}})$.

The **Koopman generator** is $\mathcal{L} = \lim_{t \to 0}(\mathcal{K}^t - I)/t$, satisfying $\mathcal{K}^t = e^{t\mathcal{L}}$. Its spectral decomposition encodes the dynamics:

$$f(\Phi^t(x)) = \sum_k \phi_k(x)\, e^{\lambda_k t}\, v_k(f)$$

where $\{\phi_k\}$ are Koopman eigenfunctions, $\{\lambda_k\}$ are Koopman eigenvalues, and $\{v_k\}$ are Koopman modes. For Hamiltonian systems:

Orbit type	Koopman spectrum	Eigenvalues λ_k
Quasi-periodic (KAM torus)	Discrete	$\lambda_k = i\omega_k$ (pure imaginary)
Chaotic (positive Lyapunov)	Continuous	Dense on imaginary axis
Near-collision	Branch-point singularity	Accumulation at collision time

1.3 The Central Insight

The Padé approximant $P_L(z)/Q_M(z)$ to the generating function $G(z; \mathbf{v}_0)$ is a **rational approximation of the Koopman resolvent**, restricted to the orbit indexed by $\mathbf{v}_0$.

- The **poles** of Q_M approximate the **singularities** of the Koopman resolvent — which are the Koopman eigenvalues for discrete spectrum and the branch points/essential singularities for continuous spectrum.
- The **residues** encode the Koopman modes.
- The **convergence rate** ρ measures the distance from the nearest singularity to the evaluation contour — the “analyticity strip width” that appears in the USRT.

This identification makes the Padé pole landscape a **computable window into the Koopman spectrum** — and therefore into the full dynamical classification of the orbit.

1.4 Outline

Section 2 recalls the N-body Latent and the USRT generator. Section 3 develops the Koopman connection. Section 4 states and proves the four main theorems. Section 5 presents the numerical evidence. Section 6 constructs the full Latent hierarchy for deterministic systems. Section 7 discusses implications.

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2. Background

2.1 The N-Body Latent [Nagy 2026g, 2026i]

For N gravitational bodies in d dimensions, the trajectory in Jacobi coordinates $\rho(t)$ has the generating function representation:

$$\rho(t) = G_N(e^{i\omega t}; \mathbf{v}_0) = \sum_{n=-N_\varepsilon}^{N_\varepsilon} \Lambda_n(\mathbf{v}_0)\, e^{in\omega t}$$

The Latent is $\Lambda = \{\Lambda_n\}_{|n|\le N_\varepsilon}$, with: - **Size**: $(N-1)d \cdot (2N_\varepsilon + 1)$ real numbers - **Analyticity**: $G_N(z)$ is analytic on an annulus $\rho^{-1} < |z| < \rho$ with $\rho > 1$ - **Convergence**: error $\le C\rho^{-2N_\varepsilon}$ (exponential in the number of modes)

2.2 The USRT Spectral Generator [Nagy 2026b, §8.1]

For an Itô diffusion $dX_t = \mu(X_t)\,dt + \sigma(X_t)\,dW_t$, the Fokker–Planck density $p(x,t)$ has spectral coefficients $A_k(t)$ satisfying:

$$A(t) = e^{Mt} A(0)$$

The generator matrix M — a finite matrix in the Galerkin-truncated basis — is the grade-2 Latent of the process. It encodes: the stationary distribution ($A_\infty = \mathrm{null}(M)$), the spectral gap ($\lambda_2(M)$, the mixing rate), first passage times ($\mathbb{E}[\tau] = -\mathbf{1}^\top M_{\text{killed}}^{-1} A(0)$), and all spectral risk measures.

2.3 The Latent of Latents [Nagy 2026j]

The mapping $\mathcal{F} : \mathbf{v}_0 \mapsto \Lambda(\mathbf{v}_0)$ from initial conditions to orbit Latents is smooth within each dynamical regime. Its meta-Latent $\Lambda_{\mathcal{F}}$ has rank $R = O(\log(1/\varepsilon)/\log \rho_{\text{meta}})$, where ρ_{meta} is the meta-analyticity parameter of $\mathcal{F}$. Evaluating $\Lambda_{\mathcal{F}}$ at a new IC yields the orbit Latent without integration.

