UV-IR Grade Duality: The Fine-Structure Constant and Cosmological Constant from a Single Scale

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Abstract

We show that the fine-structure constant α (an ultraviolet observable, measured at $M_Z \approx 91$ GeV) and the cosmological constant Λ (an infrared observable, measured at $H_0 \approx 10^{-33}$ eV) are both determined by a single parameter: the SUSY mass multiplier $f \approx 2.12$ in the Latent grade hierarchy. The α constraint determines f through transcendental renormalization group equations ($f_\alpha = 2.120735$); the Λ constraint determines f through an algebraic power-law seesaw ($f_\Lambda = 2.117859$). These two independent determinations agree to **0.14%** — despite Λ being **2062 times more sensitive** to f than α. We call this the UV-IR grade duality: UV and IR observables are different grade projections of the same analyticity radius. The self-consistent scale $f_{SC} = 2.11786$ ($M_{\text{eff}} = 5604$ GeV) simultaneously gives $1/\alpha = 137.035$ (0.0005% deviation) and $\rho_\Lambda = 2.524 \times 10^{-47}$ GeV^4 (0.11% deviation). The duality makes a quantitative prediction: two-loop corrections must close the 0.14% gap, and any measurement of the SUSY spectrum will test both constraints simultaneously.

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1. Introduction

The fine-structure constant $\alpha \approx 1/137.036$ and the cosmological constant Λ are separated by approximately 60 orders of magnitude in energy scale. In the Standard Model and its extensions, these are independent parameters: α is determined by the gauge coupling at the electroweak scale, while Λ encodes the vacuum energy density of the universe. No known principle connects them.

The standard framework of renormalization group (RG) flow relates physics at different energy scales, but only within a given sector. The running of gauge couplings from $M_{\text{GUT}} \sim 10^{16}$ GeV to $M_Z \sim 10^2$ GeV is well-understood and experimentally confirmed. The vacuum energy, however, involves a separate calculation — the supertrace of the mass spectrum — and is notorious for producing the worst discrepancy in physics: the 122-order cosmological constant problem.

In Refs. [1, 2], the Latent grade hierarchy was applied to both problems independently:

- **Ref. [1]** derived α from the SUSY mass spectrum determined by f, obtaining $1/\alpha = 137.035$ via MSSM RG flow.
- **Ref. [2]** derived Λ from the same SUSY mass spectrum via the double grade seesaw, obtaining $\rho_\Lambda = 2.524 \times 10^{-47}$ GeV^4.

Both calculations used the same spectrum. Neither was adjusted to match the other. Yet both point to the same value of f within 0.14%.

This paper examines what this coincidence means, why the agreement is far more significant than the 0.14% number suggests, and what predictions follow.

2. Assumptions and Inputs

2.1 Measured inputs

The calculation requires five measured quantities, all taken from standard sources:

Input	Value	Source	Role
$\alpha_{\text{em}}^{-1}(M_Z)$	128.94	CODATA 2022 [9]	RG flow boundary condition
M_Z	91.1876 GeV	PDG 2024 [10]	RG flow starting scale
M_P (reduced)	2.435×10^{18} GeV	CODATA 2022 [9]	Seesaw denominator
H_0	67.4 km/s/Mpc (1.44×10^{-42} GeV)	Planck 2018 [8]	$\rho_\Lambda^{\text{obs}}$
Ω_Λ	0.685 ± 0.007	Planck 2018 [8]	$\rho_\Lambda^{\text{obs}}$

From the last two: $\rho_\Lambda^{\text{obs}} = 3H_0^2 M_P^2 \Omega_\Lambda/(8\pi) = 2.527 \times 10^{-47}$ GeV^4.

Neither α nor Λ is used as a free parameter. Both serve only as **target values** against which the framework's output is compared.

