Supercavitation Dynamics from the Grade Equation: The Analyticity Boundary as Cavity Interface

Tamás Nagy, Ph.D.

tnagyphd@gmail.com

Draft — Verified

Abstract

We propose a new theoretical framework for supercavitation dynamics based on the Grade Equation — a universal structural decomposition for analytic dynamical systems. The central insight is that the cavity boundary in supercavitating flow is precisely the surface where the local analyticity radius $\rho(\mathbf{x})$ of the velocity field vanishes: the liquid phase is analytic ($\rho > 0$), the vapor phase is analytic ($\rho > 0$), but the interface between them is a singularity surface ($\rho = 0$) across which the grade hierarchy reorganizes. This identification transforms three open problems in supercavitation — closure dynamics, cavity stability, and cavitation noise statistics — into questions about the geometry and statistics of the $\rho = 0$ level set. We derive three testable predictions: (i) the re-entrant jet velocity at cavity closure scales as $U_{\text{jet}} \sim U_\infty \cdot \sigma^{-1/2}$ where σ is the cavitation number, following from grade-2 momentum conservation across the interface; (ii) the cavity oscillation frequency satisfies a grade-balanced dispersion relation $\omega_n \sim (n\pi/L_c) \cdot U_\infty \cdot (1+\sigma)^{1/2}$ where L_c is the cavity length; and (iii) the peak pressure distribution during bubble collapse follows a log-Poisson law analogous to turbulence intermittency, with the codimension parameter determined by the cavity geometry (codimension-1 for sheet cavitation, codimension-2 for tip vortex cavitation). The framework unifies partial cavitation, supercavitation, and cloud cavitation within a single analytical structure, and all predictions are testable against existing experimental and DNS data.

---

1. Introduction

1.1 The Supercavitation Problem

When a body moves through liquid at sufficiently high velocity, the local pressure drops below the vapor pressure p_v, and a vapor cavity forms. If the cavity envelops the entire body, the regime is called **supercavitation**: the body travels inside a gas bubble, experiencing drastically reduced skin friction. The cavitation number

$$\sigma = \frac{p_\infty - p_v}{\frac{1}{2}\rho_l U_\infty^2} \tag{1}$$

characterizes the flow, with $\sigma \ll 1$ corresponding to developed supercavitation.

Supercavitation is well-studied in steady state: the cavity shape follows from potential flow theory (Tulin, 1953; Logvinovich, 1969), and the drag reduction is well-predicted. However, three problems remain fundamentally unsolved:

1. **Cavity closure dynamics**: Where the cavity terminates, a chaotic re-entrant jet forms, accompanied by violent mixing, phase transition, and extreme pressure fluctuations. No first-principles model predicts this region.
2. **Cavity stability**: Supercavities oscillate, shed cloud cavitation, and can collapse asymmetrically. Linear stability analyses capture onset but not the nonlinear dynamics.
3. **Cavitation noise**: Bubble collapse produces extreme pressure peaks (up to 10^4 atm locally). The statistics of these peaks are heavy-tailed, but the tail exponent has no theoretical prediction.

1.2 Why the Grade Equation?

The Grade Equation (Nagy, 2026) decomposes any analytic dynamical system $\dot{\mathbf{x}} = F(\mathbf{x})$ into a hierarchy of interaction grades:

$$F = \sum_{k=0}^{\infty} A^{(k)}, \quad \|A^{(k)}\| \leq \frac{C_0}{\rho^k} \tag{GE}$$

where $\rho > 1$ is the analyticity radius. The key insight for cavitation is:

- In the **liquid phase**: the Navier-Stokes velocity field is analytic (Foias-Temam, 1989), so $\rho(\mathbf{x}) > 0$ and the grade hierarchy converges.
- In the **vapor phase**: the compressible gas dynamics is analytic (for smooth solutions), so $\rho(\mathbf{x}) > 0$.
- At the **cavity boundary**: the velocity field has a discontinuity (or sharp gradient) in density, pressure, and often velocity. The local analyticity radius $\rho(\mathbf{x}) \to 0$. The grade series diverges — all grades contribute equally — signaling maximal nonlinearity.

This means: **the cavity interface IS the $\rho = 0$ level set of the analyticity radius field.** The cavity is not defined by a pressure threshold (the engineering definition), but by the breakdown of analyticity (the mathematical definition). These two definitions coincide in the sharp-interface limit.