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3. The Deterministic Spectral Generator

3.1 The Koopman Generator in Latent Coordinates

Let $\Phi^t : \mathbb{R}^{2(N-1)d} \to \mathbb{R}^{2(N-1)d}$ be the Hamiltonian flow of the N-body system. The Koopman generator acts on observables:

$$\mathcal{L} f = \lim_{t \to 0} \frac{f \circ \Phi^t - f}{t} = \sum_k \dot{q}_k \frac{\partial f}{\partial q_k} + \ddot{q}_k \frac{\partial f}{\partial \dot{q}_k}$$

Now consider the Latent coefficients as observables: $\Lambda_n(\mathbf{v}_0)$ is a function on phase space. The Koopman operator advances these coefficients:

$$\Lambda_n(\Phi^t(\mathbf{v}_0)) = (\mathcal{K}^t \Lambda_n)(\mathbf{v}_0)$$

In the Galerkin-truncated Latent basis $\{\Lambda_n\}_{|n| \le N_\varepsilon}$, the Koopman operator is represented by a **finite matrix L** satisfying:

$$\Lambda_n(t) \approx \sum_m L_{nm}(t)\, \Lambda_m(0)$$

This matrix **L** is the **deterministic spectral generator** — the direct counterpart of the Fokker–Planck matrix M.

3.2 The Parallel

Property	Stochastic (M)	Deterministic ($\mathbf{L}$)
Defines	Fokker–Planck semigroup e^{Mt}	Koopman semigroup $e^{t\mathbf{L}}$
Grade in Latent algebra	2 (matrix)	2 (matrix)
Eigenvalues	$\mathrm{Re}(\lambda_k) \le 0$ (dissipative)	$\mathrm{Re}(\lambda_k) = 0$ (Hamiltonian)
Spectrum encodes	Mixing rate, stationary law	Frequencies, KAM structure

Property	Stochastic (M)	Deterministic ($\mathbf{L}$)
Finite approximation	Galerkin truncation to N modes	Galerkin truncation to N_ε modes
Convergence	$O(\rho^{-2N})$ (USRT)	$O(\rho^{-2N_\varepsilon})$ (N-body Latent Theorem)

3.3 Connection to Padé Poles

The Padé approximant $P_L(z)/Q_M(z)$ to the Taylor series of a trajectory component $q(t) = \sum a_n t^n$ is a rational approximation. Its poles (roots of Q_M) approximate the singularities of $q(t)$ in the complex time plane.

For a signal with Koopman decomposition $q(t) = \sum_k c_k e^{\lambda_k t}$, the Padé [L/M] approximant converges to the Prony representation: the M poles converge to $\{1/\lambda_k\}_{k=1}^{M}$ (the reciprocal eigenvalues) as $L, M \to \infty$ with $L/M \to 1$.

For the N-body problem, $q(t)$ is not a finite sum of exponentials — it has infinitely many Koopman modes, and for $N = 3$, Painlevé's theorem guarantees that the complex-time singularities are exactly the collisions. The Padé poles approximate these collision singularities, which are the same objects that determine the convergence rate ρ in the Latent Theorem.

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4. Main Results

4.1 Theorem 1: Padé–Koopman Correspondence

Theorem 1. Let $\mathbf{q}(t)$ be an N-body trajectory with Koopman decomposition in the j-th coordinate:

$$q_j(t) = \sum_k c_k(\mathbf{v}_0)\, e^{\lambda_k t}\, v_k(q_j)$$

Let $\{z_1^{(L)}, \dots, z_L^{(L)}\}$ be the poles of the $[L/L]$ Padé approximant to the Taylor series of q_j around t_0. Then:

(i) For a quasi-periodic orbit on a KAM torus with frequencies $\omega_1, \dots, \omega_r$, the Padé poles satisfy:

$$\min_k |z_m^{(L)} - 2\pi i n/\omega_k| \to 0 \quad \text{as } L \to \infty$$

for each integer combination n, i.e., the poles approximate the (reciprocal) Koopman eigenvalues on the imaginary axis.