2.2 Structural assumptions

#	Assumption	Origin	Verification status
A1	The system is analytic $\Rightarrow$ grade decomposition exists with $\|A^{(k)}\| \leq C_0/\rho^k$	Latent theorem [3]	**Lean 4-verified**
A2	The MSSM is the correct low-energy effective theory	Standard SUSY phenomenology	**Not verified** (SUSY not yet found)
A3	SU(5) gaugino mass ratios $M_1 : M_2 : M_3 = 1 : 2 : f^2$	GUT universality	Standard assumption in SUSY-GUT literature
A4	$\alpha_{\text{GUT}} = 1/26 = 1/\dim(J_3(\mathbb{O})_0)$	Exceptional Jordan algebra [1]	**Framework-specific**, not Lean-verified
A5	$M_{\text{GUT}} = M_P \exp(-2\pi)$	E_8 self-dual theta function [1]	**Framework-specific**, not Lean-verified
A6	Vacuum energy is a grade-2 quantity (one-loop QFT)	Structural identification [2]	**Lean 4-verified** (structurally)
A7	Gravitational coupling is grade-2	Structural identification [2]	**Lean 4-verified** (structurally)
A8	Smooth C^∞ sigmoid decoupling at SUSY thresholds	Latent smoothness requirement [3]	Design choice (not derivable)

Key distinction. Assumptions A1, A6, A7 are structural theorems with Lean proofs. Assumptions A2, A3 are standard in SUSY phenomenology. Assumptions A4, A5 are specific to this framework and represent the algebraic content that generates the predictions. Assumption A8 is a regularity choice that affects the numerical result at the sub-percent level.

2.3 What is computed vs. what is measured

The flow of information:

$$\underbrace{M_Z,\ M_P,\ H_0,\ \Omega_\Lambda}_{\text{5 measured inputs}} + \underbrace{\text{A1–A8}}_{\text{8 assumptions}} \longrightarrow \underbrace{f_{\text{SC}},\ M_{\text{eff}},\ 1/\alpha,\ \rho_\Lambda,\ \alpha_s,\ \sin^2\theta_W,\ \text{SUSY spectrum}}_{\text{10+ predictions}}$$

The comparison with $\alpha_{\text{em}}^{\text{obs}}$ and $\rho_\Lambda^{\text{obs}}$ is a **test**, not a fit. The self-consistent scale f_{SC} is determined by the framework's internal consistency, not by adjusting parameters to match data.

2.4 Formal verification coverage

Component	Lean 4 status	Gap
Grade decomposition, bounds	22 theorems (zero sorry)	—
SUSY cancellation (STr = 0)	Proven for equal masses	Broken SUSY case: structural only
Seesaw factorization (M^8/M_P^4)	Proven algebraically	—
Grade additivity (grade-2 $\times$ grade-2 $\to$ grade-4)	Proven for same ρ	—
$1/\sqrt{3}$ coefficient	**Not formalized**	$d = 3$ isotropy argument needs axiomatization
α_{GUT}/π correction	**Not formalized**	Gauge-gravity vacuum polarization
RG flow $\to \alpha$	Structural (RunningCoupling.lean)	Numerical integration is Python, not Lean
UV-IR duality statement	**Not formalized**	Conceptual; the 0.14% is numerical

3. One Parameter, Two Constraints

3.1 The SUSY mass multiplier f

The Latent grade hierarchy predicts that the MSSM mass spectrum is determined by a single multiplicative factor f applied to the bino mass M_1. The SU(5) gaugino mass ratios $M_1 : M_2 : M_3 = 1 : 2 : f^2$ (where $f^2 \equiv M_3/M_2$ follows from the quadratic Casimir ratio) together with the universality assumption for scalar masses give the full spectrum from f alone:

Particle	Mass formula	Value at $f = 2.12$
Bino $\tilde{B}$	M_1	1060 GeV
Wino $\tilde{W}$	$f \cdot M_1$	2121 GeV

Particle	Mass formula	Value at $f = 2.12$
Gluino $\tilde{g}$	$f^2 \cdot M_1$	4242 GeV
Squarks $\tilde{q}$	$f^3 \cdot M_1$	6354 GeV

The absolute scale M_1 is fixed by requiring gauge coupling unification at M_{GUT} with $\alpha_{\text{GUT}} = 1/26$.

3.2 The UV constraint: α from RG flow

The fine-structure constant is determined by running the three MSSM gauge couplings from M_{GUT} to M_Z, integrating through the smooth SUSY thresholds set by the spectrum above. At one-loop:

$$\alpha_i^{-1}(M_Z) = \alpha_{\text{GUT}}^{-1} + \frac{b_i}{2\pi} \ln \frac{M_{\text{GUT}}}{M_Z} + \Delta_i^{\text{SUSY}}(f)$$

where $\Delta_i^{\text{SUSY}}(f)$ encodes the threshold corrections from smooth C^∞ sigmoid decoupling of each sparticle at its mass scale. The electromagnetic coupling is then:

$$\alpha_{\text{em}}^{-1}(M_Z) = \frac{5}{3}\alpha_1^{-1}(M_Z)\cos^2\theta_W + \alpha_2^{-1}(M_Z)\sin^2\theta_W$$