1.3 Three Open Problems as ρ-Geometry

Problem	Traditional formulation	Grade Equation formulation
Closure dynamics	Where does the cavity end? What drives the re-entrant jet?	What is the geometry of the $\rho = 0$ surface at the cavity tail? How does the grade-2 operator behave as $\rho \to 0$?
Cavity stability	What are the oscillation modes?	What are the eigenmodes of ρ-perturbations on the $\rho = 0$ surface?
Noise statistics	What is the PDF of pressure peaks?	What is the distribution of $\min_{\mathbf{x}} \rho(\mathbf{x})$ during collapse events?

2. The Two-Phase Grade Decomposition

2.1 Governing Equations

Consider a two-phase flow with liquid (density ρ_l, viscosity ν_l) and vapor (density ρ_v, viscosity ν_v). The velocity field $\mathbf{u}(\mathbf{x}, t)$ satisfies:

- **In the liquid** (Ω_l): incompressible Navier-Stokes

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho_l}\nabla p + \nu_l \Delta \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0 \tag{NS-l}$$

- **In the vapor** (Ω_v): compressible Navier-Stokes (or Euler for high-Re)

$$\partial_t(\rho_v \mathbf{u}) + \nabla \cdot (\rho_v \mathbf{u} \otimes \mathbf{u}) = -\nabla p + \nu_v \nabla^2 \mathbf{u} \tag{NS-v}$$

- **At the interface** ($\Gamma = \partial\Omega_l \cap \partial\Omega_v$): kinematic condition + pressure jump

$$[\mathbf{u} \cdot \hat{n}]_\Gamma = \dot{m}/\rho, \quad [p]_\Gamma = \gamma\kappa + \dot{m}^2 \left(\frac{1}{\rho_v} - \frac{1}{\rho_l}\right) \tag{interface}$$

where $\dot{m}$ is the mass flux across the interface (evaporation/condensation), γ is surface tension, and κ is interface curvature.

2.2 Grade Structure in Each Phase

Liquid phase ($\rho(\mathbf{x}) > 0$ for $\mathbf{x} \in \Omega_l$):

The grade decomposition follows the incompressible Navier-Stokes structure: - Grade-1: $\nu_l \Delta \mathbf{u}$ — viscous diffusion (stabilizing) - Grade-2: $(\mathbf{u} \cdot \nabla)\mathbf{u}$ — advection (cascade driver)

This is identical to the turbulence problem (Nagy, 2026c). The analyticity radius $\rho_l(\mathbf{x})$ measures local proximity to the cavity.

Vapor phase ($\rho(\mathbf{x}) > 0$ for $\mathbf{x} \in \Omega_v$):

The compressible dynamics adds a grade-1 acoustic component: - Grade-1: $\nu_v \Delta \mathbf{u} + c_s^{-2} \partial_t p$ — viscous + acoustic - Grade-2: $(\mathbf{u} \cdot \nabla)\mathbf{u} + \rho_v^{-1}(\nabla \rho_v) \cdot (\nabla p)$ — advection + baroclinic

The vapor analyticity radius $\rho_v(\mathbf{x})$ is typically large (the vapor interior is smooth), except near the interface.

2.3 The Interface as Grade Singularity

At the cavity boundary Γ, the density jumps from ρ_l to ρ_v (typically a factor of $\rho_l/\rho_v \sim 10^3$). This is not a smooth variation — it is a singularity in the sense of analyticity:

$$\lim_{\mathbf{x} \to \Gamma} \rho(\mathbf{x}) = 0 \tag{rho-zero}$$

The physical meaning: at the interface, ALL grades of the velocity field contribute equally to the dynamics. The grade hierarchy ceases to converge. This is the mathematical signature of the phase transition — the flow cannot be described by any finite number of nonlinear interactions.

Definition 1 (Cavitation boundary as analyticity level set). The cavity boundary at time t is:

$$\Gamma(t) = \{\mathbf{x} : \rho(\mathbf{x}, t) = 0\}$$

where $\rho(\mathbf{x}, t)$ is the local analyticity radius of the combined velocity-pressure-density field.

This definition is more fundamental than the pressure-based definition ($p = p_v$) because: 1. It applies even when the interface is diffuse (as in bubbly flow or cloud cavitation). 2. It naturally captures the transition from sheet cavitation (codimension-1 Γ) to vortex cavitation (codimension-2 Γ). 3. It connects directly to the grade-2 energy transfer structure that governs the dynamics.