(ii) For a chaotic orbit with positive maximal Lyapunov exponent $\lambda_{\max} > 0$, the Padé poles do not converge to isolated points but instead fill a region of the complex plane whose diameter grows with L.

(iii) For a near-collision orbit approaching distance $d_{\min}$ from a binary collision, the nearest Padé pole to the real axis satisfies:

$$|\mathrm{Im}(z_{\mathrm{nearest}})| = O(d_{\min}^{3/2})$$

reflecting the Painlevé branch-point singularity structure $q \sim (t - t^*)^{2/3}$.

Proof sketch. Part (i) follows from the convergence theory of Padé approximants for meromorphic functions (Baker and Graves-Morris 1996): if $q(t)$ is meromorphic in a disk of radius R, the Padé $[L/L]$ poles converge to the poles of q in the disk. For quasi-periodic orbits, the Koopman decomposition gives a convergent sum of exponentials; the Padé poles converge to the associated singularities. Part (ii) follows from the characterization of Padé approximants for functions with natural boundaries (Stahl 1997): the spurious poles fill the region beyond the natural boundary. Part (iii) follows from the known behavior $q(t) - q(t^*) \sim (t - t^*)^{2/3}$ near a binary collision (Levi-Civita 1920), which gives a branch point at complex distance proportional to $d_{\min}^{3/2}$ from the real axis. □

4.2 Theorem 2: Spectral Complexity

Definition. The **Padé spectral entropy** of a trajectory at Padé order L is:

$$H_L = -\sum_k p_k \log p_k, \qquad p_k = \frac{|c_k|^2}{\sum_j |c_j|^2}$$

where $\{c_k\}$ are the residues of the Padé approximant at its poles, and the normalized entropy is $\hat{H}_L = H_L / \log L$.

Theorem 2. For the planar three-body problem with energy $E = -h < 0$:

(i) If the orbit is quasi-periodic (lies on a 2-torus), then $\hat{H}_L \to 0$ as $L \to \infty$.

(ii) If the orbit has positive topological entropy $h_{\mathrm{top}} > 0$, then $\liminf_{L\to\infty} \hat{H}_L > 0$.

Proof sketch. Part (i): a quasi-periodic orbit with r incommensurable frequencies has Koopman modes concentrated on r fundamental frequencies and their harmonics. As $L \to \infty$, the Padé residues concentrate on these frequencies, giving $H_L = O(\log r)$ while $\log L \to \infty$. Part (ii): positive topological entropy implies the orbit visits exponentially many dynamically distinct regions, requiring spectral energy to spread across many modes. The Padé residues cannot concentrate on finitely many poles, giving $\hat{H}_L > 0$. □

Connection to Direction 2. The U(t) spectral entropy computed from the FFT of the potential energy trace is a proxy for $\hat{H}_L$. The numerical results (4,905 bounded orbits, 5 mass sets) show clear separation between low-entropy (regular) and high-entropy (complex) orbits, with entropy correlating with virial crossing count (a proxy for dynamical complexity).

4.3 Theorem 3: Quantitative Sundman

Theorem 3 (Quantitative Convergence Rate). Let $\mathbf{q}(t)$ be an N-body trajectory on $[0, T]$ with minimum interparticle distance $d_{\min} > 0$. The Padé $[L/L]$ approximant to $q_j(t)$ around t_0 satisfies:

$$|q_j(t) - P_L(t)/Q_L(t)| \le C \cdot \rho^{-2L}$$

where:

$$\rho = \exp(2\pi\tau_{\min}/T)$$

and $\tau_{\min}$ is the distance from the nearest complex-time singularity to the real axis. For binary near-collision with minimum approach $d_{\min}$:

$$\tau_{\min} = \Theta(d_{\min}^{3/2})$$

giving:

$$\rho = \exp\left(\Theta\left(\frac{d_{\min}^{3/2}}{T}\right)\right)$$

Significance. This is the quantitative version of the Sundman–Wang existence theorem. Sundman (1912) and Wang (1991) proved convergent power series solutions exist; our result gives the **practical convergence rate** in terms of computable orbit parameters.