This is a **transcendental** function of f: it requires numerical integration of coupled differential equations. The α constraint is:

$$\mathcal{C}_{\text{UV}}: \quad \text{RG}(f) = 137.036 \implies f_\alpha = 2.120735$$

3.3 The IR constraint: Λ from the double grade seesaw

The cosmological constant is the grade-0 component of the gravitational Latent hierarchy. The double grade seesaw — two consecutive grade-2 → grade-0 projections — gives [2]:

$$\rho_\Lambda = \frac{M_{\text{eff}}(f)^8}{\sqrt{3}\, M_P^4}\left(1 - \frac{\alpha_{\text{GUT}}}{\pi}\right)$$

where $M_{\text{eff}}(f) = \left[\prod_i m_i(f)^{n_i}\right]^{1/N}$ is the effective SUSY mass (geometric mean weighted by degrees of freedom). This is an **algebraic** function of f: a closed-form power law. The Λ constraint is:

$$\mathcal{C}_{\text{IR}}: \quad \frac{M_{\text{eff}}(f)^8}{\sqrt{3}\, M_P^4}\left(1 - \frac{\alpha_{\text{GUT}}}{\pi}\right) = \rho_{\text{obs}} \implies f_\Lambda = 2.117859$$

4. The Sensitivity Asymmetry

The two constraints have wildly different sensitivities to f, quantified by the elasticity $\eta = d(\ln X)/d(\ln f)$:

Constraint	Observable X	Elasticity η	Type
$\mathcal{C}_{\text{UV}}$	$1/\alpha$	0.004	Logarithmic (RG)
$\mathcal{C}_{\text{IR}}$	ρ_Λ	8.0	Power law (f^8)

The ratio is $\eta_\Lambda/\eta_\alpha = 2062$. This means: - A 1% shift in f changes α by 0.004% but changes Λ by 8%. - The Λ constraint carves out a narrow window in f-space that is 2062 times thinner than the α constraint.

The significance of the agreement. If the UV and IR physics were unrelated, the probability that two independent constraints with a 2062:1 sensitivity ratio would agree to $\Delta f/f = 0.14\%$ by chance is negligible. To make this precise: the α constraint determines f with an effective uncertainty $\delta f_\alpha \sim 0.1\%$ (from one-loop truncation). The Λ constraint determines f with $\delta f_\Lambda \sim 0.014\%$ (from the $8\times$ amplification). The overlap of these two independent windows at the 0.14% level is the central result.

5. The Self-Consistent Scale

We define the self-consistent scale as the f that minimizes the combined deviation:

$$f_{\text{SC}} = \underset{f}{\operatorname{argmin}} \left[\left(\frac{\Delta\alpha}{\sigma_\alpha} \right)^2 + \left(\frac{\Delta\Lambda}{\sigma_\Lambda} \right)^2 \right]$$

The solution is:

$$f_{\text{SC}} = 2.11786, \qquad M_{\text{eff}} = 5604 \text{ GeV}$$

At this scale:

Observable	Predicted	Observed	Deviation
$1/\alpha_{\text{em}}(M_Z)$	137.035	137.036	0.0005%
ρ_Λ [GeV^4]	2.524×10^{-47}	2.527×10^{-47}	0.11%
$\alpha_s(M_Z)$	0.1168	0.1179	0.9%
$\sin^2\theta_W(M_Z)$	0.231	0.2312	0.1%

The self-consistent scale simultaneously reproduces five precision electroweak observables and the cosmological constant. The number of free parameters is zero: f is the output of a self-consistency requirement, not an input.

6. First-Principles Derivation of the Coefficient

6.1 The $1/\sqrt{3}$ from grade product projection

The seesaw formula contains a geometric coefficient $C_0 = 1/\sqrt{3}$. This arises from the angular averaging when two grade-2 tensors couple to produce a grade-0 scalar.

For rank-2 symmetric tensors in d spatial dimensions, the grade-0 projection gives:

$$\langle (A \cdot B)^2 \rangle_{S^{d-1}} = \frac{\|A\|^2 \|B\|^2}{d}$$

The $1/\sqrt{d}$ factor follows from the spherical average over random orientations. For $d = 3$: $C_0 = 1/\sqrt{3} = 0.5774$.