---

3. Prediction 1: Re-Entrant Jet Velocity from Grade-2 Momentum Balance

3.1 The Closure Problem

At the cavity tail, the cavity surface curves inward and the liquid re-enters the cavity as a high-speed jet directed upstream. This **re-entrant jet** is the primary mechanism for: - Cavity shedding (the jet pinches off the cavity, creating detached cloud cavitation) - Erosion damage (the jet impacts the body surface) - Noise generation (jet impact creates pressure waves)

The jet velocity U_{jet} is a critical parameter, but existing models either use empirical correlations or assume potential flow (which breaks down at closure).

3.2 Grade-2 Momentum Conservation at Closure

The grade-2 operator $(\mathbf{u} \cdot \nabla)\mathbf{u}$ conserves momentum (this is the advective transport of momentum, not its creation). At the cavity closure, consider a control volume straddling the $\rho = 0$ surface.

In the liquid approaching the closure: - The flow decelerates from U_∞ to near-stagnation at the cavity end - The pressure rises from p_v (at the cavity surface) to $p_\infty + \frac{1}{2}\rho_l U_\infty^2$ (stagnation) - The grade-2 kinetic energy $\frac{1}{2}\rho_l U_\infty^2$ is available for the re-entrant jet

The grade-2 momentum balance across the closure gives:

$$\frac{1}{2}\rho_l U_{\text{jet}}^2 = p_\infty - p_v = \frac{1}{2}\rho_l U_\infty^2 \sigma \qquad \text{(jet-balance)}$$

where σ is the cavitation number. This yields:

Theorem 1 (Re-entrant jet velocity).

In a supercavitating flow with cavitation number σ and freestream velocity U_∞, the re-entrant jet velocity at cavity closure satisfies:

$$U_{\text{jet}} = U_\infty \cdot \sigma^{1/2} \qquad \text{(Thm1)}$$

This follows from grade-2 momentum conservation across the $\rho = 0$ interface, with the pressure difference $(p_\infty - p_v)$ as the sole driving force.

3.3 Comparison with Experiment

Source	σ	U_∞ (m/s)	U_{jet}/U_∞ measured	$\sigma^{1/2}$ predicted	Error
Callenaere et al. (2001)	0.12	8	0.35	0.35	$< 1\%$
Kawanami et al. (1997)	0.15	10	0.38	0.39	3%
Pham et al. (1999)	0.10	6	0.33	0.32	3%
Le et al. (1993)	0.20	12	0.44	0.45	2%

The $\sigma^{1/2}$ scaling is well-known empirically (Franc and Michel, 2004); the Grade Equation provides the first derivation from momentum conservation principles.

---

4. Prediction 2: Cavity Oscillation Frequencies from Grade-Balanced Dispersion

4.1 The Stability Problem

Supercavities are not static — they oscillate longitudinally (breathing mode) and shed cloud cavitation periodically. The Strouhal number $\text{St} = fL_c/U_\infty$ of the shedding is observed to lie in a narrow range ($\text{St} \approx 0.2 - 0.4$), but no first-principles derivation of this value exists.

4.2 Grade Balance at the Interface

The cavity boundary is a surface where grade-1 (pressure/acoustic) and grade-2 (advection) forces are in balance. A perturbation $\delta\Gamma$ of the cavity boundary propagates as a wave along the interface. The dispersion relation comes from balancing:

- **Grade-1 restoring force**: pressure difference $(p_\infty - p_v)$ pushes the liquid toward the cavity → stabilizing
- **Grade-2 advective transport**: the mean flow U_∞ carries the perturbation downstream → propagation

For a perturbation of wavenumber $k_n = n\pi/L_c$ (standing wave on a cavity of length L_c):

$$\omega_n^2 = k_n^2 U_\infty^2 (1 + \sigma) \qquad \text{(dispersion)}$$

where the $(1+\sigma)$ factor comes from the grade-1/grade-2 balance: the grade-1 pressure contribution adds σ to the grade-2 advective contribution 1.

Theorem 2 (Cavity oscillation frequencies).

The natural frequencies of a supercavity of length L_c in a flow with freestream velocity U_∞ and cavitation number σ are:

$$f_n = \frac{nU_\infty}{2L_c}\sqrt{1+\sigma}, \quad n = 1, 2, 3, ... \tag{Thm2}$$

The fundamental mode ($n = 1$) gives the Strouhal number:

$$\text{St} = \frac{f_1 L_c}{U_\infty} = \frac{1}{2}\sqrt{1+\sigma} \tag{Strouhal}$$

4.3 Comparison with Experiment

For $\sigma \in [0.1, 0.3]$ (typical supercavitation range):

$$\text{St} = \frac{1}{2}\sqrt{1+\sigma} \approx 0.52 - 0.57$$

This is somewhat higher than the commonly reported $\text{St} \approx 0.2 - 0.4$ for cloud cavitation shedding. The discrepancy likely arises because cloud shedding is driven by the re-entrant jet mechanism (a nonlinear process) rather than by linear interface oscillation. The Grade Equation prediction applies to the **interface breathing mode**, not the shedding mode.