Connection to Direction 4. The numerical experiments (5 orbit types, Padé orders $L = 2, \dots, 16$) show: - $d_{\min} = 0.005 \Rightarrow \rho \approx 1.08$ (slow convergence — near collision) - $d_{\min} = 0.65 \Rightarrow \rho \approx 2.70$ (fast convergence — smooth orbit)

The exponential convergence is visible as straight lines on log-scale plots, confirming the ρ^{-2L} rate.

4.4 Theorem 4: Dynamical Classification from Generator Topology

Theorem 4. For the planar three-body problem at energy $E = -h$, the Padé pole landscape of a trajectory classifies it into one of three dynamical regimes:

Regime	Padé pole signature	Koopman spectrum
Bounded regular	Poles symmetric about real axis, well-separated, $\lvert\mathrm{Im}\rvert \gg 0$	Discrete, pure imaginary
Bounded chaotic	Poles scattered, clustered, poor separation	Continuous, dense on imaginary axis
Ejection	Poles asymmetric, one dominant real pole	Isolated real eigenvalue (escape rate)
Collision	Pole on or very near real axis at $t \approx t_{\text{collision}}$	Singularity at collision time

Connection to Direction 1. The pole landscape from 44 orbits (880 poles) shows clear separation: bounded orbits have symmetric pole distributions far from the real axis; ejection orbits have asymmetric clusters; collision orbits have poles on the real axis. The correlation between $d_{\min}$ and $\lvert\mathrm{Im}(\text{nearest pole})\rvert$ is confirmed numerically.

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5. Numerical Evidence

The four theorems are supported by two computational campaigns.

5.1 Campaign 1: Brake Orbit Scan (J = 0)

- 12,900 brake orbits at $E = -1$ across 5 mass sets
- DOP5(4) adaptive integrator, rtol = 10^{-12}, $T_{\max} = 120$
- Max energy drift: 3.4×10^{-6}
- 4,905 bounded, 1,890 ejection, 6,105 collision

5.2 Campaign 2: Q2 Deep Study (J 0 + RE Perturbations)

- 18,840 orbits including non-zero angular momentum and RE perturbations
- 9 perturbation magnitudes ($\varepsilon = 10^{-6}$ to 0.2), 100 random directions each
- Max energy drift: 1.3×10^{-6}
- 11,477 bounded, 6,928 ejection, 435 collision

5.3 Four Direction Analyses

Direction 1 (Padé pole landscape). 44 orbits, [10/10] Padé, 880 poles in the complex time plane. Confirms Theorem 1(iii): |Im(nearest pole)| correlates with $d_{\min}$ on a log-log plot.

Direction 2 (Spectral entropy). 4,905 bounded orbits, FFT of U(t) traces. Confirms Theorem 2: spectral entropy correlates with virial crossing count ($\rho = 0.42$, $p < 10^{-10}$). Mass asymmetry increases spectral complexity.

Direction 3 (Virial bounds). 16,302 bounded orbits. Empirical bounds: $\min(U)$ median = 1.12, max/min ratio median = 19.9. Mass asymmetry raises $\min(U)$.

Direction 4 (Sundman convergence). 4 orbits, Padé $[L/L]$ for $L = 2, \ldots, 16$. Exponential convergence confirmed. Rate ρ scales with $d_{\min}$: from $\rho = 1.08$ ($d_{\min} = 0.005$) to $\rho = 2.70$ ($d_{\min} = 0.65$).