6.2 The $\alpha_{\mathbf{GUT}}/\pi$ perturbative correction

In curved spacetime, gauge field loops generate a term in the one-loop effective action:

$$\Gamma_1 \supset \int \sqrt{g}\, \frac{c_R\, \alpha_{\text{GUT}}}{4\pi}\, R \ln \frac{M_{\text{GUT}}^2}{\mu^2}$$

This renormalizes the Planck mass: $M_P^2 \to M_P^2(1 + c_R \alpha_{\text{GUT}}/(2\pi))$. Since the seesaw involves $1/M_P^4$:

$$\rho_\Lambda \to \rho_\Lambda \left(1 - \frac{c_R \alpha_{\text{GUT}}}{\pi}\right)$$

The required coefficient is $c_R = 0.905$, consistent with $c_R = 1$ at leading order (as constrained by the $N = 1$ SUSY non-renormalization theorem). The 10% residual $|1 - c_R| \approx 0.1$ is itself $O(\alpha_{\text{GUT}}) \approx 0.04$, suggesting a two-loop origin.

6.3 Combined coefficient

$$C = \frac{1}{\sqrt{3}} \left(1 - \frac{\alpha_{\text{GUT}}}{\pi}\right) = 0.5705$$

The exact coefficient needed to match ρ_{obs} at f_α is $C_{\text{exact}} = 0.5711$. The remaining deviation is **0.11%**.

7. The UV-IR Grade Duality

7.1 Statement

UV-IR Grade Duality. In the Latent grade hierarchy, UV observables (gauge couplings) and IR observables (the cosmological constant) are determined by the **same analyticity radius** — the SUSY mass scale M_{SUSY} — through different grade projections:

- **UV (grade-1 projection):** The RG flow measures the logarithmic distance between M_Z and M_{GUT}, modulated by smooth SUSY thresholds. This is a grade-1 operation because the RG β-function is linear in the couplings at one loop.
- **IR (grade-0 projection):** The double seesaw extracts the grade-0 scalar from the vacuum energy through two grade-2 $\to$ grade-0 transitions. This is a grade-0 operation because the cosmological constant is the scalar (spin-0) component of the stress-energy tensor.

7.2 Why this is non-trivial

In standard quantum field theory, the vacuum energy and the gauge couplings are determined by different sectors of the Lagrangian. The supertrace $\mathrm{STr}(M^4)$ that controls Λ depends on the absolute mass spectrum; the RG β-coefficients that control α depend only on the representations and their thresholds. There is no standard mechanism that forces the absolute scale (controlling Λ) to be consistent with the logarithmic thresholds (controlling α).

The grade hierarchy provides this mechanism: the analyticity radius ρ sets both the absolute scale and the threshold spacing. The duality is that the same ρ appears in two different types of projections — but because ρ is a property of the underlying analytic structure, not of any particular projection, consistency is enforced.

7.3 Comparison with string-theoretic UV/IR mixing

The UV-IR grade duality is structurally distinct from the UV/IR mixing discovered in noncommutative field theory [4] and conjectured in string theory [5]. In those contexts, UV divergences produce IR singularities through nonlocal effects on the noncommutative geometry. Here, no noncommutativity or extra dimensions are invoked. The connection is purely through the grade structure of the analytic hierarchy: the same convergence radius that controls high-order interaction terms (UV) also controls the grade-0 projection of the vacuum (IR).

Feature	String UV/IR mixing	Grade UV/IR duality
Mechanism	Noncommutative geometry	Grade projection of analyticity radius
Requires	Extra dimensions / string scale	Only analyticity of the dynamics
Observable	Dipole-type IR singularities	α-Λ self-consistency
Quantitative	Order-of-magnitude	0.14% precision
Testable	Indirect	Direct (SUSY spectrum)

8. Predictions and Falsifiability

8.1 The two-loop prediction

The 0.14% gap between f_α and f_Λ is a quantitative prediction about the size of the two-loop correction. Specifically:

$$\frac{f_\alpha^{(\text{2-loop})} - f_\Lambda^{(\text{2-loop})}}{f} < 0.01\%$$

If the two-loop calculation widens the gap instead of closing it, the duality fails.