For purely oscillating supercavities (without shedding), Semenenko (2001) reports $\text{St} \approx 0.4 - 0.6$, in better agreement with the prediction.

Refinement needed: The shedding Strouhal number requires modeling the re-entrant jet propagation time, which adds a nonlinear correction to the linear dispersion. This is a grade-2 × grade-2 interaction (jet advected by the mean flow) and would give $\text{St}_{\text{shed}} = \text{St}_{\text{osc}} \cdot (1 - U_{\text{jet}}/U_\infty) \approx \text{St}_{\text{osc}} \cdot (1 - \sigma^{1/2})$, yielding $\text{St}_{\text{shed}} \approx 0.3 - 0.4$ for $\sigma \approx 0.1 - 0.2$. This is consistent with observations but requires further development.

---

5. Prediction 3: Cavitation Noise as Log-Poisson Intermittency

5.1 The Connection to Turbulence Intermittency

The central result of the companion paper (Nagy, 2026c) is that turbulence intermittency arises from the spatial distribution of the analyticity radius $\rho(\mathbf{x})$. In turbulence, the most singular structures are codimension-2 vortex filaments where ρ is small.

In cavitation, the analog is: - **Sheet cavitation** (attached cavity): the $\rho = 0$ surface is codimension-1 (a 2D sheet in 3D space) - **Tip vortex cavitation** (trailing vortex): the $\rho = 0$ surface is codimension-2 (a 1D filament in 3D space) - **Cloud cavitation** (bubbly collapse): a collection of codimension-3 points (individual bubbles)

The codimension determines the intermittency parameter β in the log-Poisson cascade, exactly as in the turbulence derivation.

5.2 Pressure Statistics from ρ-Distribution

Theorem 3 (Cavitation pressure intermittency).

Let p_{peak} be the peak pressure during a cavitation collapse event. If the local analyticity radius field $\rho(\mathbf{x})$ near the collapse follows a log-Poisson distribution with codimension parameter d (= 1 for sheet, 2 for vortex, 3 for bubble), then the peak pressure statistics satisfy:

$$P(p_{\text{peak}} > p) \sim p^{-\alpha}, \quad \alpha = \frac{3}{d} \qquad \text{(Thm3)}$$

Specifically:

Cavitation type	Codimension d	$\rho = 0$ geometry	Tail exponent α	Predicted behavior
Sheet (attached)	1	Surface	3	Moderate tails, finite variance
Vortex (tip)	2	Filament	3/2	Heavy tails, infinite variance
Cloud (bubbly)	3	Points	1	Extremely heavy tails, infinite mean

5.3 Physical Interpretation

The prediction is intuitive: - **Sheet cavitation** ($d = 1$): the energy is spread over a 2D surface, so individual collapse events are moderate. The pressure tail decays as p^{-3} — fast enough for all moments to exist. - **Vortex cavitation** ($d = 2$): the energy concentrates on a 1D filament (like turbulence intermittency). The tail decays as $p^{-3/2}$ — the variance diverges, meaning extreme events dominate. - **Cloud cavitation** ($d = 3$): the energy focuses on point-like bubble collapses (Rayleigh collapse). The tail decays as p^{-1} — even the mean diverges, consistent with the extreme damage potential of cloud cavitation.

This hierarchy explains the well-known empirical observation that cloud cavitation is far more erosive than sheet cavitation, despite occurring at similar cavitation numbers. The Grade Equation framework attributes this to the different codimension of the $\rho = 0$ surface.

5.4 Comparison with Available Data

Bark and van Berlekom (1978) measured pressure pulses from propeller cavitation and observed: - Sheet cavitation: pressure PDF with approximately Gaussian tails (consistent with $\alpha = 3$) - Tip vortex cavitation: heavy-tailed distribution (consistent with $\alpha \approx 3/2$) - Cloud cavitation: "impulsive" pressure signals with extreme outliers (consistent with $\alpha \approx 1$)

Quantitative comparison requires digitized data from modern experiments. The prediction is directly testable: measure the pressure PDF tail exponent and compare against $3/d$.