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6. The Full Latent Hierarchy for Deterministic Systems

Combining the results of this paper with the Latent framework [Nagy 2026e], the USRT [Nagy 2026b], and the N-body Latent [Nagy 2026g, 2026i], we can now state the complete hierarchy:

6.1 Five Levels

Level	Object	Mathematical identity	Encodes
0	State $\mathbf{q}(t)$	Point in phase space	Position at one instant
1	**Trajectory Latent** $\Lambda(\mathbf{v}_0)$	Fourier coefficients of $G(z; \mathbf{v}_0)$	One complete orbit
2	**Spectral Generator** $\mathbf{L}$	Koopman generator in Latent basis	Temporal dynamics of the Latent

Level	Object	Mathematical identity	Encodes
3	**Meta-Latent** $\Lambda_{\mathcal{F}}$	Latent of the map $\mathbf{v}_0 \mapsto \Lambda(\mathbf{v}_0)$	Entire family of orbits
4	**Classification Latent**	Topology of Spec($\mathbf{L}$) over IC space	Dynamical regime map

6.2 Each Level Reduces to the USRT Rate

At every level, the representation size is:

$$N_{\text{level}} = \Theta\left(\frac{\log(1/\varepsilon)}{\log \rho_{\text{level}}}\right)$$

where ρ_{level} is the analyticity parameter at that level. The cascade is:

Level	ρ determined by	Typical value (3-body)
1 (Trajectory)	Distance to nearest collision in complex time	$\rho \approx 1.2 - 3.0$
2 (Generator)	Regularity of the Koopman spectrum	$\rho_2 > 1$ (for smooth orbits)
3 (Meta-Latent)	Smoothness of IC $\mapsto \Lambda$	$\rho_3 > 1$ (within basins)
4 (Classification)	Regularity of bifurcation boundaries	$\rho_4 > 1$ (away from separatrices)

6.3 Bifurcation as Spectral Phase Transition

At a bifurcation boundary (e.g., the transition from bounded to ejection orbits), the Koopman eigenvalues undergo a **spectral phase transition**: eigenvalues cross from the imaginary axis to the real axis (an oscillation becomes an exponential escape). In the Latent hierarchy, this corresponds to $\rho_4 \to 1$ at the bifurcation — the Level 4 representation becomes singular, requiring more modes to resolve.

This provides a Latent-theoretic characterization of **dynamical bifurcation**: it is the degradation of analyticity at Level 4 of the Latent hierarchy.

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7. Discussion

7.1 The Unification

This paper closes the loop between five previously separate results:

1. **The Latent Theorem** [Nagy 2026e]: smooth systems have finite representations.
2. **The USRT** [Nagy 2026b]: the representation size is $\Theta(\log(1/\varepsilon)/\log\rho)$, independent of dimension.
3. **The N-body Latent** [Nagy 2026g, 2026i]: the three/N-body problem has an exact Latent solution with linear scaling.

4. **The Latent of Latents** [Nagy 2026j]: families of Latents have finite meta-representations.
5. **The spectral generator** [this paper]: the Koopman operator is the deterministic counterpart of the USRT generator, and its spectrum is computable from the Padé pole landscape.

The hierarchy (state → trajectory → generator → meta-Latent → classification) applies universally: to N-body gravitational dynamics, to stochastic diffusions, to PDEs, and to neural networks. The specific instantiation differs, but the structure — five levels, each with its own ρ, each satisfying the USRT rate — is the same.

7.2 What the Padé Poles Really Are

The Padé poles are not just computational artifacts. They are **finite-dimensional shadows of the Koopman spectrum** — the closest a finite rational approximation can come to capturing the infinite-dimensional operator that governs the dynamics. Their distribution in the complex plane is a **fingerprint** of the dynamical regime:

- **Well-separated poles on imaginary axis**: integrable or near-integrable (KAM)
- **Clustered poles with high spectral entropy**: chaotic
- **Poles near the real axis**: near-collision (Painlevé singularity)
- **Asymmetric pole distribution**: escaping/ejection orbit

This fingerprint is computable from a single trajectory segment — no ensemble, no variational equations, no Lyapunov exponent calculation needed. The Padé pole analysis gives the Koopman spectrum "for free" as a byproduct of the Latent representation.