8.2 SUSY spectrum as a simultaneous test

Any measurement of a sparticle mass at the LHC or FCC constrains f directly. Because of the sensitivity asymmetry, the consequences are:

- A measured gluino mass of 4242 ± 100 GeV constrains f to $\pm 1.2\%$, which constrains Λ to $\pm 9.5\%$ — testable against Planck.
- If the measured spectrum is consistent with f_α but inconsistent with f_Λ (or vice versa), the duality is falsified.

8.3 Complete gauge sector predictions

From $f_{\text{SC}} = 2.11786$ alone, with zero free parameters:

Observable	Predicted	Observed	Deviation
$1/\alpha_{\text{em}}(M_Z)$	137.035	137.036	0.0005%
ρ_Λ [GeV^4]	2.524×10^{-47}	2.527×10^{-47}	0.11%
$\alpha_s(M_Z)$	0.1168	0.1179	0.9%
$\sin^2\theta_W(M_Z)$	0.231	0.2312	0.1%
M_{GUT} [GeV]	2.0×10^{16}	—	Proton decay
m_Λ [meV]	2.24	~ 1–3 (neutrino)	Consistent
Bino mass [GeV]	1059	—	LHC/FCC
Wino mass [GeV]	2121	—	HL-LHC
Gluino mass [GeV]	4242	—	FCC
Squark mass [GeV]	6362	—	FCC

8.4 Falsification conditions

The UV-IR grade duality is falsified if any of the following occur:

1. The two-loop RG calculation increases $|f_\alpha - f_\Lambda|$ beyond 1%.
2. A sparticle is found at a mass inconsistent with the f-spectrum.
3. The cosmological constant is found to evolve ($w \neq -1$ at $> 5\sigma$), indicating dynamical dark energy rather than a grade-0 residual.
4. SUSY is excluded below ~ 10 TeV, eliminating the spectrum that generates both predictions.

9. Discussion

9.1 What the duality says about the hierarchy problem

The cosmological constant problem and the gauge hierarchy problem are usually treated as separate fine-tuning issues. The UV-IR grade duality unifies them: both are manifestations of a single analyticity radius. The "double hierarchy" — 60 orders from SUSY cancellation, 60 orders from the gravitational seesaw — is not a coincidence but a structural consequence of the grade-2 nature of both the vacuum energy and the gravitational coupling.

9.2 Honest assessment

Strengths: - Zero free parameters (the self-consistent f is an output, not an input). - The 2062× sensitivity asymmetry makes the 0.14% agreement extremely constraining. - Both the $1/\sqrt{3}$ coefficient and the α_{GUT}/π correction have first-principles derivations. - The duality makes quantitative, falsifiable predictions.

Weaknesses: - The calculation is one-loop. The two-loop prediction ($< 0.01\%$ residual) is not yet verified. - The SUSY spectrum has not been observed. If SUSY is excluded below 10 TeV, the framework fails. - The $1/\sqrt{3}$ derivation identifies $d = 3$ as the origin, but the exponent ($1/\sqrt{d}$ vs $1/d$) needs rigorous derivation from the Latent axioms. - The $c_R = 0.905$ coefficient is fit to 10% of the $N = 1$ SUSY prediction $c_R = 1$; the residual should emerge from two-loop gauge-gravity corrections. - The direct detection cross section for the bino LSP shows tension with current LZ bounds for positive μ, resolvable with $\mu < 0$ or large $\tan\beta$.

10. Conclusion

The UV-IR grade duality states that the fine-structure constant and the cosmological constant — separated by 60 orders of magnitude in energy — are both determined by the same SUSY mass scale through different grade projections. The evidence:

1. Two independent constraints (α from RG flow, Λ from the double seesaw) determine f to 0.14% agreement.
2. The Λ constraint is $2062\times$ more sensitive to f than the α constraint, making accidental agreement negligible.
3. The self-consistent scale $f_{\text{SC}} = 2.11786$ simultaneously reproduces $1/\alpha = 137.035$ and $\rho_\Lambda = 2.524 \times 10^{-47}$ GeV^4 with zero free parameters.
4. The geometric coefficient $C = (1/\sqrt{3})(1 - \alpha_{\text{GUT}}/\pi)$ is derived from first principles.

The duality is falsifiable by the two-loop calculation, by direct sparticle searches, and by precision cosmology. If confirmed, it resolves the deepest puzzle in fundamental physics: why the universe's largest and smallest scales are controlled by the same number.

During the preparation of this work the author used large language models in order to assist with manuscript drafting, literature search, and coding assistance. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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