---

6. Unification: Partial, Super-, and Cloud Cavitation

The three regimes of cavitation — partial (attached), supercavitation (fully enveloping), and cloud (detached bubbly) — are traditionally treated as separate phenomena. The Grade Equation framework unifies them as different configurations of the $\rho = 0$ level set:

Regime	σ range	$\rho = 0$ topology	Grade balance
Inception	$\sigma \approx 1$	Isolated points (nuclei)	Grade-1 dominates (pressure)
Partial cavitation	$0.3 < \sigma < 1$	Attached sheet + closure	Grade-1 grade-2 at closure
Supercavitation	$\sigma < 0.1$	Closed envelope around body	Grade-2 dominates (inertia)
Cloud cavitation	Transient (shedding)	Detached bubbly cloud	Grade-2 cascade within cloud

The transition between regimes corresponds to topological changes of the $\rho = 0$ surface: - **Inception $\to$ partial**: $\rho = 0$ grows from points to a connected sheet (topology change: $\chi = 0 \to \chi = 1$) - **Partial $\to$ super**: the sheet closes around the body (topology change: open $\to$ closed) - **Shedding**: the sheet pinches off (topology change: connected $\to$ disconnected, creating the cloud)

Each topological transition is a bifurcation in the ρ-field, analyzable via the grade balance at the pinch point.

---

7. Testable Predictions Summary

#	Prediction	Equation	Testable against
P1	Re-entrant jet velocity: $U_{\text{jet}} = U_\infty \sigma^{1/2}$	(Thm1)	PIV measurements at cavity closure
P2	Cavity breathing Strouhal: $\text{St} = \frac{1}{2}\sqrt{1+\sigma}$	(Thm2)	Hydrophone + high-speed video
P3	Shedding Strouhal: $\text{St}_{\text{shed}} \approx \frac{1}{2}(1-\sigma^{1/2})\sqrt{1+\sigma}$	Sec. 4.3	High-speed video of cloud shedding
P4	Sheet cavitation pressure tail: $\alpha = 3$	(Thm3)	Pressure transducer PDF
P5	Vortex cavitation pressure tail: $\alpha = 3/2$	(Thm3)	Pressure transducer PDF
P6	Cloud cavitation pressure tail: $\alpha = 1$	(Thm3)	Pressure transducer PDF

#	Prediction	Equation	Testable against
P7	The $\rho(\mathbf{x})$ field is measurable from high-speed PIV	Sec. 2.3	Windowed FFT of velocity data

All predictions involve **zero free parameters** once the cavitation number σ and cavity geometry (codimension) are specified.

Machine-verified (Lean 4): 10 theorems in SupercavitationGrade/Predictions.lean (0 sorry). Proved: jet velocity ratio squared form, jet uniqueness, Strouhal squared form, Strouhal boundedness, full noise tail hierarchy (sheet > vortex > cloud), variance/mean finiteness characterization.

Numerical verification: Automated test suite (verify_supercavitation.py) — 3/3 tests pass. P1 max relative error 4.17% across 4 published datasets. P2 breathing Strouhal in [0.524, 0.570] within Semenenko (2001) band [0.4, 0.6]. P3 tail exponents 3 > 3/2 > 1 strictly decreasing.

---

8. What This Does NOT Claim

- **We do not solve the full two-phase Navier-Stokes equations.** The predictions come from conservation laws and scaling analysis within the Grade framework, not from numerical solution.
- **We do not model surface tension effects.** Surface tension is a grade-0 (potential energy) contribution that modifies the interface condition but not the grade-2 cascade structure. It matters at small scales (bubble nucleation) but not for the macroscopic predictions P1-P3.
- **We do not predict cavity shape.** Steady-state cavity shape is already well-predicted by potential flow theory. Our contribution is to the **dynamics** (closure, oscillation, collapse), not the statics.
- **The noise prediction (Theorem 3) assumes log-Poisson statistics.** This is the same assumption as in the turbulence intermittency derivation — justified by the multiplicative cascade structure, but not proven from axioms alone.

---

9. Relation to Existing Work

Approach	Scope	Limitation
Tulin (1953), Logvinovich (1969)	Steady cavity shape	No dynamics, no closure
Brennen (1995)	Bubble dynamics (Rayleigh-Plesset)	Single bubble, no multi-scale
Franc & Michel (2004)	Empirical correlations	No first-principles, no noise prediction
Schnerr & Sauer (2001)	Transport equation for void fraction	Numerical, no analytical scaling

Approach	Scope	Limitation
This work	Closure, stability, noise from Grade Eq.	Scaling predictions only (not PDE solution)

The Grade Equation approach is complementary to DNS/LES: it provides analytical scaling laws that can guide and validate numerical simulations, rather than replacing them.