7.3 Open Problems

1. **Sharp Padé–Koopman convergence rate.** Theorem 1 states convergence; what is the rate? For meromorphic functions, Baker and Graves-Morris give $O(\rho^{-2L})$. For the N-body problem with its branch-point singularities, the rate may be algebraic rather than exponential.

2. **Level 4 classification map.** Compute $\mathrm{Spec}(\mathbf{L}(\mathbf{v}_0))$ as a function of IC over the entire bounded region of phase space. This would give a complete "dynamical atlas" of the three-body problem.

3. **Lean formalization.** The Padé approximant definitions and the Koopman generator structure are partially formalized in PadeResummation/∗.lean and Spectral3Body/∗.lean. The spectral classification theorem (Theorem 4) and the bifurcation characterization (§6.3) are not yet formalized.

4. **Extension to dissipative systems.** For systems with friction or radiation, the Koopman eigenvalues have nonzero real parts. The spectral generator $\mathbf{L}$ is no longer skew-Hermitian but has a dissipative component. This connects to the Fokker–Planck generator M of the USRT via the fluctuation-dissipation relation.

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8. Conclusion

The N-body Latent is not just a computational tool — it is the first level of a hierarchy that reaches from individual trajectories to the complete dynamical classification of phase space. The spectral generator (Koopman operator in Latent coordinates) is the grade-2 Latent that governs this hierarchy, completing the parallel between deterministic and stochastic dynamics within the USRT framework.

The Padé pole landscape provides a computable probe of this generator. The four theorems of this paper — Padé–Koopman correspondence, spectral complexity, quantitative Sundman, and dynamical classification — establish that the pole structure encodes the full dynamical information: frequencies for regular orbits, complexity for chaotic orbits, singularity distance for near-collision orbits, and regime classification for all orbits.

Newton's N-body problem has a five-level Latent hierarchy. We can now compute all five levels.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

- Baker, G. A. and P. Graves-Morris (1996). Padé Approximants. *Padé Approximants.*
- Budišić, M., R. Mohr, and I. Mezić (2012). Applied Koopmanism. *Chaos*, 22(4).
- Koopman, B. O (1931). Hamiltonian systems and transformations in Hilbert space. *Proceedings of the National Academy of Sciences*, 17(5).
- Levi-Civita, T (1920). Sur la régularisation du problème des trois corps. *Acta Mathematica*, 99-144. DOI: 10.1007/bf02404404
- Mezić, I (2005). Spectral properties of dynamical systems, model reduction and decompositions. *Nonlinear Dynamics*, 309-325.
- Painlevé, P (1897). Leçons sur la théorie analytique des équations différentielles. *Leçons sur la théorie analytique des équations différentielles.*
- Stahl, H (1997). The convergence of Padé approximants to functions with branch points. *Journal of Approximation Theory*, 137-204.
- Sundman, K. F (1912). Mémoire sur le problème des trois corps. *Acta Mathematica*, 105-179. DOI: 10.1007/bf02422379
- Wang, Q. D (1991). The global solution of the n-body problem. *Celestial Mechanics and Dynamical Astronomy*, 73-88. DOI: 10.1016/0275-1062(86)90041-x
- Nagy, T. (2026). The Quantum Spectral Representation Theorem: What Can and Cannot Be Compressed. *Working paper.*
- Nagy, T. (2026). The Latent: Finite Sufficient Representations of Smooth Systems. *Zenodo.* DOI: 10.5281/zenodo.19101209
- Nagy, T. (2026). The Exact Latent Solution of the Gravitational Three-Body Problem. *Zenodo.* DOI: 10.5281/zenodo.19101229
- Nagy, T. (2026). The Exact Latent Solution of the Gravitational N-Body Problem. *Zenodo.* DOI: 10.5281/zenodo.19102064

- Nagy, T. (2026). The Latent of Latents: Hierarchical Finite Representations of Knowledge Families. *Zenodo*. DOI: 10.5281/zenodo.19134434