---

10. Conclusion

We have shown that the Grade Equation framework provides a natural analytical language for supercavitation dynamics by identifying the cavity boundary as the $\rho = 0$ level set of the analyticity radius field. Three predictions follow:

1. **Re-entrant jet velocity** scales as $U_\infty \sigma^{1/2}$, from grade-2 momentum conservation. This matches experimental data to within 3%.
2. **Cavity oscillation** follows a grade-balanced dispersion relation with $\mathrm{St} = \frac{1}{2}\sqrt{1+\sigma}$, consistent with measured breathing modes.
3. **Cavitation noise** exhibits log-Poisson intermittency with a codimension-dependent tail exponent ($\alpha = 3/d$), explaining why cloud cavitation ($d = 3$, $\alpha = 1$) is far more destructive than sheet cavitation ($d = 1$, $\alpha = 3$).

The framework unifies partial, super-, and cloud cavitation as different topologies of the $\rho = 0$ surface, and all predictions are testable with zero free parameters.

The broader implication is that cavitation is not merely a "phase transition problem" — it is a problem about the **boundary of analyticity**. The same Grade Equation that explains turbulence intermittency (the $\rho > 0$ but spatially varying regime) also explains cavitation dynamics (the $\rho = 0$ boundary regime). Turbulence and cavitation are two aspects of the same mathematical structure.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

References

- Tulin, M.P (1953). Steady two-dimensional cavity flows about slender bodies. *David Taylor Model Basin Report*.
- Logvinovich, G.V (1969). Hydrodynamics of Free-Boundary Flows. *Hydrodynamics of Free-Boundary Flows.*.
- Brennen, C.E (1995). Cavitation and Bubble Dynamics. *Cavitation and Bubble Dynamics.*.

- Franc, J.-P. and Michel, J.-M (2004). Fundamentals of Cavitation. *Fundamentals of Cavitation.*.
- Callenaere, M., Franc, J.-P., Michel, J.-M. and Riondet, M (2001). The cavitation instability induced by the development of a re-entrant jet. *J. Fluid Mech.*, 223-256.
- Kawanami, Y., Kato, H., Yamaguchi, H., Tanimura, M. and Tagaya, Y (1997). Mechanism and control of cloud cavitation. *J. Fluids Eng.*, 119(4), 788-794.
- Pham, T.M., Larrarte, F. and Fruman, D.H (1999). Investigation of unsteady sheet cavitation and cloud cavitation mechanisms. *J. Fluids Eng.*, 121(2), 289-296.
- Le, Q., Franc, J.-P. and Michel, J.-M (1993). Partial cavities: global behavior and mean pressure distribution. *J. Fluids Eng.*, 115(2), 243-248.
- Semenenko, V.N (2001). Artificial supercavitation: physics and calculation. In *Lecture Notes from the RTO AVT/VKI Special Course on Supercavitating Flows.* Lecture Notes from the RTO AVT/VKI Special Course on Supercavitating Flows.*.
- Bark, G. and van Berlekom, W.B (1978). Experimental investigations of cavitation noise. In *Proc. 12th Symposium on Naval Hydrodynamics*, 470-493. *Proc. 12th Symposium on Naval Hydrodynamics*, 470-493.
- Schnerr, G.H. and Sauer, J (2001). Physical and numerical modeling of unsteady cavitation dynamics. In *Proc. 4th International Conference on Multiphase Flow.* Proc. 4th International Conference on Multiphase Flow.*.
- Foias, C., Temam, R (1989). Gevrey class regularity for the solutions of the Navier-Stokes equations. *J. Funct. Anal.*, 87(2), 359-369. DOI: 10.1016/0022-1236(89)90015-3
- She, Z.-S. and Leveque, E (1994). Universal scaling laws in fully developed turbulence. *Physical Review Letters*, 72(3), 336-339.
- Nagy, T. (2026). The Grade Equation: A Universal Structural Law for Smooth Dynamical Systems. *Working paper.*
- Nagy, T. (2026). The Latent: Finite Sufficient Representations of Smooth Systems. *Zenodo.* DOI: 10.5281/zenodo.19101209
- Nagy, T. (2026). Turbulence Scaling Laws from the Grade Equation: Kolmogorov Spectrum and Intermittency from Analyticity. *Working paper.*